Reading Assignment for
The Physics of Galaxy Clusters (Summer Term 2020)

Lecturer: Christoph Pfrommer

in preparation of lecture 3

Next online meeting May 5, 2020, 12:00

Please read and work through the script, covering the following topics:

2.2.2 Hierarchical Structure Formation

2.2.3 Non-linear Evolution

2.3 Spherical Collapse

I prepared the following questions that should help you to understand the topics. Please read a topic first, think about it and then work through my set of questions on this topic. Some questions are going beyond what you have read in the lecture notes. I do not expect you to answer these questions as well, but I would like you to start thinking about them and they will certainly be the starting point for our next zoom meeting. Ideally you can come up with many more questions yourself!

- **Hierarchical Structure Formation**
  - The particular form of the linear matter power spectrum for a cold dark matter (CDM) cosmology is given by
    $$P(k) \propto \begin{cases} k & (k < k_0) \\ k^{-3} & (k \gg k_0). \end{cases} \tag{1}$$
    Here, $k_0 = 2\pi a_{\mathrm{eq}}/\lambda_0$ is the comoving wave number of the particle horizon at matter-radiation equality. Plot the variance of the matter density fluctuations as a function of wave number, $\sigma^2(k)$, and as a function of enclosed mass of a perturbation, $\sigma^2(M)$.
  - Initially, $\sigma^2(k) < \sigma_*^2 = 1$ on all scales and structure grows linearly. At which scales do the fluctuations become first non-linear ($\sigma^2(k) > 1$) and why? Plot $\sigma^2(k)$ as a function of wave number for two consecutive times (in the linear regime) and determine the critical wave numbers at which the fluctuation strength becomes non-linear, respectively? Does this scale become smaller or larger with time?
  - What does this imply for the collapse of the first objects? Does structure formation in CDM cosmologies proceed top-down (first on large scales, then on progressively smaller ones) or bottom-up with smaller structures to form first?
  - How would you have to change the initial linear power spectrum in order for structure formation to proceed differently? *Bonus:* What physics would you have to change in such a universe?
  - By looking at the potential fluctuations, argue why the Harrison-Zeldovich-Peebles spectrum is distinguished from any other power spectrum and why this is physically preferred. How would you have to modify the primordial power spectrum if you adopted a different spectral index for the Harrison-Zeldovich-Peebles spectrum to make it physical?
  - *Bonus:* How are the power spectrum of the cosmic microwave background and that of the density fluctuations related?

    * Describe how they differ mathematically. You do not need to write down equations, but should explain the different concepts. Hint: you will find the answer by considering over which spaces the two power spectra are defined.
    * How are those related physically? (You can only answer this if you have already taken the Cosmology class.)

- **Non-linear Evolution**
  - Why do you need numerical simulations to study the non-linear phase of structure formation?
  - *Bonus:* What does a "particle" in these numerical N-body simulations represent? What are the considerations when choosing the number of particles and how is it limited to the maximum and minimum numbers of particles?
  - Describe in your own words which algorithms are improving the scaling properties of numerical codes with the numbers of particles.
  - *Bonus:* Plot the probability distribution function of the density contrast at early and at late times after non-linear structure formation has already begun? What is the reason that the distribution becomes skewed at late times?
  - Watch a few movies of structure formation simulations, that you can find e.g., on the following web page:
    `https://www.tng-project.org/media/`
    Which properties that we talked about so far can you recognize in these simulations? Which things do you not understand or would like to know more about?

- **Spherical Collapse**
  - Summarize the assumptions of the spherical collapse model. What is the benefit of doing this calculation if you have to assume these numbers of simplifications?
  - Repeat the derivation of Eqns. (2.47) and (2.48) and plot the parametric solution of $R(\theta)$.
  - The solution is periodic beyond $\theta = 2\pi$. Is this completely unphysical? Why is the solution for $\theta > 2\pi$ not realized in nature?
  - Why does the sphere remain uniform as it collapses if the sphere has a uniform initial overdensity (δ_i) at some early time (t_i)?
  - Why do perturbations collapse earlier if they are initially more over overdense?
  - We find values for the density contrast at collapse ($t = t_\mathrm{c} = 2t_\mathrm{max}$) of

$$\delta_\mathrm{c} \equiv \delta_\mathrm{lin}(t_\mathrm{c}) = \frac{3}{20}(12\pi)^{2/3} \approx 1.686, \tag{2}$$

$$\delta_\mathrm{v} \equiv \delta_\mathrm{coll} = 18\pi^2 - 1 = 177\ . \tag{3}$$

    Explain the difference of these results that describe the same quantity at the same time.
  - We will later on use both results. Under which circumstances would you use the first and under which the second result?
  - Last week, I asked the question that, in the notes, I state that we typically find $\bar{\rho}_\mathrm{cl} \sim 10^3 \bar{\rho}_{\mathrm{m},0}$. Which processes determine this relation? What is the answer in the spherical collapse model?