Reading Assignment for
The Physics of Galaxy Clusters (Summer Term 2020)

Lecturer: Christoph Pfrommer

in preparation of lecture 11

Next online meeting June 30, 2020, 12:00

Please read and work through the script, covering the following topics:

Start with *3.3 Non-thermal Processes* and then work through

3.3.1 Magnetic Fields

I prepared the following questions that should help you to understand the topics. Please read a topic first, think about it and then work through my set of questions on this topic. Some questions are going beyond what you have read in the lecture notes (indicated by *Bonus* questions). I do not expect you to answer these questions as well, but I would like you to start thinking about them and they will certainly be the starting point for our next zoom meeting. Ideally you can come up with many more questions yourself. **If you have problems with a derivation or if something is unclear, please email me those points well before the lectures**!

- **Non-thermal processes.** The energy loss rate of a relativistic electron of energy $E_\mathrm{e} = \gamma m_\mathrm{e} c^2$ is given by

$$\dot{E}_\mathrm{e} = \frac{\sigma_\mathrm{T} c}{6\pi} (B_\mathrm{cmb}^2 + B^2)\gamma^2, \tag{1}$$

  where σ_T is the Thomson cross section, m_e is the electron rest mass, c is the light speed, γ is the Lorentz factor, B is the magnetic field strength and $B_\mathrm{cmb} \simeq 3.2\mu\mathrm{G}$ is the equivalent field of the cosmic microwave background (cmb) energy density today.

  - Review the properties of radio halos and relics.
  - Derive the cooling time $t_\mathrm{cool} = E_\mathrm{e}/\dot{E}_\mathrm{e}$ of a relativistic electron and evaluate it for $B = 1\ \mu\mathrm{G}$ and $\gamma = 10^4$.
  - Rewrite the cooling time as a function of B and ν_syn by using the formula of the synchrotron frequency (Eqn. 3.244). Show that the cooling time has a maximum and determine the magnetic field strength of this maximum. *Bonus:* how do electrons primarily cool above and below this critical magnetic field strength?
  - Determine $\max(t_\mathrm{cool})$ at $\nu_\mathrm{syn} = 1.4$ GHz.
  - Assume that the relativistic electrons are accelerated at a strong cluster merger shock and are advected with the post-shock gas. Assume that the incoming gas had a pre-shock velocity of $v_\mathrm{pre} = 1200$ km/s in the shock frame, calculate the maximum cooling length $L_\mathrm{cool} = v_\mathrm{post} t_\mathrm{cool}$ at 1.4 GHz for a shock Mach number of $\mathcal{M} = 3$. What is L_cool for 5 μG and how does this visually compare to the radial extends of radio relics?
  - Compare this cooling length of electrons to the spatial extend of radio halos. What does this imply for the acceleration process of the radio-emitting electrons?
  - Review the process of Faraday rotation measurements. If you decrease the magnetic coherence length while leaving the gas density and magnetic field strength invariant, what happens to the value of RM?
  - Explain the phenomenon of the $n\pi$ ambiguity for the observable polarization angle. What could you do to circumvent it?

- **Magnetic fields**
  - Derive the Biermann battery equation. Why can we neglect the momentum equation for protons and the time derivative on the left-hand side?
  - Explain the physical meaning of the terms in the Biermann battery equation.
  - Calculate to order of magnitude the expected magnetic field strength that is generated by the Biermann battery in a collapsing proto-galaxy.
  - Sketch an astro-physical system that allows for the Biermann battery.
  - Starting with the equations of magneto-hydrodynamics (MHD), derive the flux-freezing property of magnetic fields.
  - If a gas sphere (a filament) that is threaded by magnetic field collapses, work out the scaling $B \propto \rho^{\alpha_B}$ which depends on the magnetic topology.
  - Going back to our derivation of the dispersion relation for sound waves by perturbing the mass, momentum and entropy equation of a hydrodynamic fluid without conduction and viscosity. How many equations do you have and how many eigenvalues does the linearized system of equations allow for? Identify them.
  - Add magnetic fields to the system in the MHD approximation. How many equations and eigenvalues do you have now? Identify them.
  - Derive the scaling of the eddies in Alfvénic turbulence. What does this imply for the resulting turbulence and how does this differ from Kolmogorov turbulence?
- **Magnetic Force and Stress**
  1. Show that the Lorentz force can be written in the following ways

$$\vec{F}_{\mathrm{L}} = \frac{1}{c}\vec{j} \times \vec{B} = \frac{1}{4\pi}\left(\boldsymbol{\nabla} \times \vec{B}\right) \times \vec{B} = -\frac{1}{8\pi}\boldsymbol{\nabla} B^2 + \frac{1}{4\pi}\left(\vec{B} \cdot \boldsymbol{\nabla}\right)\vec{B} = -\boldsymbol{\nabla} \cdot \bar{\boldsymbol{M}}, \quad (2)$$

     where

$$M_{ij} = \frac{1}{8\pi} B^2 \delta_{ij} - \frac{1}{4\pi} B_i B_j \quad (3)$$

     is the magnetic stress tensor. It plays a role analogous to the fluid pressure in ordinary fluid mechanics (explaining the minus sign introduced in its definition). The first term on the right of (2) is the gradient of the *magnetic pressure* $B^2/8\pi$ and the second term is often called the magnetic *curvature or tension force* (which can also be present if the field lines are straight).
  2. Show that the surface force (per unit area) exerted *by* a bounded volume V *on* its surroundings is given by

$$\vec{F}_{\mathrm{S}} = \vec{n} \cdot \bar{\boldsymbol{M}} = \frac{1}{8\pi} B^2 \vec{n} - \frac{1}{4\pi} \vec{B} B_n, \quad (4)$$

     where $B_n = \vec{B} \cdot \vec{n}$ is the component of $\vec{B}$ along the outward normal $\vec{n}$ to the surface of the volume.
  3. To understand the meaning of magnetic stress, take a uniform magnetic field along the z-direction and compute the surface forces $\vec{F}_{\mathrm{S}}$ exerted by a rectangular volume that is aligned with the magnetic field (while there are 6 different surface elements, symmetry limits the surface forces to only two different types). Which magnetic force terms (pressure or tension) are contributing to these surface forces? Explain the action of these forces and why magnetic fields can be thought of as elastic wires.