Reading Assignment for
The Physics of Galaxy Clusters (Summer Term 2020)

Lecturer: Christoph Pfrommer

in preparation of lecture 2

Next online meeting Apr. 28, 2020, 12:00

Please read and work through the script, starting on page 10 and cover the following topics:

1.2.4 Sunyaev-Zeldovich Effect

1.2.5 Synthesis of Observational Windows

1.2.6 Relation to Average Universe

2.1 The Growth of Perturbations: Newtonian Equations and Density Perturbations

2.2.1 Power Spectra

I prepared the following questions that should help you to understand the topics. Please read a topic first, think about it and then work through my set of questions on this topic. Some questions are going beyond what you have read in the lecture notes. I do not expect you to answer these questions as well, but I would like you to start thinking about them and they will certainly be the starting point for our next zoom meeting. Ideally you can come up with many more questions yourself!

- **Sunyaev-Zeldovich Effect**
  - Why do galaxy clusters appear as holes in the cosmic microwave background sky at $\nu < \nu_0$ and as extended sources above?
  - Why does the intracluster medium (ICM) inside a galaxy cluster dominate the line-of-sight integral of the Compton-y parameter? To answer this, let's do an *order of magnitude problem:* put a galaxy cluster of mass $10^{15}\,\mathrm{M}_\odot$ and radius 3 Mpc at a distance of 1 Gpc:
    - compute the Compton-y parameter of the ICM,
    - the intergalactic medium from us to the cluster ($n_\mathrm{e} \sim 2 \times 10^{-7}\,\mathrm{cm}^{-3}$ and $T \sim 10^4$ K),
    - and our own galactic halo ($n_\mathrm{e} \sim 2 \times 10^{-4}\,\mathrm{cm}^{-3}$ and $T \sim 10^6$ K).

    Which contribution dominates and by how much?
  - Why is the Compton-y parameter independent of redshift? Is the observable (solid-angle) integrated Compton-y parameter independent of redshift?
- **Synthesis of Observational Windows**
  - Which observational method would you prefer to observe the inner parts of a cluster? which for the outer parts?
  - Which problem do you see arising for studying small clusters with galaxy observations? How reliable can you estimate the cluster mass here?
  - Which method is most powerful to do cluster cosmology? which criteria would you find most important for this?

- **Relation to Average Universe**
  - The critical density of the universe is the density that closes the universe, i.e., it would make the cosmic expansion asymptotically come to a halt in the absence of a cosmological constant or dark energy. Its value today is given by

$$\rho_{\mathrm{cr},0} = \frac{3H_0^2}{8\pi G} \approx 10^{-29}\ \mathrm{g\ cm}^{-3}, \tag{1}$$

    where $H_0 \approx 70$ km s^{-1} Mpc^{-1} is the Hubble constant. Please argue to order of magnitude (without invoking general relativity) why this form is obvious. Hint: if gravity dominates, the dynamical time of a system is $t_{\mathrm{dyn}} \sim 1/\sqrt{G\rho}$. Where does this come from?
  - Explain why clusters are rare objects.
  - In the notes, I state that we typically find $\bar{\rho}_{\mathrm{cl}} \sim 10^3 \bar{\rho}_{\mathrm{m},0}$. Which processes determine this relation?

- **The Growth of Perturbations**
  - Equations (2.5) and (2.6) are Friedmann's Equations. These are derived in general relativity but can also be motivated with Newtonian arguments as follows. Consider a spherical sub-volume of radius $r(t)$, within an (infinite) expanding, homogeneous mass density distribution $\rho(t)$ of a idealized pressureless fluid. The radius $r(t) \equiv a(t) \cdot R$ (where $R = \mathrm{const.}$) is defined to enclose constant mass as a function of time. At $t = t_0$, we have $r(t_0) = R$ and $\rho(t_0) = \rho_0$ and some expansion rate $\dot{a}(t_0)$. It can be shown that the 2nd equation is equivalent to the equation of motion for $a(t)$ for a test mass on the sphere's surface. How can you interpret the first of Friedmann's Equations? What is the physical meaning of all terms?
  - Equation (2.12) splits the velocity into the Hubble flow and a peculiar velocity. Explain in your own words the physical meaning of this separation. Why does this insight help in deriving the perturbation equations?
  - *Bonus:* if you have a lot of free time, you may want to derive equation (2.13) in the notes. To make things a little easier, you may want to follow my set of cosmology notes:
    `https://pages.aip.de/pfrommer/Lectures/cosmology.pdf`
    Equations (2.1) to (2.19) guide your through this (lengthy) derivation.
  - After this, you deserve a break. You may watch the following movies of the Millennium simulation, which visualize cosmic scales:
    `https://www.youtube.com/watch?v=Y9yQOb94yl0`
    and show a fly-through the highly structure large-scale structure of the universe:
    `https://www.youtube.com/watch?v=JNIXAKkuShQ`
  - Equation (2.13) is an ordinary differential equation for the density contrast δ. Which mathematical form has this equation? What is the physical meaning of the term with H?
  - I mentioned in my notes that the growing mode solutions of equation (2.22) are responsible for growing a cluster? which mode grows cosmic voids (huge volumes of nearly empty space)? Are decaying modes responsible for growing these? Justify your answer.

- In the derivation, the pressure term provides the restoring force. It communicates the pressure gradients via collisions and sound waves to the gas. However, dark matter does not interact via sound waves? How would you need to conceptually change the derivation to account for a collisionless dark matter component? How does dark matter influence structure formation?

- **Power Spectra**
  - Why does the Dirac delta distribution appear in the definition of the power spectrum in equation (2.26)? Why does the power spectrum only depend on the magnitude of the wave vector and not its direction?
  - How does the window function influence the density contrast? Draw some one-dimensional function that varies widely. How does this look after you applied a top-hat or a Gaussian window? Imagine you have field δ with two spatial scales (one with a large and one with a small wavelength). Then you apply the filter which has a scale in between. Which of the two scales survives (if any)? How does the power spectrum look if you draw it before and after filtering?