Reading Assignment for
The Physics of Galaxy Clusters (Winter Term 2021/22)

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in preparation of lecture 8

Next lecture Dec 9, 2021, 16:15

Please read and work through the script, covering the following topic:

3.1.7 Entropy Generation by Accretion

3.1.8 Cluster Scaling Relations

I prepared the following questions that should help you to understand the topics. Please read a topic first, think about it and then work through my set of questions on this topic. Some questions are going beyond what you have read in the lecture notes (indicated by *Bonus* questions). I do not expect you to answer these questions as well, but I would like you to start thinking about them and they will certainly be the starting point for our next lecture. Ideally you can come up with many more questions yourself!

- **Turbulent Scaling Laws.** This completes the *turbulence* topic we started two weeks ago.

  Consider the Navier-Stokes equation in the following compact form

$$\frac{\mathrm{d}\boldsymbol{v}}{\mathrm{d}t} = -\frac{\boldsymbol{\nabla} P}{\rho} + \nu \boldsymbol{\nabla}^2 \boldsymbol{v}, \qquad (1)$$

  where $\mathrm{d}/\mathrm{d}t = \partial/\partial t + \boldsymbol{v} \cdot \boldsymbol{\nabla}$ is the Lagrangian time derivative and $\nu = \eta/\rho$ is the kinematic viscosity.

  1. By introducing characteristic length L_0, velocity V_0, and density ρ_0 scales, rewrite the Navier-Stokes equation into dimensionless form. *Hint:* you also have to introduce a dimensionless time and a dimensionless Nabla operator using these three characteristic scales.

     You will find, that the dimensionless equation involves one number, the Reynolds number $\mathrm{Re} \equiv L_0 V_0/\nu$, that characterizes the flow and determines the structure of the solutions to this equation. What is the meaning of this number?

  2. In the lectures, we introduced an energy flow rate per unit mass, $\dot{\epsilon} = v_\lambda^3/\lambda$, that is valid on all scales λ and constant (because energy does not accumulate at any intermediate scale). Hence, $\dot{\epsilon} = V^3/L$ has also the meaning of an energy injection rate into the turbulent cascade at the outer scale L. Defining the *Kolmogorov length* ℓ, show that this defines corresponding velocity and time scales,

$$\ell \equiv \left(\frac{\nu^3}{\dot{\epsilon}}\right)^{1/4}, \quad v_\ell = (\dot{\epsilon}\nu)^{1/4}, \quad \tau_\ell = \left(\frac{\nu}{\dot{\epsilon}}\right)^{1/2}. \qquad (2)$$

     What value has the Reynolds number Re at the *Kolmogorov length* ℓ and why?

     Work out the scaling of the following ratios, L_0/ℓ, V_0/v_ℓ, τ/τ_ℓ, and ϵ_0/ϵ_ℓ with the Reynolds number. We speak about a turbulent flow if the Reynolds number at the outer scale is $\mathrm{Re}(L) \gtrsim 10^3$. Interpret your ratios in the light of this requirement.

3. Do we have turbulent flows in the hot ($k_\mathrm{B}T = 10$ keV) intracluster medium ($n = 10^{-3}\,\mathrm{cm}^{-3}$), if the outer scale is $L_0 = 300$ kpc and we consider it to be a purely hydrodynamical system? Argue qualitatively, what you would expect to change, if we did add magnetic fields to the system.

The following two bullets are the questions for your reading assigment:

- **Entropy Generation by Accretion**
  - Motivate the connection between the phase space density and entropy. When is a gas degenerate and why is the ICM very far from this?
  - To describe accretion onto a galaxy cluster, we put ourselves into the post-shock rest system – why?
  - Derive each of the governing equations (3.140) to (3.143).
  - Derive the entropy generated at the cluster accretion shock in dimensional and dimensionless form, Eqns. (3.147) and (3.152).
  - What happens to the cluster entropy profile for lumpy accretion?
- **Ideal Cluster Scaling Relations**
  - Derive the ideal cluster scaling relations $k_\mathrm{B}T$-M_Δ, M_gas-M_Δ, $M_\star$-M_Δ, Y-M_Δ, and L_X-M_Δ, where Δ denotes the overdensity with respect to the critical density of the universe (typically taken to be $\Delta = 200$).
  - Using our order of magnitude numbers for a $10^{15}\,\mathrm{M}_\odot$ cluster of Section 1.2, derive $k_\mathrm{B}T$, M_gas, $M_\star$, and Y for a $10^{14}\,\mathrm{M}_\odot$ cluster and a group of $10^{13}\,\mathrm{M}_\odot$, each at a redshift of $z = 0$ and $z = 1$. To this end assume $\Omega_\mathrm{m0} = 0.3$, $\Omega_\Lambda = 0.7$, which implies that the universe has zero curvature $\Omega_\mathrm{K} = 0$ and we can neglect the radiation term at late times.
  - Argue for each wave band, why the halo mass scale of $\sim 10^{14}\,\mathrm{M}_\odot$ is a good choice for calling an object a galaxy cluster.
  - Sketch observed and theoretically expected scaling relations of clusters. Note that they agree at the high-mass end of around $10^{15}\,\mathrm{M}_\odot$.
- **Real Cluster Scaling Relations**
  - Assume an isothermal gas in a cluster with a beta profile of the gas density:

    $$\rho = \rho_0 \left[\frac{1}{1 + (r/r_\mathrm{c})^2)}\right]^{3/2\beta},$$

    where ρ_0 is the central density, r_c is the core radius, and β is the scaling parameter, calculate the X-ray luminosity for $\beta = 2/3$ and 1 (typical values in the X-ray literature). Compare your result to our approximation of Eqn. (3.172) and discuss it.
  - Explain how pre-heating the gas before it gets accreted onto the cluster can solve the problem of the X-ray luminosity scaling relation.
  - Explain how AGN feedback and radiative cooling can theoretically also solve the problem of the X-ray luminosity scaling relation.