Reading Assignment for
The Physics of Galaxy Clusters (Winter Term 2021/22)

Lecturer: Christoph Pfrommer

in preparation of lecture 12

Next lecture Jan 27, 2021, 16:15

Please read and work through the script, covering the following topics:

3.3.2 Cosmic Rays

I prepared the following questions that should help you to understand the topics. Please read a topic first, think about it and then work through my set of questions on this topic. Some questions are going beyond what you have read in the lecture notes (indicated by *Bonus* questions). I do not expect you to answer these questions as well, but I would like you to start thinking about them and they will certainly be the starting point for our next zoom meeting. Ideally you can come up with many more questions yourself. **If you have problems with a derivation or if something is unclear, please email me those points well before the lectures**!

- **Cosmic Rays.**
  - Please work through all the derivations in this chapter.
  - Describe how cosmic rays (CRs) interact with Alfvén waves. Please explain the physics of the wave-particle interactions in configuration and in Fourier space.
  - CRs are virtually colisionless (they only colide on average once per Hubble time with the ambient gas protons in clusters). How can they excert a pressure if they do not collide with the gas?
  - Which processes can accelerate and deaccelerate CRs?
  - Which hadronic and leptonic non-thermal emission processes do you know? What is the underlying physics of these emission processes?
  - Why is Fermi-II acceleration with Alfvénic turbulence inefficient? Please explain the reason for the inefficient acceleration in configuration and in Fourier space.
  - Explain how CRs can be accelerated via the Fermi-II process? What is the physical reason behind the more efficient interactions with fast magnetosonic modes in comparison to Alfvén modes.
  - Compare the pros and cons of Fermi-I vs. Fermi-II acceleration.
- **Adiabatic Cosmic Rays**

  Introducing the dimensionless momentum $p = P_\mathrm{p}/(m_\mathrm{p}\, c)$, we assume that the differential cosmic ray (CR) particle momentum spectrum per volume element can be approximated by a single momentum power law above the minimum momentum q:

  $$f(p) = \frac{\mathrm{d}^2 N}{\mathrm{d}p\,\mathrm{d}V} = C\, p^{-\alpha}\, \theta(p-q), \tag{1}$$

  where $\theta(x)$ denotes the Heaviside step function. The CR pressure is then given by

  $$P_\mathrm{CR} = \frac{m_\mathrm{p}c^2}{3} \int_0^\infty \mathrm{d}p\, f(p)\, \beta\, p = \frac{C\, m_\mathrm{p}c^2}{6}\, \mathcal{B}_{\frac{1}{1+q^2}} \left(\frac{\alpha-2}{2}, \frac{3-\alpha}{2}\right), \tag{2}$$

  where $\beta = v/c = p/\sqrt{1+p^2}$ is the dimensionless velocity of the CR particle and $\mathcal{B}_x(a,b)$ denotes the incomplete beta function, and $\alpha > 2$ is assumed.

1. Using Liouville's theorem, work out how the low-momentum cutoff q and CR normalization C changes upon an adiabatic density change from ρ_0 to ρ.
2. Calculate the CR adiabatic index $\gamma_{\rm CR} = {\rm d}\ln P_{\rm CR}/{\rm d}\ln\rho$ and take the non-relativistic limit ($q \ll 1$ and $\alpha > 3$) and the ultra-relativistic limit ($q \to \infty$) of $\gamma_{\rm CR}$.
3. Imagine that CRs are accelerated at a strong cosmological structure formation shock with a relative CR pressure of $X_{\rm CR} = P_{\rm CR}/P_{\rm th} = 0.1$. Calculate $X_{\rm CR}$ in the ultra-relativistic limit after the composite of CRs and thermal gas has experienced adiabatic density increase by a factor of 10^3 from the warm-hot intergalactic medium to the cluster center.

- **Diffusive Shock Acceleration**

  Restricting to one spatial dimension, the steady-state CR transport equation for the isotropic CR distribution function $f(x,p)$ reads in the limit of negligible Fermi-II acceleration and radiative losses

$$v(x)\frac{\partial f}{\partial x} - \frac{1}{3}\frac{\partial v}{\partial x}p\frac{\partial f}{\partial p} = \frac{\partial}{\partial x}\left[D(x,p)\frac{\partial f}{\partial x}\right], \qquad (3)$$

  where D is the CR diffusion coefficient and $v(x)$ is the mean gas velocity. We assume a sharp shock transition of the velocity field (as seen in the shock frame),

$$v(x) = v_1 + (v_2 - v_1)\theta(x), \qquad (4)$$

  where the subscripts 1 and 2 denote the gas velocity in the up- and downstream region of the shock. Assume a pre-existing relativistic population in the upstream, f_1, and solve the CR transport equation for $f_2(p)$ in the downstream region of the shock.