The Physics of Galaxy Clusters

8th Lecture

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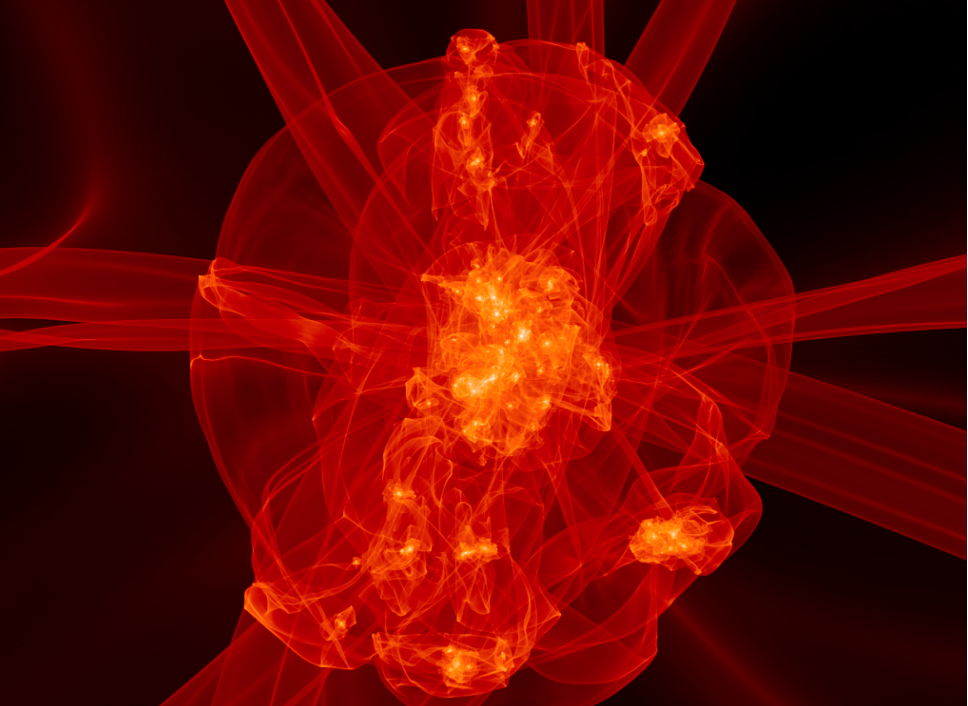

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  - shocks are produced via sound wave steepening or via a supersonic compressible disturbance (non-linearity of Navier Stokes equations)
  - shocks: sudden change in density, temperature, and pressure that decelerates a supersonic flow
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- **Astrophysical collisionless shocks** can
  - * accelerate particles (electrons and ions) $\rightarrow$ cosmic rays (CRs)
  - * amplify magnetic fields (or generate them from scratch)
  - * exchange energy between electrons and ions

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Entropy generation by accretion – 1

Motivate the connection between the phase space density and entropy. When is a gas degenerate and why is the ICM very far from this?

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- The Uncertainty Principle is $\Delta p_x \Delta x = h$, and statistical mechanics counts the number of states with $h^{-3}\mathrm{d}^3x\mathrm{d}^3p$. Hence the phase space density of cluster gas is

$$f \sim \frac{h^3 \mathrm{d}^6 N}{\mathrm{d}^3 x \mathrm{d}^3 p} \sim n \left(\frac{h}{m_\mathrm{p} v}\right)^3$$
$$\sim 6 \times 10^{-35} \left(\frac{n}{10^{-3}\ \mathrm{cm}^{-3}}\right) \left(\frac{v}{10^3\ \mathrm{km\ s}^{-1}}\right)^{-3} \propto K^{-3/2}.$$

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- If this was unity, we would deal with a degenerate gas. Instead, this is extremely small, making it the least degenerate (non-relativistic) gas in the Universe or equivalently, the highest entropy gas (of equilibrium systems)!

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- Because the ICM in the cluster reflects the post-shock state, which is at rest.
- We first consider the case of ***cold*** accretion (P and K of the incoming gas are negligible) which implies the strong-shock regime ($\mathcal{M}_1 \gg 1$). Conveniently, we transform our Rankine-Hugoniot jump conditions to the rest frame of the post-shock gas (where $v_{2,\mathrm{post}} = 0$), i.e., the cluster rest frame.

shock frame: $$v_2 = \frac{v_1}{4}$$

post-shock rest frame: $$v_\mathrm{acc} = v_1 - v_2 = v_1 \left(1 - \frac{1}{4}\right) = \frac{3}{4} v_1$$

$$kT_2 = \frac{3}{16} \bar{m} v_1^2 = \frac{1}{3} \bar{m} v_\mathrm{acc}^2$$

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- If instead, the incoming gas has been heated before passing through the accretion shock, the Mach number is smaller and the cluster entropy level reflects both, the amount of pre-heating and entropy production at the accretion shock.

Entropy generation by accretion – 3

- Suppose that mass accretes in a series of concentric shells, each with a baryon fraction f_b, that initially comoves with the Hubble flow as in the spherical collapse model. In this simple model, a shell that initially encloses total mass M reaches zero velocity at the turnaround radius r_ta and falls back through an accretion shock at radius r_acc in the neighborhood of the virial radius $r_\mathrm{ta}/2$.

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- The system of governing equations are

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\begin{aligned}
\dot{M}_\mathrm{g} &= \frac{\mathrm{d}M_\mathrm{g}}{\mathrm{d}t} = 4\pi\rho_1 r_\mathrm{acc}^2 \left.\frac{\mathrm{d}r}{\mathrm{d}t}\right|_\mathrm{acc} = 4\pi\rho_1 r_\mathrm{acc}^2 v_\mathrm{acc} = f_\mathrm{b}\dot{M}, && (1)\\
v_\mathrm{acc}^2 &= \frac{GM}{r_\mathrm{acc}} = \frac{2GM}{r_\mathrm{ta}}, \quad (\text{assuming } \Omega_\Lambda = 0), && (2)\\
k_\mathrm{B}T_2 &= \frac{1}{3}\bar{m}v_\mathrm{acc}^2, \quad \bar{m} = \mu m_\mathrm{p}, && (3)\\
\rho_2 &= 4\rho_1. && (4)
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Here, ρ_1 is the pre-shock density, ρ_2 and T_2 are the post-shock density and temperature, $r_\mathrm{acc} = r_\mathrm{ta}/2$ is the accretion radius.

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  Here, ρ_1 is the pre-shock density, ρ_2 and T_2 are the post-shock density and temperature, $r_\mathrm{acc} = r_\mathrm{ta}/2$ is the accretion radius.
- In the post-shock frame, the post-shock thermal energy equals the pre-shock ram pressure (+ initial thermal energy that we neglect here) and Eqn. (3) implies

$$\epsilon_2 = \frac{3}{2}\frac{k_\mathrm{B}T_2}{\bar{m}} = \frac{3}{2}\frac{1}{3}v_\mathrm{acc}^2 = \frac{v_\mathrm{acc}^2}{2} = \epsilon_{1,\,\mathrm{kin}}.$$

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Entropy generation by accretion – 4

- Using Eqns. (3) and (4), we can compute the post-shock entropy that is produced by smooth accretion

$$K_{2,\mathrm{sm}} \equiv \frac{k_\mathrm{B} T_2}{\bar{m}\rho_2^{2/3}} = \frac{v_\mathrm{acc}^2}{3(4\rho_1)^{2/3}}. \tag{5}$$

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$$\dot{M}_\mathrm{g} = 4\pi r_\mathrm{acc}^2 \rho_1 \left(\frac{GM}{r_\mathrm{acc}}\right)^{1/2} = f_\mathrm{b}\dot{M} \qquad \Longrightarrow \qquad \rho_1 = \frac{\dot{M} f_\mathrm{b}}{4\pi r_\mathrm{acc}^{3/2}\sqrt{GM}}.$$

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- Inserting this equation and Eqn. (2) into Eqn. (5) yields

$$\begin{aligned} K_{2,\mathrm{sm}} &= \frac{v_\mathrm{acc}^2}{3(4\rho_1)^{2/3}} = \frac{1}{3}\left[\frac{\pi(GM)^2}{f_\mathrm{b}\dot{M}}\right]^{2/3} \\ &= \frac{1}{3}\left(\frac{\pi G^2}{f_\mathrm{b}}\right)^{2/3}\left(\frac{\mathrm{d}\ln M}{\mathrm{d}\ln t}\right)^{-2/3}(Mt)^{2/3}. \qquad (6) \end{aligned}$$

Because the entropy generated at the shock front increases monotonically with time, such an ideal, smoothly accreting cluster never convects but rather accretes shells of baryons as if they were onion skins.

Entropy generation by accretion – 5

- It is useful to cast Eqn. (6) into dimensionless form using a characteristic cluster entropy K_{200},

$$K_{200} \equiv \frac{k_{\mathrm{B}} T_{200}}{\bar{m}(200 f_{\mathrm{b}} \rho_{\mathrm{cr}})^{2/3}} = \frac{1}{2}\left[\frac{2\pi}{15}\frac{G^2 M_{200}}{f_{\mathrm{b}} H(z)}\right]^{2/3}.$$

  Note that we have adopted here the critical density of the Universe, ρ_{cr}, and the characteristic temperature of a singular isothermal sphere, $k_{\mathrm{B}} T_{200}$,

$$\begin{aligned} \rho_{\mathrm{cr}} &\equiv \frac{3H^2(z)}{8\pi G}, \\ k_{\mathrm{B}} T_{200} &= \frac{G M_{200} \bar{m}}{2 r_{200}} = \frac{\bar{m}}{2}\left[10 G M_{200} H(z)\right]^{2/3}. \end{aligned}$$

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- Note that the mass of the *singular isothermal sphere (SIS)* is given by

$$M = \frac{2\sigma^2 r}{G} = \frac{2 r k_B T}{G \bar{m}} \quad \Longrightarrow \quad k_B T = \frac{G M \bar{m}}{2r}. \tag{7}$$

Entropy generation by accretion – 6

- Let's recap the equations for $K_{2,\mathrm{sm}}$ (Eqn. 6) and K_{200}

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\begin{aligned}
K_{2,\mathrm{sm}} &= \frac{1}{3}\left(\frac{\pi G^2}{f_\mathrm{b}}\right)^{2/3}\left(\frac{\mathrm{d}\ln M}{\mathrm{d}\ln t}\right)^{-2/3}(Mt)^{2/3}, \\
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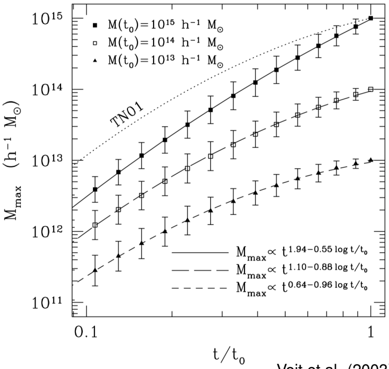
- Combining those equations allows to cast Eqn. (6) into dimensionless form

$$\frac{K_{2,\mathrm{sm}}}{K_{200}} = \frac{2}{3}\left(\frac{15}{2}\,\frac{H_0}{M_{200}}\right)^{2/3}\left(\frac{\mathrm{d}\ln M}{\mathrm{d}\ln t}\right)^{-2/3}(Mt)^{2/3}. \qquad (8)$$

Thus, the entropy profile due to smooth accretion of cold gas depends entirely on the mass accretion history $M(t)$, and the entropy profiles of objects with similar $M(t)$ should be self-similar with respect to K_{200}.



Voit et al. (2003)

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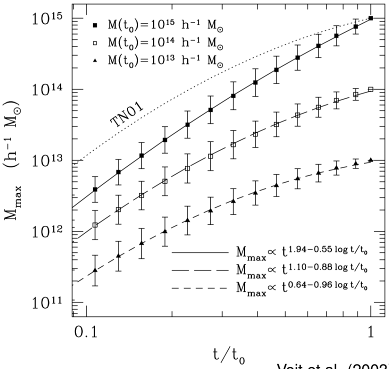
Entropy generation by accretion – 7



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- Inserting these growth times $t \propto M^{0.5\ldots1}$ into Eqn. (8) yields

$$K_{2,\mathrm{sm}} \propto (Mt)^{2/3} \propto M^{1\ldots4/3}.$$

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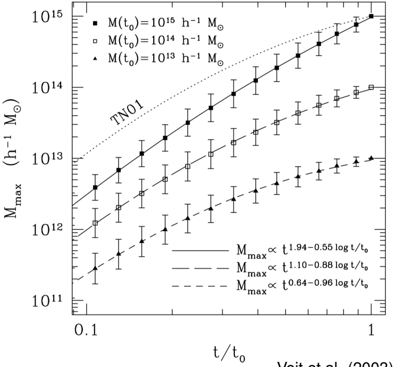

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- Throughout a cluster, M encompassed by a given radius is approximately $M \propto r$ (which is exact for the singular isothermal sphere, see Eqn. 7).
- Assuming $M \propto M_\mathrm{g}$ at large radii, we obtain the following radial entropy profile

$$K_{2,\mathrm{sm}} \propto r^{1\ldots4/3} \qquad (9)$$

which compares well with numerical simulations $K_{2,\mathrm{sm}} \propto r^{1.1}$.

Entropy generation by accretion – 8

- In real clusters the accreting gas is lumpy and not smooth which transforms the nature of entropy generation. Incoming gas that is bound to accreting sublumps of matter enters the cluster with a wide range of densities. There is no well-defined accretion shock but rather a complex network of shocks as different lumps of infalling gas mix with the intracluster medium of the main cluster.

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- Numerical simulations of cluster formation all agree on the baseline profile in non-radiative gas simulations for $r > 0.1 r_{200}$,

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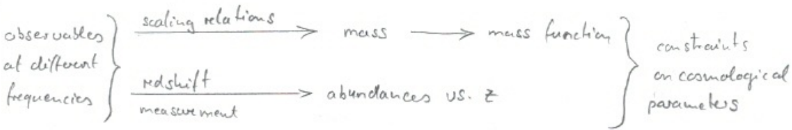
- For $r < 0.1 r_{200}$, there is more dispersion among the simulations and the answer depends on the numerical technique, with grid codes showing an elevated entropy core due to efficient "entropy mixing" (regime of radiative physics).
- While the simulated entropy profiles resemble that from smooth accretion models, there is a problem: the normalization of the smooth models is higher. The likely reason is that smooth accretion maximizes the entropy production as smoothing does not change the accretion velocity but reduces the mean density of accreting gas lumps. Since the post-shock entropy scales as $K_2 \propto v_{\mathrm{acc}}^2 \rho_1^{-2/3}$, a smaller (smoothed) density implies larger entropy everywhere.

Ideal cluster scaling relations – 1



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observables at different frequencies $\left\{ \begin{array}{l} \xrightarrow{\text{scaling relations}} \text{mass} \longrightarrow \text{mass function} \\ \xrightarrow[\text{measurement}]{\text{redshift}} \text{abundances vs. } z \end{array} \right\}$ constraints on cosmological parameters

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- The main assumption underlying this approach is the choice of an average density of the matter so that this implicitly defines a cluster "radius" by

$$M_\Delta = \frac{4}{3}\pi r_\Delta^3 \Delta \rho_{\rm cr}(z), \quad \Delta = 100 \ldots 500,$$

which also relates the temperature to this definition, $T \sim T_\Delta$.

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  which also relates the temperature to this definition, $T \sim T_\Delta$.
- Cautionary remarks: when considering X-ray emission, we encounter ρ_{X}, T_{X} which is not necessarily identical to $\bar{\rho} = \Delta\rho_{\mathrm{cr}}$ and T_Δ as it is degenerate with observational biases.
- Not accounting for these would break self-similarity as e.g., the presence of a clumped multiphase medium may bias T_{X} towards the dense, colder phase with a higher X-ray emissivity.

Ideal cluster scaling relations – 2

- Recall that the definition of the *critical* density at scale factor a and today are defined as

$$\rho_{\rm cr}(a) \equiv \frac{3H^2(a)}{8\pi G}\,, \quad \rho_{\rm cr0} \equiv \frac{3H_0^2}{8\pi G}\,, \quad \text{and}$$
$$H^2(a) \equiv \left(\frac{\dot a}{a}\right)^2 \equiv H_0^2 E^2(a) = H_0^2\left[\Omega_{\rm r0}a^{-4} + \Omega_{\rm m0}a^{-3} + \Omega_{\Lambda 0} + \Omega_{\rm K}a^{-2}\right]$$

is the Hubble function that derives from Friedmann's equation and describes the expansion rate of the universe and the cosmological parameters Ω_i are defined earlier. Note that scale factor and redshift are related via $a = 1/(1+z)$.

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- We assume hydrostatic equilibrium and obtain the scaling

$$\frac{k_{\mathrm{B}}T}{m} \sim v^2 \sim \frac{GM_\Delta}{r_\Delta} \propto M_\Delta^{2/3}\rho_{\mathrm{cr}}^{1/3},$$

  where we used $r_\Delta \propto M_\Delta^{1/3}\rho_{\mathrm{cr}}^{-1/3}(z)$ in the last step. This immediately yields the temperature-mass scaling

$$T_\Delta \propto M_\Delta^{2/3}\rho_{\mathrm{cr}}^{1/3}(z) \propto [M_\Delta E(z)]^{2/3}.$$

Ideal cluster scaling relations – 3

- We assume that clusters are self-similar objects that only scale with $M_\Delta = M_{\rm tot}$. Consequently, the gas fraction, $f_{\rm gas}(< r_\Delta) = M_{\rm gas}/M_{\rm tot}$, and stellar mass fraction, $f_\star(< r_\Delta) = M_\star/M_{\rm tot}$, do not scale with cluster mass. Here, $M_{\rm tot} = M_{\rm tot}(< r_\Delta) = M_{\rm dm} + M_{\rm gas} + M_\star$ is the gravitational mass.

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- We get the gas and stellar mass scaling,

$$\begin{aligned} M_\text{gas} &= \int_0^{r_\Delta} \rho_\text{gas} \mathrm{d}V \approx M_\Delta f_\text{gas} \propto M_\Delta, \\ M_\star &\approx M_\Delta f_\star \propto M_\Delta \implies N_\text{gals} \propto M_\Delta. \end{aligned}$$

  Especially $N_\text{gals} \propto M_\Delta$ assumes a fair sampling of the luminosity function which is not anymore the case on group scales with $\mathcal{O}(10)$ galaxies.

AIP

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- To obtain the Sunyaev-Zel'dovich scaling relation, we integrate the Compton-y parameter over the solid angle Ω subtended by the cluster,

$$\begin{aligned} Y &= \int_\Omega y \mathrm{d}A = \frac{\sigma_T}{m_e c^2} \int_V n_e k_B T_e \mathrm{d}V, \\ Y &\propto M_{\rm gas} T_\Delta \propto M_\Delta^{5/3} E(z)^{2/3}. \end{aligned}$$

Ideal cluster scaling relations – 4

- Finally, the X-ray scaling relation is obtained by assuming that free-free emission (a two-body process) is the dominating radiative process. In this case, the emissivity per unit volume is $j_\mathrm{X} \propto n_\mathrm{e} n_\mathrm{ion} T^{1/2}$ and we obtain the following scaling of X-ray luminosity with cluster mass,

$$\begin{aligned} L_\mathrm{X} &\propto \int_V n_\mathrm{e} n_\mathrm{ion} \sqrt{k_\mathrm{B} T_\mathrm{e}}\,\mathrm{d}V \propto M_\mathrm{gas} T_\Delta^{1/2}, \\ L_\mathrm{X} &\propto M_\Delta^{4/3} E(z)^{1/3} \propto T_\Delta^2 E(z)^{-1}, \end{aligned}$$

where we use the temperature-mass relation, $T_\Delta \propto [M_\Delta E(z)]^{2/3}$.

Cluster scaling relations: order of magnitudes – 1

- Let's recap all ideal cluster mass scaling relations:

$$T_\Delta \propto [M_\Delta E(z)]^{2/3}, \qquad Y \propto M_\Delta^{5/3} E(z)^{2/3},$$
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- A $10^{15}\,\mathrm{M}_\odot$ cluster has a temperature of $kT \approx 6$ keV, a gas mass of $M_{\rm gas} \approx 1.4 \times 10^{14}\mathrm{M}_\odot$ and a stellar mass of $M_\star \approx 2 \times 10^{13}\mathrm{M}_\odot$ which corresponds to $N_{\rm gal} \approx 200$ Milky-Way sized galaxies ($M_{\star,\,\rm MW} \approx 1 \times 10^{11}\mathrm{M}_\odot$).

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- Scaling these numbers at $z = 0$ yields

$$
\begin{aligned}
T_{200} &\approx 1.3\,\mathrm{keV} \times \left(\frac{M_{200}}{10^{14}\mathrm{M}_\odot}\right)^{2/3} &&\approx 0.3\,\mathrm{keV} \times \left(\frac{M_{200}}{10^{13}\mathrm{M}_\odot}\right)^{2/3}, \\
M_{\rm gas} &\approx 1.4 \times 10^{13}\mathrm{M}_\odot \times \left(\frac{M_{200}}{10^{14}\mathrm{M}_\odot}\right) &&\approx 1.4 \times 10^{12}\mathrm{M}_\odot \times \left(\frac{M_{200}}{10^{13}\mathrm{M}_\odot}\right), \\
M_\star &\approx 2 \times 10^{12}\mathrm{M}_\odot \times \left(\frac{M_{200}}{10^{14}\mathrm{M}_\odot}\right) &&\approx 2 \times 10^{11}\mathrm{M}_\odot \times \left(\frac{M_{200}}{10^{13}\mathrm{M}_\odot}\right), \\
N_{\rm gal} &\approx 20 \times \left(\frac{M_{200}}{10^{14}\mathrm{M}_\odot}\right) &&\approx 2 \times \left(\frac{M_{200}}{10^{13}\mathrm{M}_\odot}\right).
\end{aligned}
$$

- At $z = 1$ the temperatures are increased by 45% (since $E(z = 1) = 1.76$).

Cluster scaling relations: order of magnitudes – 2

Argue for each wave band, why the halo mass scale of $\sim 10^{14}\,\mathrm{M}_\odot$ is a good choice for calling an object a galaxy cluster.

Cluster scaling relations: order of magnitudes – 2

Argue for each wave band, why the halo mass scale of $\sim 10^{14}\,\mathrm{M}_\odot$ is a good choice for calling an object a galaxy cluster.

- A $10^{13}\mathrm{M}_\odot$ halo contains 2 Milky-Way sized galaxies, which cannot be called a cluster, but a group. In fact, the Local Group consists of the Milky Way and Andromeda galaxy of nearly equal size. Typically one starts to speak of clusters at the mass scale around $10^{14}\mathrm{M}_\odot$ which implies ~ 20 galaxies.
- Similarly, at around $10^{14}\mathrm{M}_\odot$, the ICM exceeds X-ray emitting particle energies of keV so that one sensibly can speak of an X-ray emitting cluster.
- A halo of $10^{13}\mathrm{M}_\odot$ has an SZ flux that is down by a factor of $0.01^{5/3} \approx 1/2000$ in comparison to a $10^{15}\mathrm{M}_\odot$ cluster. This cannot any more be detected on an individual basis and many SZ decrements at the positions of optically identified groups need to be stacked to detect a signal statistically.

AIP

Real cluster scaling relations – 1

- Observational scaling relations do not follow the self-similar prediction above. One finds

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\begin{aligned}
L_X &\propto T_X^3, \\
\frac{\mathrm{d}}{\mathrm{d}M}\left(\frac{M_{\mathrm{gas}}}{M}\right) &> 0, \\
\frac{\mathrm{d}}{\mathrm{d}M}\left(\frac{M_\star}{M}\right) &< 0,
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  where M is some observational proxy for M_Δ.
- There is less gas and more stars in smaller cluster than predicted by the self-similar scaling, and smaller clusters are dimmer in comparison to the self-similar expectation: $L_X \propto T_\Delta^2$.
- $Y(M)$ and $T_X(M)$ are roughly in agreement with the self-similar prediction. It appears that gas physics modifies these simple scale-invariant models and imposes new scales to the otherwise scale-free gravity and hydrodynamics!

AIP

Real cluster scaling relations – 2

Assume an isothermal gas in a cluster with a beta profile of the gas density:

$$\rho = \rho_0 \left[\frac{1}{1 + (r/r_\mathrm{c})^2}\right]^{3/2\beta},$$

where ρ_0 is the central density, r_c is the core radius, and β is the scaling parameter, calculate the X-ray luminosity for $\beta = 2/3$ and 1 (typical values in the X-ray literature). Compare your result to our approximation of Eqn. (3.170) and discuss it.

AIP

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- Let's perform the integral for the X-ray luminosity rigorously, assuming spherical symmetry and that bremsstrahlung (free-free emission) is the dominating emission process, $j_\mathrm{X}(r) = AT^{1/2}\rho^2(r)$:

$$L_\mathrm{X} = \int_V j_\mathrm{X} \mathrm{d}V = 4\pi A T^{1/2}\rho_0^2 \int_0^\infty \frac{r^2 \mathrm{d}r}{\left[1+(r/r_\mathrm{c})^2\right]^{3\beta}}$$
$$= 4\pi A T^{1/2}\rho_0^2 r_\mathrm{c}^3 \int_0^\infty \frac{x^2 \mathrm{d}x}{\left(1+x^2\right)^{3\beta}} = 4\pi A T^{1/2}\rho_0^2 r_\mathrm{c}^3 f(\beta),$$

  where $f(\beta=1) = \pi/16$ and $f(\beta = 2/3) = \pi/4$.
- Interestingly, in the notes we obtained this scaling without so much effort.

Real cluster scaling relations – 3

- Rigorous computation of the X-ray luminosity:

$$L_\mathrm{X} = \int_V j_\mathrm{X} \mathrm{d}V = 4\pi A T^{1/2} \rho_0^2 r_\mathrm{c}^3 f(\beta),$$

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- In the notes we assume $\beta = 2/3$ and study the limiting cases for the X-ray luminosity:

$$\begin{aligned} L_X &\propto \int_0^R \rho^2 T^{1/2} r^2 \mathrm{d}r, \\ \frac{\mathrm{d}L_X}{\mathrm{d}\ln r} &\propto r^3 \rho^2 T^{1/2} \sim \begin{cases} \rho_0^2 r^3 & \text{for } r < r_c, \\ \rho_0^2 r^{-1} T^{1/2} & \text{for } r > r_c \quad \text{and} \quad \beta = \dfrac{2}{3}. \end{cases} \end{aligned}$$

Thus, the contribution to the X-ray luminosity per logarithmic bin in radius increases steeply toward r_c (because of the larger available volume) and then drops beyond r_c (after realizing that $T \sim r^{-1/2}$ in the peripheral cluster parts). The radii around r_c dominate L_X and thus, we expect

$$L_X \propto \rho_0^2 T^{1/2} r_c^3,$$

which is the same scaling with ρ_0 and r_c.

Real cluster scaling relations – 4

- The density profile with $\beta = 2/3$ follows

$$\rho(r) = \rho_\Delta \left(\frac{r}{r_\Delta}\right)^{-2} \quad \text{for } r > r_\mathrm{c}$$

- We define the cluster concentration parameter $c = r_\Delta / r_\mathrm{c}$, implying $\rho(r_\mathrm{c}) \equiv \rho_\mathrm{c} = c^2 \rho_\Delta$ and rewrite L_X in terms of the concentration c:

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- Using self-similar scalings $T_\Delta \propto M_\Delta^{2/3}$ and $M_\mathrm{gas} \propto M_\Delta \propto r_\Delta^3$, we obtain

$$L_\mathrm{X} \propto c M_\Delta^{4/3} E(z)^{1/3} \propto c T_\Delta^2 E(z)^{-1}.$$

$\Rightarrow$ If c is independent of mass, we recover the self-similar scaling. However, radiative gas physics modifies c so that it assumes a mass dependence.

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- There have been three classes of models suggested to explain the deviation from scale invariance:
  1. "pre-heating" the gas by supernova winds or some other feedback mechanism *before* falling into clusters,
  2. heating the gas in the cluster, potentially through feedback by active galactic nuclei (AGN),
  3. cooling out the low-entropy gas at the cluster center and fueling central star formation selectively removes the low-entropy gas.

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Real scaling relations – pre-heating

We define the cluster concentration parameter $c = r_\Delta / r_\mathrm{c}$, implying $\rho(r_\mathrm{c}) \equiv \rho_\mathrm{c} = c^2 \rho_\Delta$ and found

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- **Solution 1.** "Pre-heating" the gas by supernova winds or some other feedback mechanism *before* falling into clusters imprints an "entropy floor" in the gas distribution – a minimum entropy level K_min below which it cannot fall.
- The clusters' central entropy is $K_0 \propto T \rho_0^{-2/3} \propto T c^{-4/3}$. If all clusters have $K_0 = K_\mathrm{min} = \mathrm{const.}$, then

$$c \propto T^{3/4} K_\mathrm{min}^{-3/4} \implies L_X \propto T^{2.75}.$$

- Thus, an entropy floor leads to larger cores (relative to r_Δ), $r_c = r_\Delta / c \propto (K_\mathrm{min}/T)^{3/4} r_\Delta$, which is larger for smaller clusters (lower T) and thus to a steeper L_X-T relation close to the observations.

Real scaling relations – AGN feedback and cooling

- **Solution 2.** An alternative possibility is that the gas gets heated after falling into the cluster, potentially through feedback by active galactic nuclei (AGN).
- This is however energetically much more expensive: to reach the same entropy as in the pre-heating case, one needs more energy if it is injected at the center by a factor

$$\frac{k_\mathrm{B}T_\mathrm{center}}{k_\mathrm{B}T_\mathrm{pre-heat}} \sim \frac{K_\mathrm{center}}{K_\mathrm{pre-heat}}\left[\frac{n_\mathrm{center}}{n_\mathrm{pre-heat}}\right]^{2/3} \sim 10^2.$$

  Here, we adopted typical values for $n_\mathrm{center} \sim 2 \times 10^{-3}\,\mathrm{cm}^{-3}$ and $n_\mathrm{pre-heat} \sim 10\bar{n} \sim 2 \times 10^{-6}\,\mathrm{cm}^{-3}$.
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- However, AGNs can provide this energy if the energy can be effectively coupled into the ICM.
- **Solution 3.** Cooling out the low-entropy gas at the cluster center and fueling central star formation selectively removes the low-entropy gas. The gas at larger radii (and on higher adiabatic curves) flows in adiabatically and replaces the condensed gas which leads to the formation of an entropy floor.
- This process is observed to happen, but the star formation rate is only 10% of what would be needed to explain the steeper L_X-T slope.

- **Entropy generation by accretion**
  * entropy of accreted cluster gas increases with time as $K \propto t^{2/3}$
  * smoothly accreting cluster never convects but builds stable atmosphere
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- **Cluster scaling relations**
  * scaling relations relate different observables to mass that depends on cosmological parameters via the mass function: **clusters as cosmological probes**
  * observed $Y(M)$ and $T_\mathrm{X}(M)$ obey self-similar relations
  * $M_\star(M)$, $M_\mathrm{gas}(M)$, $L_\mathrm{X}(M)$ show deviations due to baryonic physics
  * pre-heating, AGN feedback, and radiative cooling are suggested solutions

AIP