

# Reading Assignment for The Physics of Galaxy Clusters (Winter Term 2022/23)

Lecturer: Christoph Pfrommer

in preparation of lecture 3

Next lecture Nov 2, 2022, 14:00 (**note that we will start at 2pm sharp!**)

Please read and work through the script, covering the following topics:

*2.2.2 Hierarchical Structure Formation*

*2.2.3 Non-linear Evolution*

*2.3 Spherical Collapse*

I prepared the following questions that should help you to understand the topics. Please read a topic first, think about it and then work through my set of questions on this topic. Some questions are going beyond what you have read in the lecture notes. I do not expect you to answer these questions as well, but I would like you to start thinking about them and they will certainly be the starting point for our next lecture. Ideally you can come up with many more questions yourself!

## • Hierarchical Structure Formation

- The particular form of the linear matter power spectrum for a cold dark matter (CDM) cosmology is given by

$$P(k) \propto \begin{cases} k & (k < k_0) \\ k^{-3} & (k \gg k_0). \end{cases} \quad (1)$$

Here,  $k_0 = 2\pi a_{\text{eq}}/\lambda_0$  is the comoving wave number of the particle horizon at matter-radiation equality. Plot the variance of the matter density fluctuations as a function of wave number,  $\sigma^2(k)$ , and as a function of enclosed mass of a perturbation,  $\sigma^2(M)$ .

- Initially,  $\sigma^2(k) < \sigma_*^2 = 1$  on all scales and structure grows linearly. At which scales do the fluctuations become first non-linear ( $\sigma^2(k) > 1$ ) and why? Plot  $\sigma^2(k)$  as a function of wave number for two consecutive times (in the linear regime) and determine the critical wave numbers at which the fluctuation strength becomes non-linear, respectively? Does this scale become smaller or larger with time?
- What does this imply for the collapse of the first objects? Does structure formation in CDM cosmologies proceed top-down (first on large scales, then on progressively smaller ones) or bottom-up with smaller structures to form first?
- How would you have to change the initial linear power spectrum in order for structure formation to proceed differently? *Bonus:* What physics would you have to change in such a universe?
- By looking at the potential fluctuations, argue why the Harrison-Zeldovich-Peebles spectrum is distinguished from any other power spectrum and why this is physically preferred. How would you have to modify the primordial power spectrum if you adopted a different spectral index for the Harrison-Zeldovich-Peebles spectrum to make it physical?
- *Bonus:* How are the power spectrum of the cosmic microwave background and that of the density fluctuations related?

- \* Describe how they differ mathematically. You do not need to write down equations, but should explain the different concepts. Hint: you will find the answer by considering over which spaces the two power spectra are defined.
- \* How are those related physically? (You can only answer this if you have already taken the Cosmology class.)

• **Non-linear Evolution**

- Why do you need numerical simulations to study the non-linear phase of structure formation?
- *Bonus:* What does a “particle” in these numerical N-body simulations represent? What are the considerations when choosing the number of particles and how is it limited to the maximum and minimum numbers of particles?
- Describe in your own words which algorithms are improving the scaling properties of numerical codes with the numbers of particles.
- *Bonus:* Plot the probability distribution function of the density contrast at early and at late times after non-linear structure formation has already begun? What is the reason that the distribution becomes skewed at late times?
- Watch a few movies of structure formation simulations, that you can find e.g., on the following web page:  
<https://www.tng-project.org/media/>  
 Which properties that we talked about so far can you recognize in these simulations? Which things do you not understand or would like to know more about?

• **Spherical Collapse**

- Summarize the assumptions of the spherical collapse model. What is the benefit of doing this calculation if you have to assume these numbers of simplifications?
- Repeat the derivation of Eqns. (2.47) and (2.48) and plot the parametric solution of  $R(\theta)$ .
- The solution is periodic beyond  $\theta = 2\pi$ . Is this completely unphysical? Why is the solution for  $\theta > 2\pi$  not realized in nature?
- Why does the sphere remain uniform as it collapses if the sphere has a uniform initial overdensity ( $\delta_i$ ) at some early time ( $t_i$ )?
- Why do perturbations collapse earlier if they are initially more over overdense?
- We find values for the density contrast at collapse ( $t = t_c = 2t_{\max}$ ) of

$$\delta_c \equiv \delta_{\text{lin}}(t_c) = \frac{3}{20}(12\pi)^{2/3} \approx 1.686, \tag{2}$$

$$\delta_v \equiv \delta_{\text{coll}} = 18\pi^2 - 1 = 177. \tag{3}$$

Explain the difference of these results that describe the same quantity at the same time.

- We will later on use both results. Under which circumstances would you use the first and under which the second result?
- Last week, I asked the question that, in the notes, I state that we typically find  $\bar{\rho}_{\text{cl}} \sim 10^3 \bar{\rho}_{\text{m},0}$ . Which processes determine this relation? What is the answer in the spherical collapse model?