Reading Assignment for
The Physics of Galaxy Clusters (Winter Term 2021/22)

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in preparation of lecture 11

Next lecture Jan 20, 2021, 16:15

Please read and work through the script, covering the following topics:

3.3.1 Non-thermal Processes and

3.3.2 Magnetic Fields

I prepared the following questions that should help you to understand the topics. Please read a topic first, think about it and then work through my set of questions on this topic. Some questions are going beyond what you have read in the lecture notes (indicated by *Bonus* questions). I do not expect you to answer these questions as well, but I would like you to start thinking about them and they will certainly be the starting point for our next zoom meeting. Ideally you can come up with many more questions yourself. **If you have problems with a derivation or if something is unclear, please email me those points well before the lectures**!

- **Non-thermal processes.**
  - Explain the physics of synchrotron emission.
  - Review the observational properties of radio halos and relics.
  - Review the process of Faraday rotation measurements. If you decrease the magnetic coherence length while leaving the gas density and magnetic field strength invariant, what happens to the value of RM?
  - Explain the phenomenon of the $n\pi$ ambiguity for the observable polarization angle. What could you do to circumvent it?

- **Magnetic Forces and Stresses**

  1. Show that the Lorentz force density can be written in the following ways

$$\boldsymbol{f}_\mathrm{L} = \frac{1}{c}\boldsymbol{j}\times\boldsymbol{B} = \frac{1}{4\pi}\left(\boldsymbol{\nabla}\times\boldsymbol{B}\right)\times\boldsymbol{B} = \frac{1}{4\pi}\left(\boldsymbol{B}\cdot\boldsymbol{\nabla}\right)\boldsymbol{B} - \frac{1}{8\pi}\boldsymbol{\nabla}B^2 = -\boldsymbol{\nabla}\cdot\bar{\boldsymbol{M}}, \tag{1}$$

  where

$$M_{ij} = -\frac{1}{4\pi}B_iB_j + \frac{1}{8\pi}B^2\delta_{ij} \tag{2}$$

  is the magnetic stress tensor. It plays a role analogous to the fluid pressure in ordinary fluid mechanics (explaining the minus sign introduced in its definition).

  2. Adopt $\boldsymbol{B} = B\boldsymbol{b}$ and show that the Lorentz force density can be written as

$$\boldsymbol{f}_\mathrm{L} = \frac{B^2}{4\pi}\left(\boldsymbol{b}\cdot\boldsymbol{\nabla}\right)\boldsymbol{b} - \frac{1}{8\pi}\boldsymbol{\nabla}_\perp B^2. \tag{3}$$

  Demonstrate that the first term of Eq. (3) is the magnetic *curvature or tension force* and the second term represents an anisotropic *magnetic pressure.* In which direction relative to the magnetc field do they act?

3. Show that the surface force (per unit area) exerted *by* a bounded volume V *on* its surroundings is given by

$$\boldsymbol{f}_{\mathrm{S}} = \boldsymbol{n} \cdot \bar{\boldsymbol{M}} = \frac{1}{8\pi} B^2 \boldsymbol{n} - \frac{1}{4\pi} \boldsymbol{B} B_n, \tag{4}$$

where $B_n = \boldsymbol{B} \cdot \boldsymbol{n}$ is the component of $\boldsymbol{B}$ along the outward normal $\boldsymbol{n}$ to the surface of the volume.

4. To understand the meaning of magnetic stress, take a uniform magnetic field along the z-direction and compute the surface forces $\boldsymbol{F}_{\mathrm{S}}$ exerted by a rectangular volume that is aligned with the magnetic field (while there are 6 different surface elements, symmetry limits the surface forces to only two different types). Which magnetic force terms (pressure or tension) are contributing to these surface forces? Explain the action of these forces and why magnetic fields can be thought of as elastic wires. Which of these surface forces are not acting in reality and why?

- **Magnetic fields**
  - Derive the Biermann battery equation. Why can we neglect the momentum equation for protons and the time derivative on the left-hand side?
  - Explain the physical meaning of the terms in the Biermann battery equation.
  - Calculate to order of magnitude the expected magnetic field strength that is generated by the Biermann battery in a collapsing proto-galaxy.
  - Sketch an astro-physical system that allows for the Biermann battery.
  - Derive the magnetic induction equation.
  - Starting with the equations of magneto-hydrodynamics (MHD), derive the flux-freezing property of magnetic fields.
  - If a gas sphere (a filament) that is threaded by magnetic field collapses, work out the scaling $B \propto \rho^{\alpha_B}$ which depends on the magnetic topology.
  - Going back to our derivation of the dispersion relation for sound waves by perturbing the mass, momentum and entropy equation of a hydrodynamic fluid without conduction and viscosity. How many equations do you have and how many eigenvalues does the linearized system of equations allow for? Identify them.
  - Add magnetic fields to the system in the MHD approximation. How many equations and eigenvalues do you have now? Identify them.
  - Derive the scaling of the eddies in Alfvénic turbulence. What does this imply for the resulting turbulence and how does this differ from Kolmogorov turbulence?