



The Physics of Galaxy Clusters

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Leibniz Institute for Astrophysics, Potsdam (AIP)

University of Potsdam

*Lectures in the International Astrophysics Masters
Program at Potsdam University*

The Physics of Galaxy Clusters

Modus operandi

- I provide you with a full set of lecture notes, freely available at:
https://pages.aip.de/pfrommer/Lectures/galaxy_clusters.html
- weekly reading assignments, which help you to prepare the class for the upcoming week
- ~10-12 questions about the reading assignment: answers to be handed in every Tuesday at noon



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- in-person lecture every week: derivations and discussions about the material that you prepared



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- in-person lecture every week: derivations and discussions about the material that you prepared
- this won't be a 90-min class the way I would have normally taught it, but rather interactive sessions where
 - **you** can ask questions
 - **we** discuss the concepts of the lectures
 - **we** do order of magnitude physics



The Physics of Galaxy Clusters

Preliminaries

- **Contact:** `cpfrommer@aip.de`
- **Literature:** lecture notes on lecture web page
- **Background:**
cosmology part from *Galaxies & Cosmology* useful but not required
- **Credits:** 50% of weekly questions and 50% of exercises, oral exam after the course
- **Success:**
talk to your fellow students and discuss! Connect online and in real life!



The Physics of Galaxy Clusters

Outline of the lectures

- *Overview*
- *Evolution of the dark component*
- *Evolution of the baryonic component*
- *Cluster physics informed by different observables*



The Physics of Galaxy Clusters

Outline of the lectures

- **Overview**

- What is a galaxy cluster?
- Why are clusters interesting?

- **Evolution of the dark component**

- When, where, and how do clusters form?
- How many clusters are there?
- Structure of a cluster?

- **Evolution of the baryonic component**

- Non-radiative physics
- Radiative physics
- Non-thermal processes

- **Cluster physics informed by different observables**

- Optical, X-rays, Sunyaev-Zel'dovich effect, radio



The Physics of Galaxy Clusters

My own motivation to study galaxy clusters

- ***Fascinating science***

- **clusters are largest collapsed objects** in the universe
- **clusters are extremely rare:** opportunity to study large-scale structure, dark energy and dark matter
- **clusters reveal rich astrophysics:** laboratories for physics, including plasma processes to collisionless dynamics



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● *Plan of the lectures*

- start with overview today and next week
- we will assemble a cluster by starting simple and adding consecutively more complex physics



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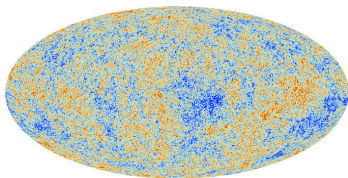
● *Main take-away points*

- theoretical astrophysics connected to multi-frequency observations
- order of magnitude physics



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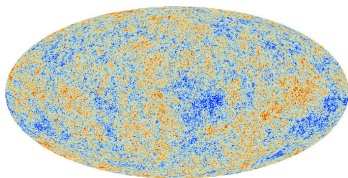
Cosmological structure formation



ESA/Planck Collaboration (2013)

- small fluctuations in cosmic microwave background are initial conditions for structure formation

Cosmological structure formation



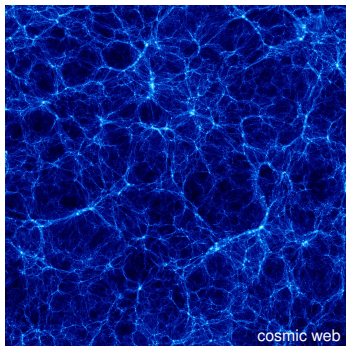
ESA/Planck Collaboration (2013)

- small fluctuations in cosmic microwave background are initial conditions for structure formation
- galaxies and clusters form at sites of constructive interference of those primordial waves



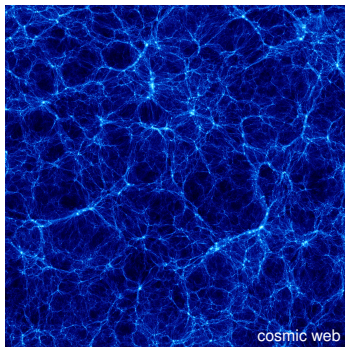
dropping pebbles into the pond generates expanding waves that interfere with each other

Cosmological structure formation



- small fluctuations in cosmic microwave background are initial conditions for structure formation
- galaxies and clusters form at sites of constructive interference of those primordial waves
- **cosmic matter assembles in the “cosmic web”** through gravitational instability
- **galaxies form as “beats on a string” along the cosmic filaments**
- **galaxy clusters form at the knots of the cosmic web** by mergers of galaxies and galaxy groups

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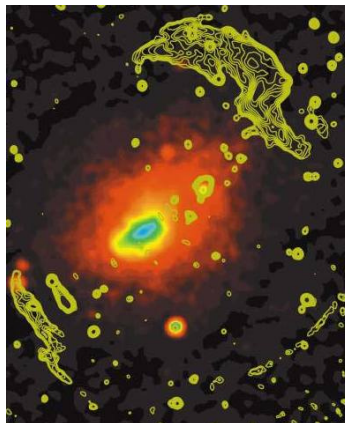
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Cluster mergers: *the* most energetic cosmic events



1E 0657-56 (“Bullet cluster”)

(X-ray: NASA/CXC/CfA/Markevitch; Optical: NASA/STScI; Magellan/U Arizona/Clowe; Lensing: NASA/STScI; ESO WFI; Magellan/U Arizona/Clowe)



Abell 3667

(radio: Johnston-Hollitt; X-ray: ROSAT/PSPC)

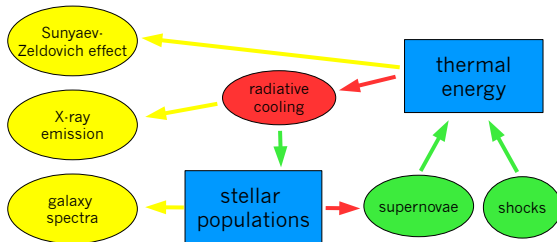


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Cosmological simulations – flowchart

Cluster observables:

Physical processes in clusters:



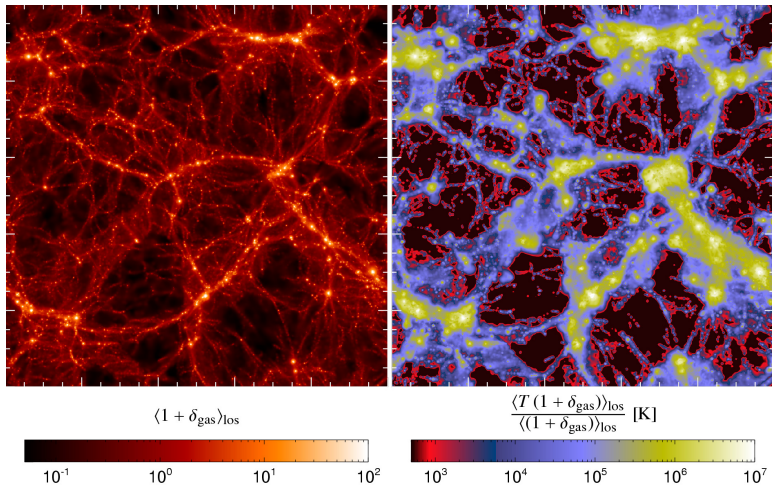
CP, EnBlin, Springel (2008)

— loss processes
— gain processes
— observables
— populations



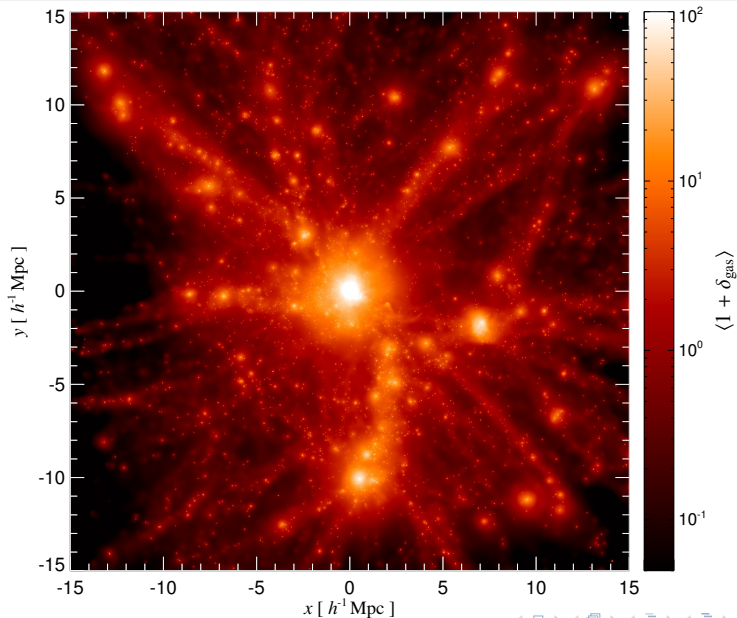
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The structure of our Universe

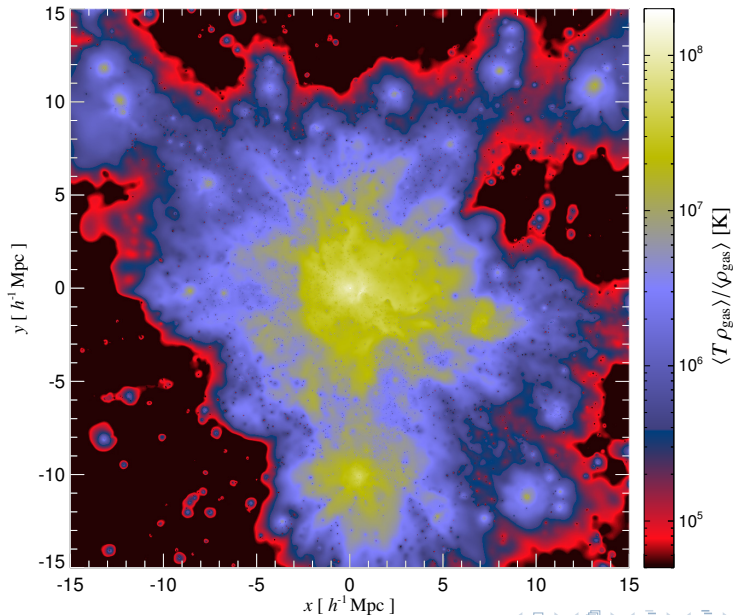


The “cosmic web” today. *Left*: the projected gas density in a cosmological simulation. *Right*: gravitationally heated intergalactic medium (CP+ 2006).

Cosmological cluster simulation: gas density



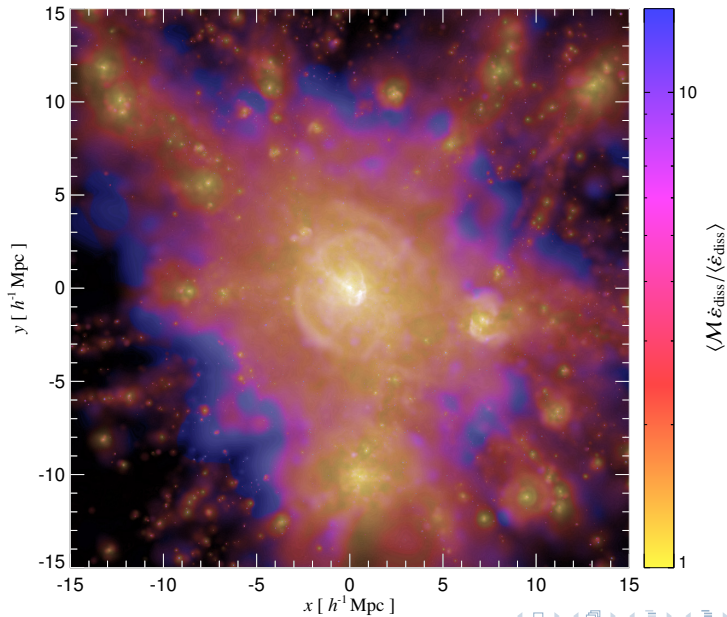
Mass weighted temperature



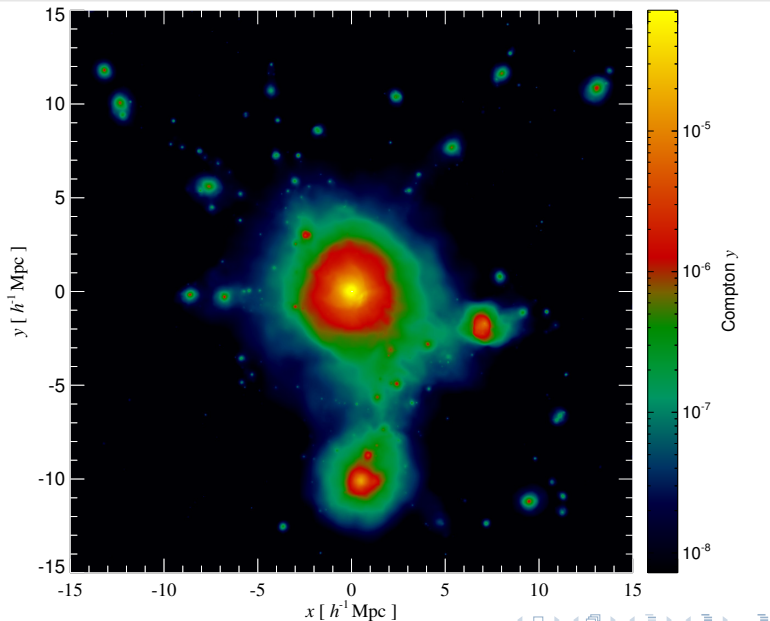
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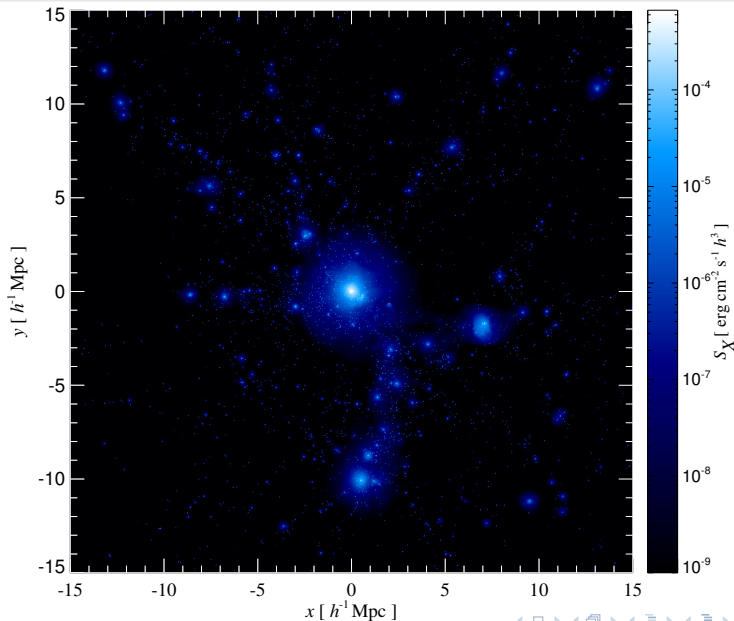
Shock strengths weighted by dissipated energy



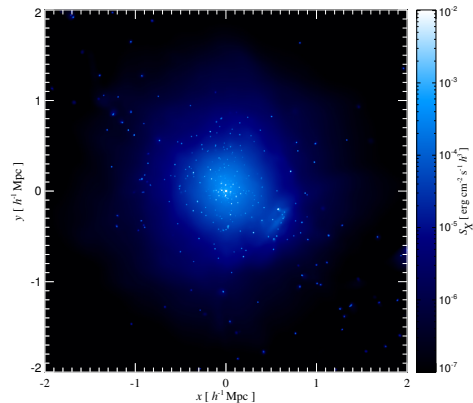
Sunyaev-Zel'dovich effect: integrated thermal pressure



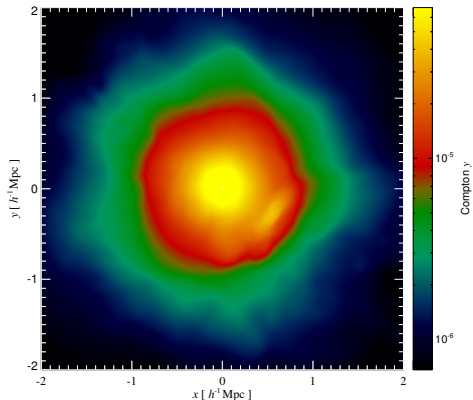
Thermal X-ray emission: gas density squared



Zooming on the cluster: thermal cluster gas

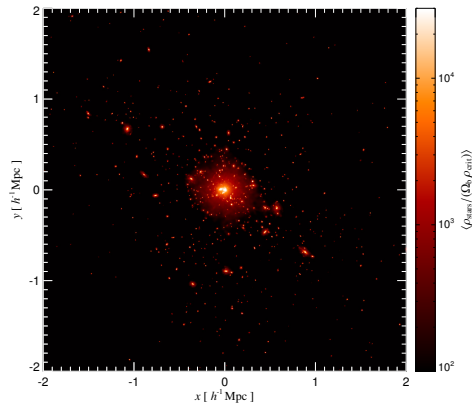


Thermal bremsstrahlung emission,
merging cluster, $M_{\text{vir}} \simeq 10^{15} M_{\odot} / h$

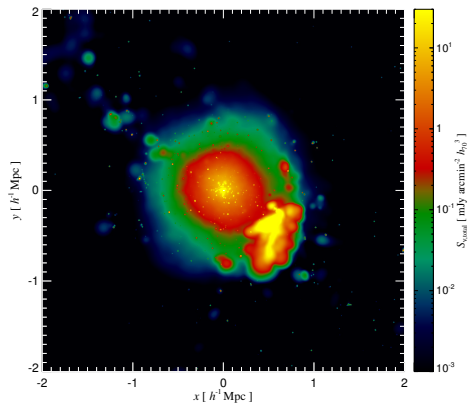


Sunyaev-Zel'dovich effect,
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Zooming on the cluster: optical vs. radio synchrotron



Stellar mass density ("cluster galaxies"),
merging cluster, $M_{\text{vir}} \simeq 10^{15} M_{\odot} / h$



Radio halo and relic emission,
merging cluster, $M_{\text{vir}} \simeq 10^{15} M_{\odot} / h$



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Order of magnitude physics

- cgs system of units: cm, gram, second



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- *astrophysical length unit*: parsecs (parallax second):
distance at which the radius of Earth's orbit subtends an angle of one second of arc,

$$1\text{pc} = \frac{\text{AU}}{\tan\left(\frac{1^\circ \cdot 2\pi}{3600 \cdot 360}\right)} \approx 2 \times 10^5 \text{AU} \approx 3 \times 10^{18} \text{cm}$$



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- *charge*: balancing electron rest mass and potential energy,

$$\frac{e^2}{r_0} = m_e c^2$$

$$1 \text{ esu} = 1 \text{ erg}^{1/2} \text{ cm}^{1/2} = 1 \text{ cm}^{3/2} \text{ g}^{1/2} \text{ s}^{-1}$$



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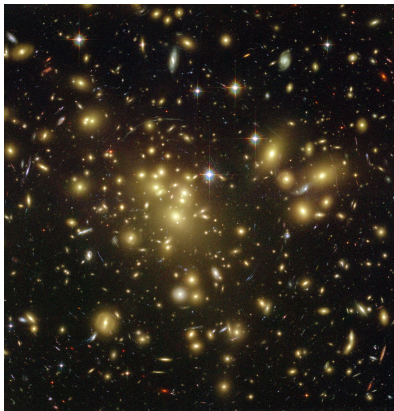
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defines classical electron radius:

$$r_0 = \frac{e^2}{m_e c^2} \approx \frac{(4.8 \times 10^{-10})^2 \text{ cm}^3 \text{ g}}{10^{-27} \text{ g} 10^{21} \text{ cm}^2 \text{ s}^{-2} \text{ s}^2} \approx \frac{2.3 \times 10^{-19}}{10^{-6}} \text{ cm} \approx 3 \times 10^{-13} \text{ cm}$$



Galaxy clusters: optical window – 1



- rich galaxy clusters have $\sim 10^3$ galaxies
- galaxies have Gaussian velocity distribution with a dispersion $\sigma_v = \sqrt{\sigma_x^2 + \sigma_y^2 + \sigma_z^2} \approx 1200 \text{ km s}^{-1}$
- the typical cluster radius is

$$r_{\text{cl}} \approx 3 \text{ Mpc} \approx 10^7 \text{ lyr} \approx 10^{25} \text{ cm}$$

- this defines a dynamical cluster timescale, $t \approx r_{\text{cl}}/\sigma_v \approx 2 \text{ Gyr}$

Galaxy clusters: optical window – 2

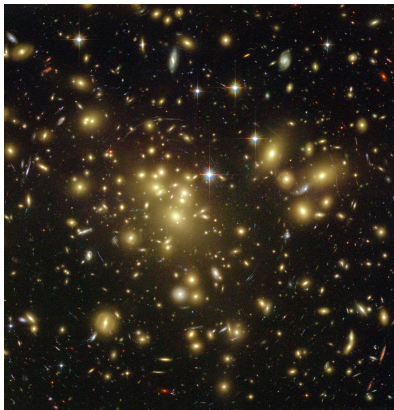


- assuming that the cluster is a closed system in dynamical equilibrium
- virial theorem relates the kinetic energy, E_{kin} , of a galaxy of mass M_{gal} to its potential energy, E_{pot} ,

$$2E_{\text{kin}} + E_{\text{pot}} = 0,$$
$$M_{\text{gal}}\sigma_v^2 - \frac{GM_{\text{cl}}M_{\text{gal}}}{r_{\text{cl}}} = 0,$$

G is Newtons gravitational constant

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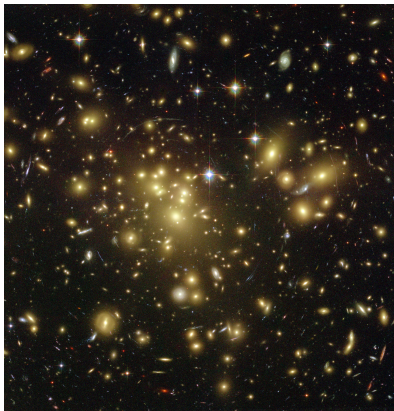
G is Newtons gravitational constant

- gravitating cluster mass, M_{cl} :

$$M_{\text{cl}} = \frac{r_{\text{cl}}\sigma_v^2}{G}$$
$$\approx \frac{10^{25} \text{ cm } 1.4 \times 10^{16} \text{ cm}^2 \text{ s}^{-2}}{7 \times 10^{-8} \text{ erg cm g}^{-2}}$$
$$\approx 2 \times 10^{48} \text{ g} \approx 10^{15} M_{\odot}$$



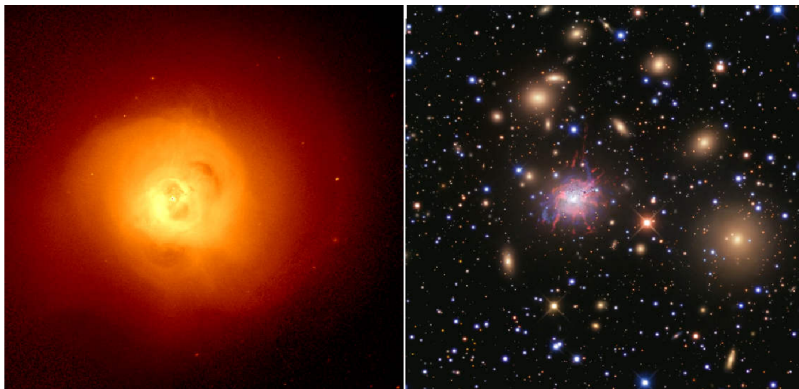
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- M_{cl} sources the high velocity dispersion of galaxies
- a typical mass range for clusters is $(10^{14} \dots 10^{15}) M_{\odot}$
- however, by adding up the all the luminous stellar mass within the galaxies, we only get $M_{*} \approx M_{\text{cl}}/50$
- this discrepancy of the gravitating and luminous mass in galaxy clusters was already noted by Fritz Zwicky in the 1930s:
he postulated the existence of dark matter more than 80 years ago!
- to be precise, back then the “dark matter” could have been baryonic in form of compact objects (such as planets) or in form of diffuse gas

Galaxy clusters: X-ray vs. optical window

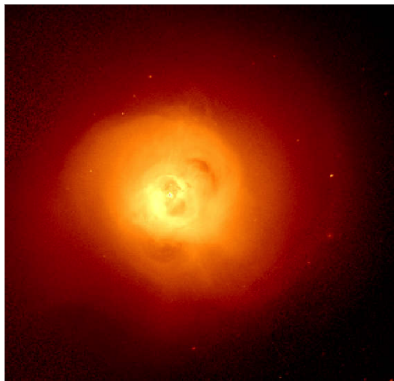
Perseus galaxy cluster



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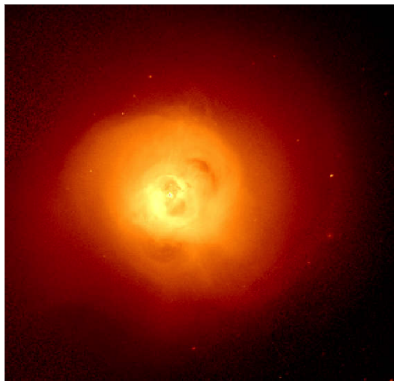


Galaxy clusters: X-ray window – 1



- 1970s: X-ray astronomy that galaxy clusters are among the brightest X-ray sources
- improved angular resolution: the entire galaxy cluster glows in X-rays, filling in the volume in between the galaxies
- which emission process produces X-rays?

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- which emission process produces X-rays?

- 1 bremsstrahlung emission of hot thermal electrons,
- 2 line emission from recombination of atoms, or
- 3 inverse Compton emission by relativistic electrons interacting with low-energy photons: electrons cool by upscattering these photons into the X-ray regime \Rightarrow power-law X-ray spectra



Galaxy clusters: X-ray window – 2



- observed X-ray spectrum is flat with an exponential decline \Rightarrow thermal bremsstrahlung emission
- bremsstrahlung emissivity scales as $j_X \propto n_e n_i \sqrt{T_e}$, where T_e , n_e , and n_i are the electron temperature, density and the ion density
- additionally, spectral lines

Galaxy clusters: X-ray window – 2



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 - additionally, spectral lines
- amount of X-rays and location of the exponential break (as well as individual lines) characterize the plasma:

$$n \approx (10^{-4} \dots 10^{-3}) \text{ cm}^{-3}$$

$$T \approx (10^7 \dots 10^8) \text{ K}$$

\Rightarrow hot, dilute, and thermal gas

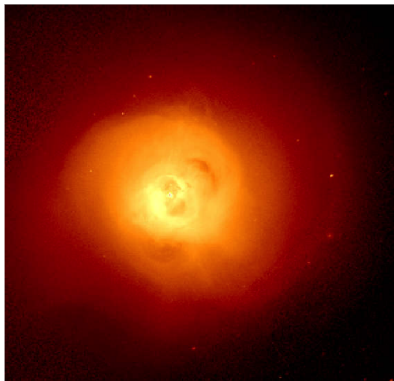


- we talk about gas temperature in terms of particle energies,

$$k_B T \approx (1 \dots 10) \text{ keV} \\ = (10^3 \dots 10^4) \text{ eV}$$

- most elements are fully ionized, except for highly-ionized iron, e.g., hydrogen-like iron (iron nucleus with one bound electron, Fe XXVI)

Galaxy clusters: X-ray window – 3



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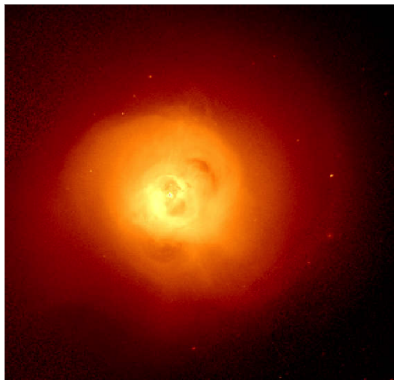
- the transition energy of such highly ionized iron is

$$\text{Fe XXV} : \quad h\nu \approx Z(Z - 1) \text{ Ry} = 26 \times 25 \times 13.6 \text{ eV} \approx 8.8 \text{ keV}$$

$$\text{Fe XXVI} : \quad h\nu \approx Z^2 \text{ Ry} = 26^2 \times 13.6 \text{ eV} \approx 9 \text{ keV},$$

i.e., the higher the temperature, the higher the ionization state

Galaxy clusters: X-ray window – 4

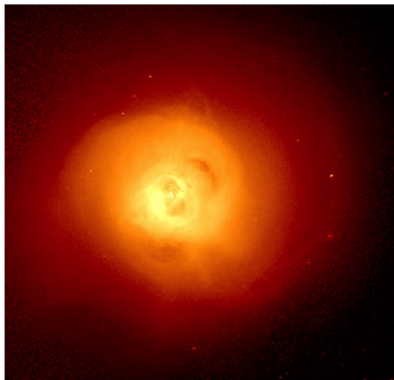


- assuming that hot gas with energy E_{th} is in hydrostatic equilibrium with the cluster potential, we have

$$E_{\text{th}} = E_{\text{pot}},$$
$$\frac{3}{2}k_{\text{B}}T = \mu m_{\text{p}} \frac{GM_{\text{cl}}}{r_{\text{cl}}},$$

m_{p} is the proton mass and $\mu = 4/(5X + 3) \approx 0.6$ is mean molecular weight with $X = 0.24$

Galaxy clusters: X-ray window – 4



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- solving for the gravitating cluster mass with $k_B T = 6$ keV yields

$$M_{\text{cl}} = \frac{3}{2} \frac{k_B T r_{\text{cl}}}{\mu m_p G} \approx \frac{1.5 \times 10^{-8} \text{ erg } 10^{25} \text{ cm}}{0.6 \times 1.7 \times 10^{-24} \text{ g } 7 \times 10^{-8} \text{ erg cm g}^{-2}}$$
$$\approx 2 \times 10^{48} \text{ g} \approx 10^{15} M_{\odot}.$$



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Galaxy clusters: X-ray window – 5



- resolved X-ray imaging of a galaxy cluster produces an X-ray surface brightness map
- deprojection enables us to back out the mass density profile
- integrating over the cluster volume yields the total gas mass, $M_{\text{gas}} \approx M_{\text{cl}}/7$

Galaxy clusters: X-ray window – 5



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-
- we found some of the matter that was “dark” in the optical by looking at a different waveband
 - the rest cannot be directly seen in any other waveband, only indirectly through its gravitational interaction \Rightarrow “dark matter”
 - it dominates the total cluster mass and is mostly responsible for the gravitational cluster potential

Galaxy clusters: inventory

- We can now summarize an inventory of cluster mass

$M_* \approx 2\%$: stars in galaxies

$M_{\text{gas}} \approx 13\%$: hot gas (1 – 10 keV)

$M_{\text{dm}} \approx 85\%$: dark matter

- the value of the baryon fraction in a cluster of $f_{\text{b,clus}} \approx 0.15$ is somewhat smaller than the cosmic mean of $f_{\text{b,clus}} \approx 0.166$



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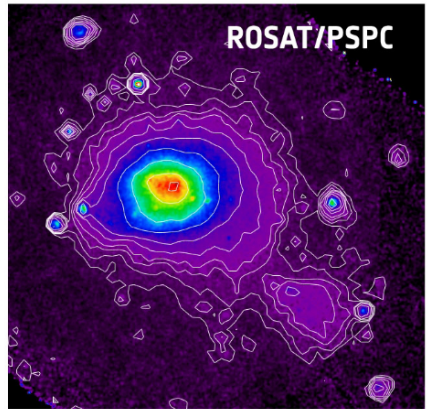
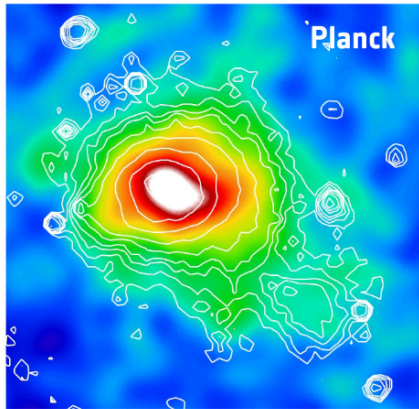
$M_{\text{gas}} \approx 13\%$: hot gas (1 – 10 keV)

$M_{\text{dm}} \approx 85\%$: dark matter

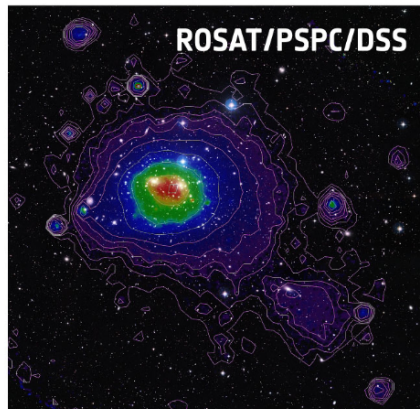
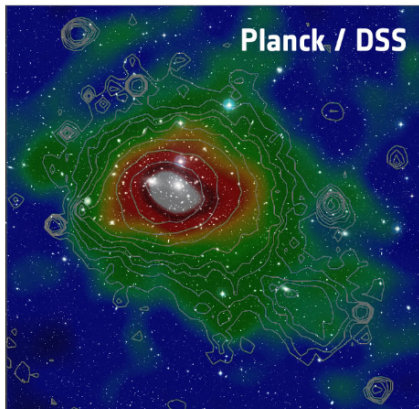
- the value of the baryon fraction in a cluster of $f_{\text{b,clus}} \approx 0.15$ is somewhat smaller than the cosmic mean of $f_{\text{b,clus}} \approx 0.166$
- this points to interesting physics, including non-gravitational energy input from supernovae and super-massive black holes
- interestingly, $f_{\text{b,clus}}$ declines toward less massive clusters in which those feedback processes have a comparably larger impact because of the shallower cluster potential



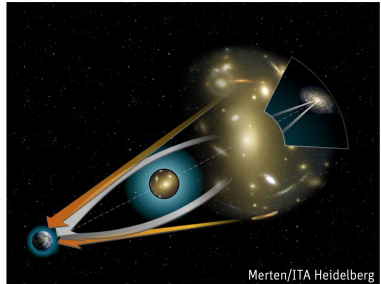
Coma galaxy cluster: Sunyaev-Zel'dovich vs. X-rays



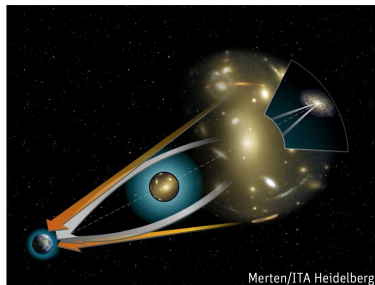
Coma galaxy cluster: SZE/X-rays vs. optical



Gravitational lensing: idea

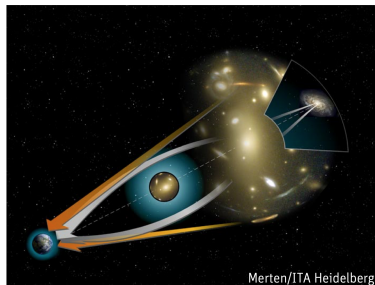


Gravitational lensing: idea



- according to general relativity, light travels on geodesics (straightest possible lines) through curved space time

Gravitational lensing: idea

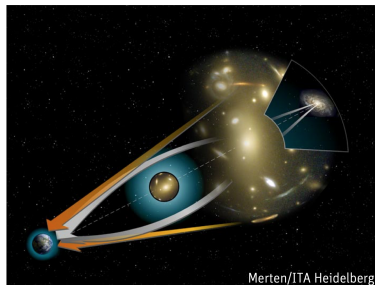


- according to general relativity, light travels on geodesics (straightest possible lines) through curved space time
- mass acts as a source of gravity, curving space time at the location of a lensing galaxy cluster \Rightarrow this causes the light rays to be deflected by the gravitational potential of the lensing object



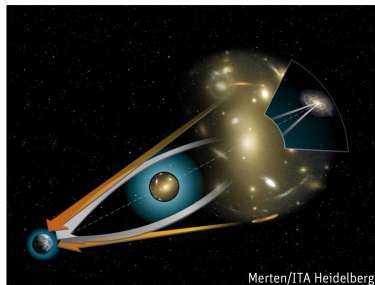
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Gravitational lensing: idea



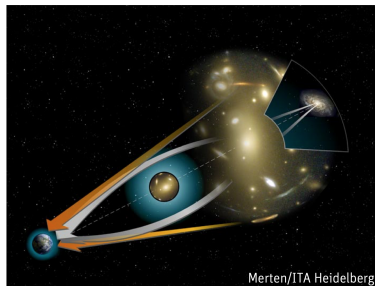
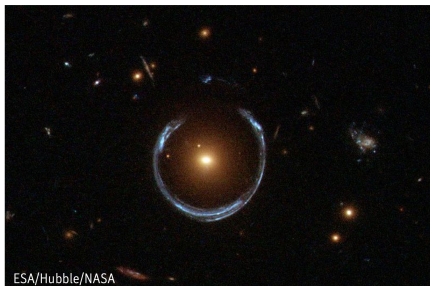
- galaxy clusters (or galaxies) can act as gravitational lenses for galaxies behind them

Gravitational lensing: idea



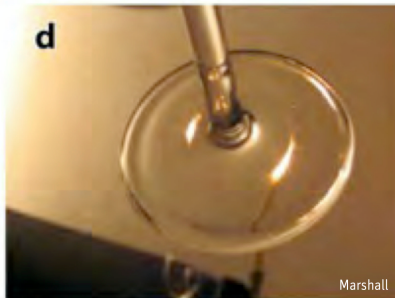
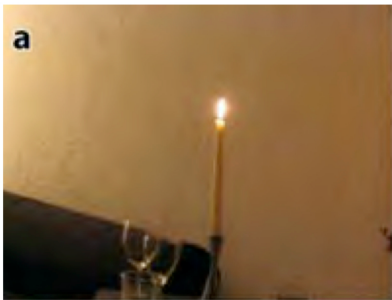
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Gravitational lensing: idea



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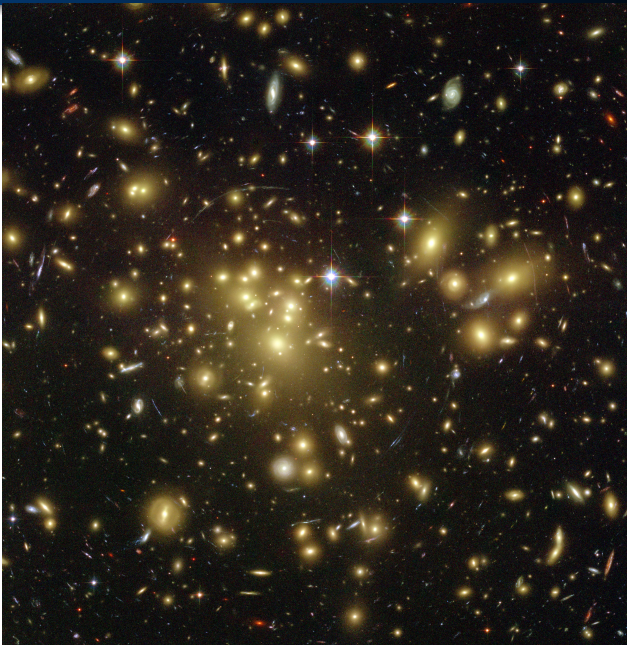
Analogy: light deflection at a wine glass



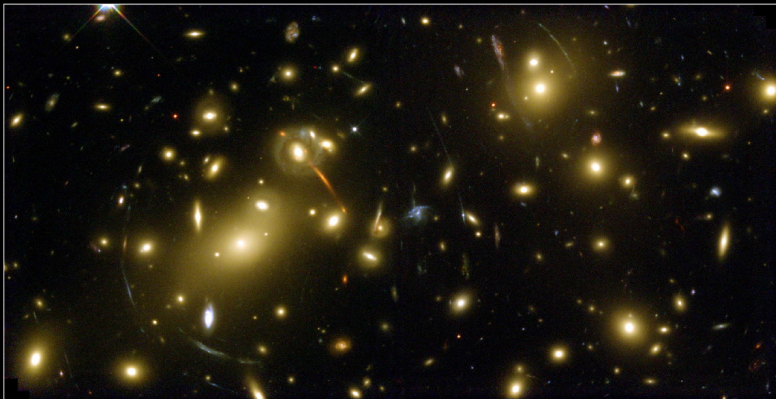
Marshall



Gravitational lensing: Abell 1689



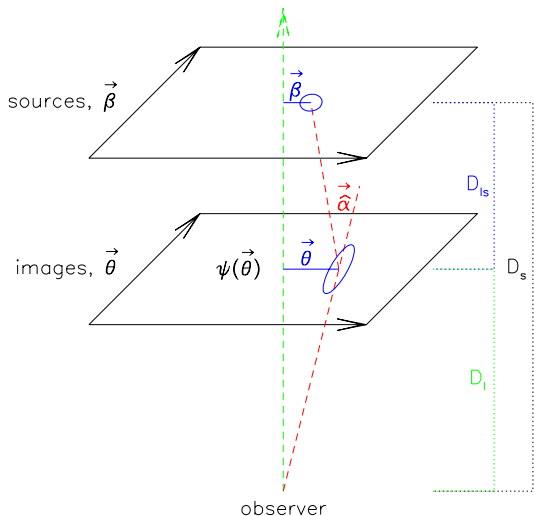
Gravitational lensing: Abell 2218



Galaxy Cluster Abell 2218
Hubble Space Telescope • WFPC2

NASA, A. Fruchter and the ERO Team (STScI, ST-ECF) • STScI-PRC00-08

Gravitational lensing: geometry



- D_l is the angular diameter distance to the light-deflecting cluster or galaxy
- D_s is the distance to the source galaxy
- D_{ls} is the angular diameter distance between deflector and source

Gravitational lensing: order of magnitudes

- later, we will derive the Einstein radius θ_E ; we state the result and insert some values to estimate the involved angular scales:

$$\begin{aligned}\theta_E &= \left[\frac{4GM(\theta_E)}{c^2} \frac{D_{ls}}{D_l D_s} \right]^{1/2} \\ &\approx 3'' \left(\frac{M}{10^{12} M_\odot} \right)^{1/2} \left(\frac{D}{1 \text{ Gpc}} \right)^{-1/2} \quad (\text{galaxy lensing}) \\ &\approx 30'' \left(\frac{M}{10^{14} M_\odot} \right)^{1/2} \left(\frac{D}{1 \text{ Gpc}} \right)^{-1/2} \quad (\text{cluster lensing}),\end{aligned}$$

where $D = D_l D_s / D_{ls}$ is the lensing efficiency distance

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- in the case of galaxy lensing, the approximation of a point lens is justified whereas for cluster lensing, the size of the lens is much larger than the size of the source
 - ⇒ requires detailed mass modeling because only the fraction of cluster mass within θ_E contributes to the lensing potential
- $\theta_E \approx 30''$ corresponds to observed giant (tangential) arcs



Gravitational lensing: two regimes

We distinguish two types of lensing:

- **strong lensing** is sensitive to the projected mass within θ_E and leads to radial arcs that are clearly visible in optical images; in this regime, a source can be imaged onto multiple different images
- **weak lensing** causes weaker distortions of a galaxy image in the tangential direction that cannot be detected on a individual basis because the effect is very small; we need to assume that the orientation of the neighboring galaxies is random and average over an aperture to detect a weak shear signal that is induced by the gravitational tidal field of the cluster lens



The Physics of Galaxy Clusters

Recap of today's lecture

Overview of galaxy clusters: observations and simulations of different observational probes of clusters

- Order of magnitudes
- Optical window: galaxies probe cluster masses; clusters provide a great “magnifying glass” for studying transformational processes of galaxies
- X-ray emission: intracluster plasma probes cluster masses and hydrodynamical flows and instabilities
- Gravitational lensing directly probes cluster potential



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- Sunyaev-Zel'dovich effect
- Relation to the average universe



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