Reading Assignment for
The Physics of Galaxy Clusters (Winter Term 2021/22)

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in preparation of lecture 3

Next lecture Nov 11, 2021, 16:15

Please read and work through the script, covering the following topics:

2.2.2 Hierarchical Structure Formation

2.2.3 Non-linear Evolution

2.3 Spherical Collapse

I prepared the following questions that should help you to understand the topics. Please read a topic first, think about it and then work through my set of questions on this topic. Some questions are going beyond what you have read in the lecture notes. I do not expect you to answer these questions as well, but I would like you to start thinking about them and they will certainly be the starting point for our next lecture. Ideally you can come up with many more questions yourself!

- **Hierarchical Structure Formation**
  - The particular form of the linear matter power spectrum for a cold dark matter (CDM) cosmology is given by
  $$P(k) \propto \begin{cases} k & (k < k_0) \\ k^{-3} & (k \gg k_0). \end{cases} \tag{1}$$
  Here, $k_0 = 2\pi a_{\rm eq}/\lambda_0$ is the comoving wave number of the particle horizon at matter-radiation equality. Plot the variance of the matter density fluctuations as a function of wave number, $\sigma^2(k)$, and as a function of enclosed mass of a perturbation, $\sigma^2(M)$.
  - Initially, $\sigma^2(k) < \sigma_*^2 = 1$ on all scales and structure grows linearly. At which scales do the fluctuations become first non-linear ($\sigma^2(k) > 1$) and why? Plot $\sigma^2(k)$ as a function of wave number for two consecutive times (in the linear regime) and determine the critical wave numbers at which the fluctuation strength becomes non-linear, respectively? Does this scale become smaller or larger with time?
  - What does this imply for the collapse of the first objects? Does structure formation in CDM cosmologies proceed top-down (first on large scales, then on progressively smaller ones) or bottom-up with smaller structures to form first?
  - How would you have to change the initial linear power spectrum in order for structure formation to proceed differently? *Bonus:* What physics would you have to change in such a universe?
  - By looking at the potential fluctuations, argue why the Harrison-Zeldovich-Peebles spectrum is distinguished from any other power spectrum and why this is physically preferred. How would you have to modify the primordial power spectrum if you adopted a different spectral index for the Harrison-Zeldovich-Peebles spectrum to make it physical?
  - *Bonus:* How are the power spectrum of the cosmic microwave background and that of the density fluctuations related?

* Describe how they differ mathematically. You do not need to write down equations, but should explain the different concepts. Hint: you will find the answer by considering over which spaces the two power spectra are defined.
* How are those related physically? (You can only answer this if you have already taken the Cosmology class.)

- **Non-linear Evolution**
  - Why do you need numerical simulations to study the non-linear phase of structure formation?
  - *Bonus:* What does a "particle" in these numerical N-body simulations represent? What are the considerations when choosing the number of particles and how is it limited to the maximum and minimum numbers of particles?
  - Describe in your own words which algorithms are improving the scaling properties of numerical codes with the numbers of particles.
  - *Bonus:* Plot the probability distribution function of the density contrast at early and at late times after non-linear structure formation has already begun? What is the reason that the distribution becomes skewed at late times?
  - Watch a few movies of structure formation simulations, that you can find e.g., on the following web page:
    `https://www.tng-project.org/media/`
    Which properties that we talked about so far can you recognize in these simulations? Which things do you not understand or would like to know more about?

- **Spherical Collapse**
  - Summarize the assumptions of the spherical collapse model. What is the benefit of doing this calculation if you have to assume these numbers of simplifications?
  - Repeat the derivation of Eqns. (2.47) and (2.48) and plot the parametric solution of $R(\theta)$.
  - The solution is periodic beyond $\theta = 2\pi$. Is this completely unphysical? Why is the solution for $\theta > 2\pi$ not realized in nature?
  - Why does the sphere remain uniform as it collapses if the sphere has a uniform initial overdensity (δ_i) at some early time (t_i)?
  - Why do perturbations collapse earlier if they are initially more over overdense?
  - We find values for the density contrast at collapse ($t = t_\mathrm{c} = 2t_\mathrm{max}$) of

$$\delta_\mathrm{c} \equiv \delta_\mathrm{lin}(t_\mathrm{c}) = \frac{3}{20}(12\pi)^{2/3} \approx 1.686, \tag{2}$$

$$\delta_\mathrm{v} \equiv \delta_\mathrm{coll} = 18\pi^2 - 1 = 177\ . \tag{3}$$

    Explain the difference of these results that describe the same quantity at the same time.
  - We will later on use both results. Under which circumstances would you use the first and under which the second result?
  - Last week, I asked the question that, in the notes, I state that we typically find $\bar{\rho}_\mathrm{cl} \sim 10^3 \bar{\rho}_\mathrm{m,0}$. Which processes determine this relation? What is the answer in the spherical collapse model?