Reading Assignment for
The Physics of Galaxy Clusters (Summer Term 2020)

Lecturer: Christoph Pfrommer

in preparation of lecture 4

Next online meeting May 12, 2020, 12:00

Please read and work through the script, covering the following topics:

2.4.1 The Press-Schechter Mass Function

2.4.2 Halo Formation as a Random Walk

2.5 Halo Density Profiles

Note that we do not cover section *2.4.3 Extended Press-Schechter Theory* in the lectures and you can skip this section.

I prepared the following questions that should help you to understand the topics. Please read a topic first, think about it and then work through my set of questions on this topic. Some questions are going beyond what you have read in the lecture notes. I do not expect you to answer these questions as well, but I would like you to start thinking about them and they will certainly be the starting point for our next zoom meeting. Ideally you can come up with many more questions yourself!

- **The Derivation of the Halo Mass Function**
  - How do the characteristic length scale $R(M)$ and its associated mass M differ from the characteristic length scale $R(M_*) \equiv R_*$ with its associated "non-linear mass" M_*?
  - *Bonus:* What is the physical basis for assuming that the density field obeys a Gaussian random process?
  - Justify the hypothesis of Press & Schechter that the probability of finding the filtered density contrast at or above the linear density contrast for spherical collapse, $\bar{\delta} > \delta_{\rm c}$, is equal to the fraction of the cosmic volume filled with haloes of mass M.
  - Which substitution do you need to adopt in order to perform the integral in Eqn. (2.74)?
  - Show explicitly, that Eqn. (2.78) is true.
  - Sketch and explain the main steps necessary to obtain the correct normalisation of the Press-Schechter mass function by considering halo formation as a random walk.
  - Explain the physical reason for the missing factor of two and why this has been missed in the first derivation.
- **Simplified Form of the Mass Function**
  Masses and length scales are related by the mean background density
  $$M = \frac{4\pi}{3}\bar{\rho}R^3. \tag{1}$$
  The mass M_* corresponding to the scale R_* on which the variance becomes unity
  $$\sigma_*^2 = 4\pi \int_0^{k_*} \frac{k^2 \mathrm{d}k}{(2\pi)^3}\, P_\delta(k) = 1 \tag{2}$$
  is called the *nonlinear mass*.

1. Show that, for a power-law power spectrum with index n, $P_\delta(k) = Ak^n$, the variance can be written in the form
$$\sigma^2 = \left(\frac{M_*}{M}\right)^{1+n/3} . \tag{3}$$
2. Discuss the special cases $n = 1$ and $n = -3$ and compare this to your qualitative drawing of σ^2 as a function of M by interpolating the asymptotic cases of the linear power spectrum.
3. Use this result to bring the Press-Schechter mass function into the form
$$f(M, a)\mathrm{d}M \equiv \frac{\partial n_{\mathrm{PS}}(M, a)}{\partial M}\mathrm{d}M = \sqrt{\frac{2}{\pi}}\frac{\alpha\bar{\rho}\delta_{\mathrm{c}}}{M_* D_+} m^{\alpha-2} \exp\left(-\frac{\delta_{\mathrm{c}}^2}{2D_+^2}\, m^{2\alpha}\right) \mathrm{d}m, \tag{4}$$
where $m = M/M_*$ and $\alpha = 1/2 + n/6$.

- **The Nonlinear Mass**

1. Using *Planck* data, the latest cosmic microwave background measurements yield $\sigma_8 \approx 0.8$ (i.e., σ on the scale of $8\,h^{-1}$ Mpc). Assume that $n = -1$ and estimate the nonlinear mass today.
2. How does the nonlinear mass evolve with time?
3. Calculate the present-day abundance of objects with mass M_* according to the Press-Schechter mass function (use $\Omega_{\mathrm{m},0} = 0.3$). Assuming, for simplicity, that these halos are randomly distributed through space, estimate the mean separation between these objects. Compare your estimate with the actual distance of the Milky Way to the Virgo cluster.

- **Halo Density Profiles**

  - **General remarks.** Scrutinize the statement that *a self-gravitating system of particles does not have an equilibrium state.* Assume that you have a globular cluster of 10^6 stars and size 10 pc. Explain what happens to the system when you eject one star after each other. What is the theoretical end state?
  - **Isothermal sphere.** Show explicitly that a power-law ansatz in $\rho(r)$ yields Eqn. (2.97).
  - Show explicitly that you can recover the analytical solution for the singular isothermal sphere (Eqn. 2.97) from the approximate form of the numerical solution of Eqn. (2.102) for $\tilde{r} \gg 3$. What is the difference of both solutions?
  - What is the problem of the singular isothermal sphere at small and larger radii? Demonstrate this explicitly by integrating the radial mass density profile over volume.
  - **Navarro-Frenk-White profile.** What is the logarithmic slope, $\mathrm{d}\log\rho/\mathrm{d}\log r$, of the NFW profile at the center, the scale radius r_{s} and at large radii?
  - Derive the mass profile $M(<r)$ of the NFW profile by using the identity $x/(1+x)^2 \equiv (1+x)^{-1} - (1+x)^{-2}$.
  - Compare the different definitions for halo mass, M_{200}, $M_{200\mathrm{m}}$, and M_{500} and order them by increasing size.
  - Sketch qualitatively the scaled NFW density profiles $\log(\rho/\rho_{200})$ for two different halos which only differ by their concentration parameters. What do you observe?