Reading Assignment for
The Physics of Galaxy Clusters (Winter Term 2022/23)

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in preparation of lecture 12

Next lecture Jan 18, 2023, 14:15

Please read and work through the script, covering the following topics:

3.3.3 Cosmic Rays

I prepared the following questions that should help you to understand the topics. Please read a topic first, think about it and then work through my set of questions on this topic. On problem is quite tricky (indicated by *Bonus* problem). I do not expect you to fully solve this as I will provide the answer in the next week. Ideally you can come up with many more questions yourself. **If you have problems with a derivation or if something is unclear, please email me those points well before the lectures**!

- **Cosmic Rays.**
  - Please work through all the derivations in this chapter.
  - Describe how cosmic rays (CRs) interact with Alfvén waves. Please explain the physics of the wave-particle interactions in configuration and in Fourier space.
  - CRs are virtually colisionless (they only colide on average once per Hubble time with the ambient gas protons in clusters). How can they excert a pressure if they do not collide with the gas?
  - Which processes can accelerate and deaccelerate CRs?
  - How does the process of *diffusive shock acceleration* work (Section 3.3.3.2)? Please follow this derivation in great detail!
    - Why does the CR velocity not experience the deceleration by the shock?
    - Why are we allowed to take the non-relativistic limit of the Lorentz transformation if CRs are relativistic particles?
    - Why are the energy gains in the downstream and upstream media identical?
    - When exactly does a CR particle experience the energy gain during this process? during the crossing of the shock or during the scattering event?
    - Which property of shocks is eventually responsible for the energy gain of CRs?
    - Argue physically (without going to the details of the derivation), why the emerging CR spectrum must be a power in momentum?
    - Does the process of *diffusive shock acceleration* work for collisional shocks in the Earth atmosphere?
  - Which hadronic and leptonic non-thermal emission processes do you know? What is the underlying physics of these emission processes?
  - Why is Fermi-II acceleration with Alfvénic turbulence inefficient? Please explain the reason for the inefficient acceleration in configuration and in Fourier space.
  - Explain how CRs can be accelerated via the Fermi-II process? What is the physical reason behind the more efficient interactions with fast magnetosonic modes in comparison to Alfvén modes.
  - Compare the pros and cons of Fermi-I vs. Fermi-II acceleration.

- **Adiabatic Cosmic Rays**

  Introducing the dimensionless momentum $p = P_\mathrm{p}/(m_\mathrm{p}\, c)$, we assume that the differential cosmic ray (CR) particle momentum spectrum per volume element can be approximated by a single momentum power law above the minimum momentum q:

$$f(p) = \frac{\mathrm{d}^2 N}{\mathrm{d}p\,\mathrm{d}V} = C\,p^{-\alpha}\,\theta(p-q), \tag{1}$$

  where $\theta(x)$ denotes the Heaviside step function. The CR pressure is then given by

$$P_\mathrm{CR} = \frac{m_\mathrm{p}c^2}{3}\int_0^\infty \mathrm{d}p\, f(p)\,\beta\, p = \frac{C\, m_\mathrm{p}c^2}{6}\,\mathcal{B}_{\frac{1}{1+q^2}}\left(\frac{\alpha-2}{2}, \frac{3-\alpha}{2}\right), \tag{2}$$

  where $\beta = v/c = p/\sqrt{1+p^2}$ is the dimensionless velocity of the CR particle and $\mathcal{B}_x(a,b)$ denotes the incomplete beta function, and $\alpha > 2$ is assumed.

  1. Using Liouville's theorem, work out how the low-momentum cutoff q and CR normalization C changes upon an adiabatic density change from ρ_0 to ρ.
  2. Calculate the CR adiabatic index $\gamma_\mathrm{CR} = \mathrm{d}\ln P_\mathrm{CR}/\mathrm{d}\ln\rho$ and take the non-relativistic limit ($q \ll 1$ and $\alpha > 3$) and the ultra-relativistic limit ($q \to \infty$) of γ_CR.
  3. Imagine that CRs are accelerated at a strong cosmological structure formation shock with a relative CR pressure of $X_\mathrm{CR} = P_\mathrm{CR}/P_\mathrm{th} = 0.1$. Calculate X_CR in the ultra-relativistic limit after the composite of CRs and thermal gas has experienced adiabatic density increase by a factor of 10^3 from the warm-hot intergalactic medium to the cluster center.

- *Bonus* problem: **Diffusive Shock Acceleration**

  Restricting to one spatial dimension, the steady-state CR transport equation for the isotropic CR distribution function $f(x,p)$ reads in the limit of negligible Fermi-II acceleration and radiative losses

$$v(x)\frac{\partial f}{\partial x} - \frac{1}{3}\frac{\partial v}{\partial x}p\frac{\partial f}{\partial p} = \frac{\partial}{\partial x}\left[D(x,p)\frac{\partial f}{\partial x}\right], \tag{3}$$

  where D is the CR diffusion coefficient and $v(x)$ is the mean gas velocity. We assume a sharp shock transition of the velocity field (as seen in the shock frame),

$$v(x) = v_1 + (v_2 - v_1)\theta(x), \tag{4}$$

  where the subscripts 1 and 2 denote the gas velocity in the up- and downstream region of the shock. Assume a pre-existing relativistic population in the upstream, f_1, and solve the CR transport equation for $f_2(p)$ in the downstream region of the shock.