

Reading Assignment for The Physics of Galaxy Clusters (Winter Term 2022/23)

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in preparation of lecture 14

Next lecture Feb 1, 2023, 14:15

In the next lecture, we will discuss a selection of the following topics: *4.3 X-ray Cluster Astrophysics* and *4.4 Sunyaev-Zel'dovich Effect*. Please prepare the following sections:

4.3.1 Hydrostatic Equilibrium Masses and Biases

4.3.2 Cluster Population and Evolution

4.3.3 Intracluster Medium Turbulence

4.3.4 Merger Shocks and Electron Equilibration

4.4.1 Thermal and Kinetic SZ Effect

4.4.2 Relativistic SZ Effect

4.4.3 Self-similar SZ Scaling Relation

4.4.4 SZ Power Spectrum

Thinking about the following questions should help you to understand the topics. Please read a topic first, think about it and then work through the set of questions on this topic. Some questions are going beyond what you have read in the lecture notes, but I would like you to start thinking about them and they will certainly be the starting point for our next discussion meeting. Ideally you can come up with many more questions yourself. **If you have problems with a derivation or if something is unclear, please email me those points well before the lectures!**

- Please read Sections 1.2.2, 1.2.4 and 1.2.5 again to recapitulate some of the general properties of X-ray and SZ emission and observations.
- **Hydrostatic Equilibrium Masses and Biases**
 - Please derive Eq. (4.119). What are typical values for the speed of sound in a galaxy cluster? How do they compare to typical galaxy velocities? What is the reason for this?
 - Eq. (4.120) can either be obtained from the Euler equation (Eq. 2.2; show it!) or is directly obtained from considering the balance of hydrodynamical and gravitational forces on a gas parcel in the intracluster medium. In the latter case, show why the hydrodynamic force per unit volume can be written as ∇P .
 - Why is the mean particle mass in the ICM (in units of the proton mass) approximately 0.588? What is assumed about the composition of the medium and the ionization state? (*Hint*: there is some information on this in Appendix A.1).
 - How should the cooling time compare to other relevant timescales (which?) for a hydrostatic atmosphere to form?
 - Please derive Eq. (4.137). How is the line-of-sight integral along z converted to an integral over r ?

- **Cluster Population and Evolution**

- Do you expect the equation for the cooling time to be accurate at lower temperatures $\lesssim 1$ keV?
- Why are entropy profiles typically rising monotonically with radius?
- How do X-ray images of cool core and non-cool core clusters differ?

- **Intracluster Medium Turbulence**

- What processes could inject turbulent motions in the intracluster medium?
- What is a typical ratio of the mean free path to the Larmor radius?
- How does the mean free path compare to the distance from the cluster center in different parts of the cluster?

- **Merger Shocks and Electron Equilibration**

- In the absence of energy exchange between ions and electrons, to which temperatures would a shock heat the ions and the electrons for a velocity change Δv of 1000 km/s across the shock?
- Why is the X-ray emission not sensitive to the ion temperature T_i ?

- **Thermal, Kinetic, and Relativistic SZ Effect**

- Show that Eq. (4.177) is a solution of Eq. (4.176).
- Show that Eq. (4.182) is a solution of Eq. (4.179).
- Show that Eq. (4.184) reduces to Eq. (4.186) for $x \ll 1$.
- What is the advantage of observing clusters in SZ (rather than in X-rays), e.g., at high redshift or large radii?
- Can we measure cluster velocities with the SZ effect?
- What kind of velocity measurements does the kinetic SZ effect allow?
- Name two processes that modify the critical frequency ν_c of the thermal SZ effect.

- **Self-similar SZ Scaling Relation**

- Derive Eq. (4.220).
- What processes could cause deviations from the derived $Y - M$ relation?
- Why do cosmological functions and parameters ($E(z), H_0$) appear in Eq. (4.220)?
- How could the $Y - M$ scaling relation be used to learn about cosmology?

- **SZ Power Spectrum**

- *Bonus:* How would the power spectrum clustering term modify the thermal SZ power spectrum?
- Please derive Eq. (4.238), e.g., starting from Eq. (4.231).
- How could the SZ power spectrum be used to learn about cosmology?