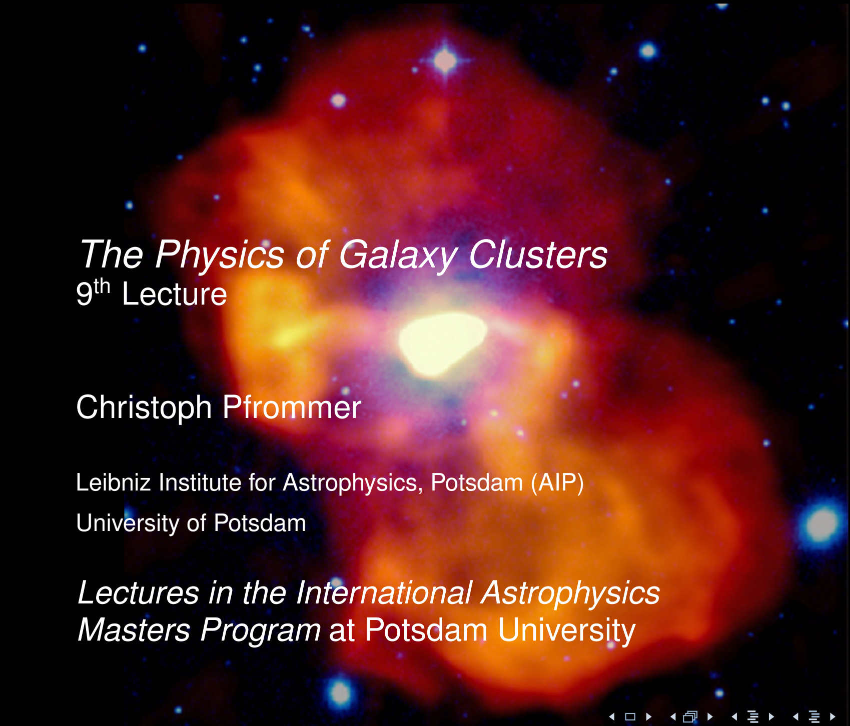

The Physics of Galaxy Clusters
9^{th} Lecture

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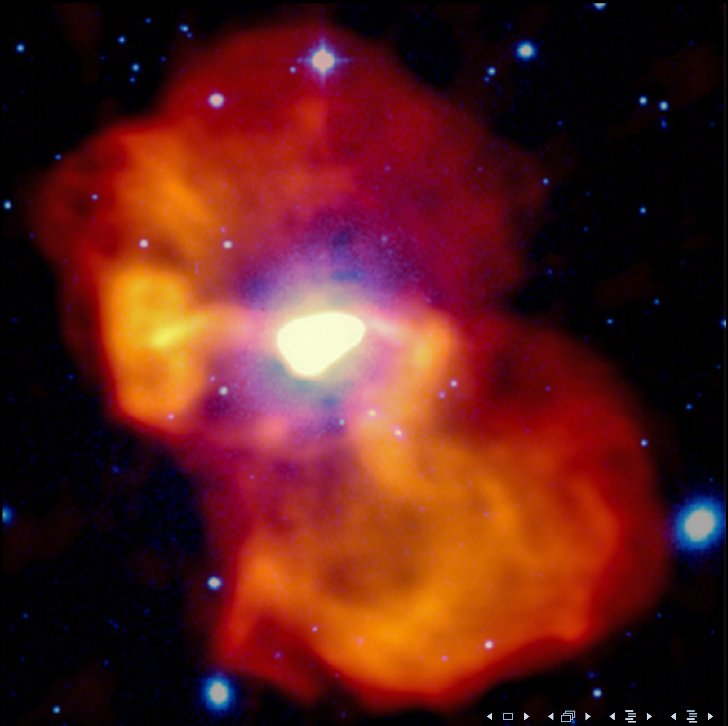

The Physics of Galaxy Clusters

Recap of last week's lecture

- **Entropy generation by accretion**
  * entropy of accreted cluster gas increases with time as $K \propto t^{2/3}$
  * smoothly accreting cluster never convects but builds stable atmosphere
  * simulated clusters show $K \propto r^{1.1}$, lower normalization due to clumpy accretion

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- **Cluster scaling relations**
  - * scaling relations relate different observables to mass that depends on cosmological parameters via the mass function: **clusters as cosmological probes**
  - * observed $Y(M)$ and $T_X(M)$ obey self-similar relations
  - * $M_\star(M)$, $M_{gas}(M)$, $L_X(M)$ show deviations due to baryonic physics
  - * suggested solutions: pre-heating, AGN feedback, and radiative cooling

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Radiative physics

- Three-dimensional hydrodynamics simulations of galaxy clusters span an enormous range in scales and track a plethora of physical processes: formidable computational challenge. Typically, we simulate a periodic box of side length L that contains a representative volume of the universe and is large enough to host enough objects of interest to provide a sufficiently large statistical sample.

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- $L \gtrsim 300\,\mathrm{Mpc} \sim 10^{27}\,\mathrm{cm}$. This large volume is necessary in order to get a few sites of constructive interference of long wavelength modes which evolve into a few massive ($M \sim 10^{15}\,\mathrm{M}_\odot$) galaxy clusters.

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- $l_\mathrm{gal} \sim 30\,\mathrm{kpc} \sim 10^{23}$ cm. The simulation needs to resolve the diameter l_gal of galaxies by at least 10 resolution elements. Such a Eulerian mesh would then have $[L/(0.1 l_\mathrm{gal})]^3 \sim 10^{15}$ individual cells—too many elements for current state-of-the-art simulations. Solution: introduce adaptive grid-refinement in Eulerian codes (which increase the numerical resolution where needed, i.e., inside collapsing objects) or Lagrangian frameworks that discretize the simulated mass rather than simulation space.

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- $l_\star \sim 3\,\mathrm{pc} \sim 10^{19}\,\mathrm{cm} \sim$ size of star forming regions. The resulting dynamical range of the simulation volume, $(L/l_\star)^3 \sim 10^{24}$, is too large to be reliably included in first-principle simulations. Instead, this requires a subgrid prescription of star formation physics to include the necessary dynamical back-reaction effects on the resolved larger scales.

Radiative cooling – 1

- At high temperatures ($k_B T \gtrsim 2$ keV) the light- and intermediate-mass elements of the ICM are fully ionized so that the only cooling process for them is free-free emission (thermal bremsstrahlung). Below $k_B T \sim 2$ keV *recombination-line cooling* of heavy elements (Fe, ...) starts to dominate the cooling process (and the associated X-ray emission, assuming typical heavy element abundances relative to hydrogen, which are ~ 0.3 times those found in the Sun).

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- The physics of bremsstrahlung emission is simple: electrons scatter off of ions and are deflected in the Coulomb field of the ions. They radiate because of their acceleration and thus lose energy, i.e., they "cool".
- The spectral X-ray emissivity j_ν is defined as as the amount of energy emitted in photons of frequency ν per unit frequency interval dν, per unit time and per unit plasma volume, $j_\nu = \mathrm{d}^3 E/(\mathrm{d}\nu \mathrm{d}t \mathrm{d}V)$. It must scale with the product of electron and ion number density (because it is a two-body interaction), with the time available for the scattering process, $t \sim l/\Delta v \sim l/\sqrt{k_B T/\bar{m}}$, where Δv is the relative velocity of electron and ion, and with the Boltzmann factor for the distribution of energy at a given temperature. Hence we get

$$j_\nu = \frac{\mathrm{d}^3 E}{\mathrm{d}\nu \mathrm{d}t \mathrm{d}V} = \tilde{C} \frac{n^2}{\sqrt{k_B T}} \mathrm{e}^{-h\nu/k_B T}, \quad \tilde{C} = \mathrm{const}.$$

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Radiative cooling – 2

- Recall the spectral X-ray emissivity j_ν:

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- The volume emissivity is the integral of j_ν over frequency ($Z = 0.3 Z_\odot$.),

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$$= C n^2 \sqrt{k_\mathrm{B} T} = 2.5 \times 10^{-23} \left(\frac{n_\mathrm{H}}{1\,\mathrm{cm}^{-3}} \right)^2 \left(\frac{T}{10^8\,\mathrm{K}} \right)^{1/2} \frac{\mathrm{erg}}{\mathrm{cm}^3\,\mathrm{s}}.$$

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- Comparing the thermal energy content to the total (frequency-integrated) X-ray emissivity defines the cooling time

$$t_\mathrm{cool} = \frac{\varepsilon_\mathrm{th}}{|\dot{\varepsilon}_\mathrm{brems}|} = \frac{3 n k_\mathrm{B} T}{2 j}$$

$$\approx 0.66 \left(\frac{k_\mathrm{B} T}{1\,\mathrm{keV}} \right)^{1/2} \left(\frac{n_\mathrm{e}}{3 \times 10^{-2}\,\mathrm{cm}^{-3}} \right)^{-1} \mathrm{Gyr} \qquad \text{(CC cluster)}$$

$$\approx 13.3 \left(\frac{k_\mathrm{B} T}{4\,\mathrm{keV}} \right)^{1/2} \left(\frac{n_\mathrm{e}}{3 \times 10^{-3}\,\mathrm{cm}^{-3}} \right)^{-1} \mathrm{Gyr} \sim t_\mathrm{Hubble}, \qquad \text{(NCC cluster)}$$

where $n = \rho/(\mu m_\mathrm{p})$, n_e is the electron number density, and $\mu = 0.588$ is the mean molecular weight of a fully ionized primordial gas.

Radiative cooling – 3

- Recall the expression for the cooling time:

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- This means that after $2t_{\mathrm{cool}}$, the thermal energy has dropped by an order of magnitude. As a result, gas would be quickly flowing from larger radii towards the center, maintaining pressure equilibrium.
- If gas in pressure equilibrium cools, it becomes denser and cools even faster. This is a run-away process that should lead to a large amount of cold gas and star formation at rates of $\sim 1000\ \mathrm{M}_{\odot}\ \mathrm{yr}^{-1}$—in conflict with observations. This is the ***famous "cooling flow problem"***.

Radiative cooling – 4

- We can gain further insight if we rewrite t_{cool} in terms of the cluster entropy $K_{\mathrm{e}} \equiv k_{\mathrm{B}} T n_{\mathrm{e}}^{-2/3}$. We define $t_0 = 2$ Gyr, $k_{\mathrm{B}} T_0 =$ keV, and $n_0 = 10^{-2}\,\mathrm{cm}^{-3}$, to obtain

$$t_{\mathrm{cool}} = t_0 \left(\frac{k_{\mathrm{B}} T}{k_{\mathrm{B}} T_0}\right)^{1/2} \frac{n_0}{n_{\mathrm{e}}} = t_0 \left(\frac{K_{\mathrm{e}}}{K_0}\right)^{3/2} \frac{k_{\mathrm{B}} T_0}{k_{\mathrm{B}} T},$$

where $K_0 = 21.5\,\mathrm{keV\,cm}^2$ is a typical value for the central entropy in CC clusters. Because $t_0 \ll t_{\mathrm{Hubble}} \approx 14\,\mathrm{Gyr}$ the cooling ICM needs additional energy injection that stabilizes it against the cooling catastrophe.

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- This demonstrates that clusters with similar temperatures (or potential depths) have longer cooling times if the central entropy is larger. We can derive a critical entropy

$$K_{\rm c}(T) \approx 80 \left(\frac{t_{\rm cool}}{14\,{\rm Gyr}}\right)^{2/3} \left(\frac{k_{\rm B} T}{\rm keV}\right)^{2/3} {\rm keV\,cm}^2,$$

  that is large enough to avoid the cooling catastrophe in galaxy groups with $k_{\rm B} T \sim$ keV.

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- Using $k_{\mathrm{B}} T = k_{\mathrm{B}} \tilde{T} (M_{200}/\tilde{M})^{2/3}$, where $\tilde{M} = 10^{15}\,\mathrm{M}_{\odot}$ and $k_{\mathrm{B}} \tilde{T} = 6$ keV, we obtain

$$t_{\mathrm{cool}} = t_0 \left(\frac{K_{\mathrm{e}}}{K_0}\right)^{3/2} \frac{k_{\mathrm{B}} T_0}{k_{\mathrm{B}} T} = 0.33 \left(\frac{K_{\mathrm{e}}}{K_0}\right)^{3/2} \left(\frac{M_{200}}{10^{15}\,\mathrm{M}_{\odot}}\right)^{-2/3} \mathrm{Gyr}.$$

Cooling versus heating – 1

- We have seen that the cooling time in the core region of cool core clusters is smaller than the Hubble time which would imply a cooling catastrophe if not countered by energy feedback. To see how much feedback is needed, we first compute the cooling rate and redefine the X-ray emissivity as an energy cooling rate $\Lambda(T)$ according to

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\begin{aligned}
j &= C n_\mathrm{H}^2 \sqrt{k_\mathrm{B} T} = \Lambda(T) n_\mathrm{H}^2, \quad \text{where} \\
\Lambda(T) &= \Lambda_0 \left(\frac{k_\mathrm{B} T}{k_\mathrm{B} T_0}\right)^{1/2} \\
&= 2.5 \times 10^{-23} \left(\frac{T}{10^8\,\mathrm{K}}\right)^{1/2} \frac{\mathrm{erg\,cm}^3}{\mathrm{s}}, \quad (Z = 0.3 Z_\odot). \qquad (1)
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- We adopt a typical gas density profile as found in X-ray observations, the so-called beta profile which is simply a King profile with the outer slope parametrized by $\beta \approx 2/3 \ldots 1$:

$$
n(r) = n_0 \left[1 + \left(\frac{r}{r_\mathrm{c}}\right)^2\right]^{-3\beta/2}.
$$

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- We consider the X-ray luminosity as a proxy for the cooling luminosity:

$$\begin{aligned} L_X &= \int_0^\infty j \mathrm{d}V = \Lambda_0 \sqrt{\frac{k_B T}{k_B T_0}}\, 4\pi \int_0^\infty n^2(r) r^2 \mathrm{d}r \\ &= \frac{4\pi}{3} r_c^3 n_0^2 \Lambda_0 \sqrt{\frac{k_B T}{k_B T_0}} \times 3 \int_0^\infty \frac{x^2 \mathrm{d}x}{(1+x^2)^{3\beta}} \\ &= \frac{4\pi}{3} r_c^3 n_0^2 \Lambda_0 \sqrt{\frac{k_B T}{k_B T_0}} \times \begin{cases} \frac{3\pi}{16} & \text{for } \beta = 1 \\ \frac{3\pi}{4} & \text{for } \beta = 2/3 \end{cases} \\ &\sim 10^{44} \left(\frac{r_c}{100\,\mathrm{kpc}}\right)^3 \left(\frac{n_0}{10^{-2}\,\mathrm{cm}^{-3}}\right)^2 \left(\frac{k_B T}{3\,\mathrm{keV}}\right)^{1/2} \mathrm{erg\ s}^{-1}, \end{aligned}$$

where we adopted $\beta = 1$ in the last step. Note that to order of magnitude, it suffices to assume a homogeneous sphere with radius r_c and a density that is equal to that of the core region to calculate L_X.

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- This corresponds to our finding last week that radii around r_c dominate L_X:

$$
\frac{\mathrm{d}L_X}{\mathrm{d}\ln r} \propto r^3 \rho^2 T^{1/2} \sim \begin{cases} \rho_0^2 r^3 & \text{for } r < r_c, \\ \rho_0^2 r^{-1} T^{1/2} & \text{for } r > r_c. \end{cases}
$$

  This means that the density drops too fast for $r > r_c$ so that the virial radius cannot be observed in the X-rays and the average density does not enter L_X.

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&\sim 10^{44} \left(\frac{r_c}{100\,\mathrm{kpc}}\right)^3 \left(\frac{n_0}{10^{-2}\,\mathrm{cm}^{-3}}\right)^2 \left(\frac{k_B T}{3\,\mathrm{keV}}\right)^{1/2} \mathrm{erg\ s}^{-1},
\end{aligned}
$$

  where we adopted $\beta = 1$ in the last step.
- Hence, a successful feedback process has to heat the ICM at an average rate of 10^{44} erg s^{-1} to balance the cooling losses.

Cooling versus heating – 4

- The density distribution of a **cool core cluster** is characterized by a double beta profile,

$$n(r) = \sum_{i=1,2} n_i \left[1 + \left(\frac{r}{r_{\mathrm{c},\,i}}\right)^2\right]^{-3\beta_i/2} .$$

Plot the density profile of this cluster as a function of radius in a double-logarithmic representation for $\beta_{1,2} = 1$, $(n_1, n_2) = (10^{-1}, 10^{-2})\,\mathrm{cm}^{-3}$, and $r_{\mathrm{c},\,1}, r_{\mathrm{c},\,2} = (10, 100)$ kpc.

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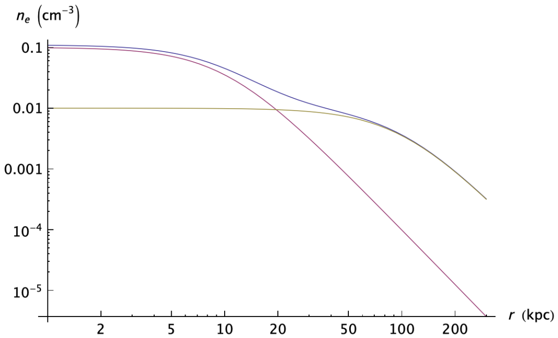

Cooling versus heating – 5

- Recall the cooling time:

$$t_{\mathrm{cool}} \approx 2.8 \left(\frac{k_\mathrm{B} T}{2\,\mathrm{keV}}\right)^{1/2} \left(\frac{n_\mathrm{e}}{10^{-2}\,\mathrm{cm}^{-3}}\right)^{-1}\ \mathrm{Gyr}.$$

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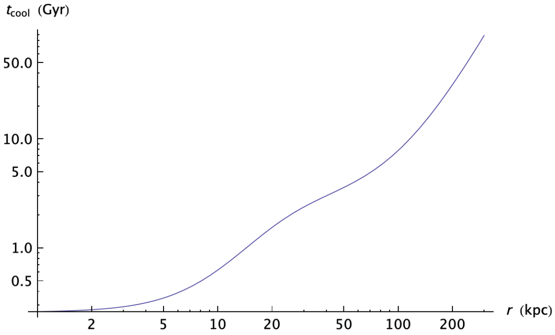

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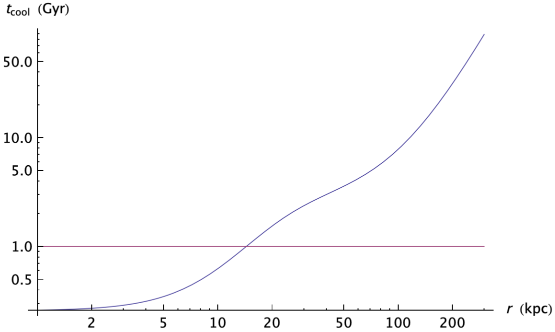
- Plot the cooling time as a function of radius in a double-logarithmic representation and assume a constant temperature of 2 keV.
- The cooling radius for this cluster (which is the radius where the cooling time equals 1 Gyr) is 14.4 kpc.



Cooling versus heating – 7

- The density distribution of a **non-cool core cluster** obeys a single beta profile. If you drop the first term and only account for the second term (with subscripts 2), plot the corresponding cooling time profile.

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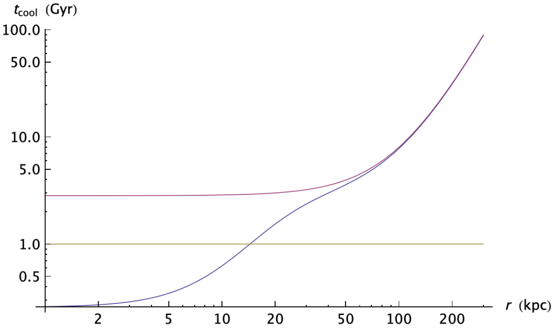

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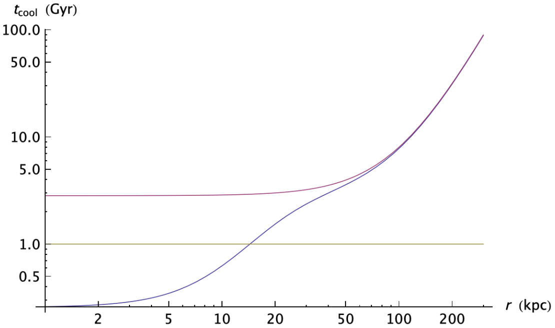

- Hence, there is no cooling radius in a non-cool core cluster because $t_{\rm cool} > 1$ Gyr everywhere. Unlike cool-core clusters, there is no severe overcooling problem in a non-cool core cluster.

Supernova explosion scenarios

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- **Thermonuclear SNe.** The progenitor system of a type Ia supernova consists of a binary with at least one massive ($\approx 1\,\mathrm{M}_\odot$) carbon-oxygen white dwarf. The *double-degenerate scenario* is a binary of two carbon-oxygen white dwarfs. At the end of their evolution, they merge and cause a thermonuclear runaway burning of carbon and oxygen in the more massive progenitor. Like core-collapse SNe, the type Ia supernovae explosion delivers 10^{51} erg of energy to the surrounding interstellar medium.

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- The *single-degenerate scenario* assumes that the companion of the white dwarf is an evolved star. When the companion star becomes a red giant, it grows over its Roche volume and transfers mass to the white dwarf. White dwarfs are stabilized by the Fermi pressure of a degenerate electrons gas. This can only stabilize masses up to $1.4\,\mathrm{M}_\odot$ against gravity. When the companion star feeds the white dwarf beyond this limit, a thermonuclear runaway burning is eventually triggered, which explodes the white dwarf. This scenario appears to be ruled out for explaining the majority of type Ia supernovae.

Feedback by supernovae – 1

- To estimate the effect of SNe heating on the ICM, we make three simplifications. We assume that 1. metals are fully mixed within the ICM, 2. neglect radiative losses, and 3. assume solar abundances.
- Since the metallicity Z of clusters is typically $0.3Z_\odot$ and radiative losses cause a large fraction of this SNe energy to be radiated away, these numbers represent the absolute upper limit that SNe can contribute to the heating which is plausibly not reachable in the ICM.

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- **Core-collapse SNe.** The mass fraction of α elements for a gas of solar abundance is

$$\frac{M_\alpha}{M_\text{gas}} \approx 0.02.$$

Hence the supernova energy per α element that is created by the SN is given by

$$\frac{E_\text{SN}\,m_\text{p}}{M_\alpha} \sim \frac{10^{51}\,\text{erg}\,m_\text{p}}{10\,\text{M}_\odot} \sim \frac{10^{51-24-34}}{2}\,\frac{\text{erg}}{\text{nucleon}} \sim 50\,\frac{\text{keV}}{\text{nucleon}}.$$

Mixing this energy into the ICM (and neglecting radiative losses), we get

$$\frac{E_\text{SN}\,m_\text{p}}{M_\text{gas}} \sim 1\,\frac{\text{keV}}{\text{nucleon}}.$$

Feedback by supernovae – 2

- We use the same assumptions as above to estimate the energetics of feedback by *thermonuclear SNe*. We assume that 1. metals are fully mixed within the ICM, 2. neglect radiative losses, and 3. assume solar abundances.
- Since the metallicity Z of clusters is typically $0.3Z_{\odot}$ and radiative losses cause a large fraction of this SNe energy to be radiated away, these numbers represent the absolute upper limit that SNe can contribute to the heating which is plausibly no reachable in the ICM.

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- **Thermonuclear SNe.** The mass fraction of iron elements for a gas of solar abundance is

$$\begin{aligned}
\frac{M_\mathrm{Fe}}{M_\mathrm{gas}} &\approx 0.001 \quad \text{(solar abundance)}, \\
\frac{E_\mathrm{SN}\, m_\mathrm{p}}{M_\mathrm{Fe}} &\sim \frac{10^{51}\,\mathrm{erg}\, m_\mathrm{p}}{1\,\mathrm{M}_\odot} \sim 500\,\frac{\mathrm{keV}}{\mathrm{nucleon}}, \\
\frac{E_\mathrm{SN}\, m_\mathrm{p}}{M_\mathrm{gas}} &\sim 0.5\,\frac{\mathrm{keV}}{\mathrm{nucleon}}.
\end{aligned}$$

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Problems. As we will now show, there are two problems with this hypothetical picture in which SNe provide the feedback energy: 1. the energetics is not sufficient and 2. the radiative losses are too strong to solve the “cooling flow problem”.

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- For comparison we estimate the gravitational energy of a Milky Way-type galaxy and a massive galaxy cluster

$$\begin{aligned} E_\mathrm{gal} &\approx \frac{m_\mathrm{p}}{2} v_\mathrm{gal}^2 \approx 0.25 \left(\frac{v_\mathrm{gal}}{220\,\mathrm{km\,s}^{-1}}\right)^2 \mathrm{keV}, \\ E_\mathrm{cluster} &\approx \frac{m_\mathrm{p}}{2} \sigma_\mathrm{cluster}^2 \approx 8 \left(\frac{\sigma_\mathrm{cluster}}{1200\,\mathrm{km\,s}^{-1}}\right)^2 \mathrm{keV}. \end{aligned}$$

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- To avoid radiative losses, SNe heating has to raise the entropy of the gas it heats to $\gtrsim 100\,{\rm keV\,cm^2}$. An evenly distributed thermal energy input of order 1 keV would have to go into gas significantly less dense than $10^{-3}\,{\rm cm^{-3}}$ to avoid such losses. But gas near the centers of present-day cluster (not to mention the densities of the interstellar medium within galaxies where SNe occur) is denser than that with average densities $\bar{n}_{\rm ISM} \sim 1\,{\rm cm^{-3}}$, particularly at earlier times when most of the star formation happened. Simulations that spread SNe feedback evenly thus produce too many stars in clusters!

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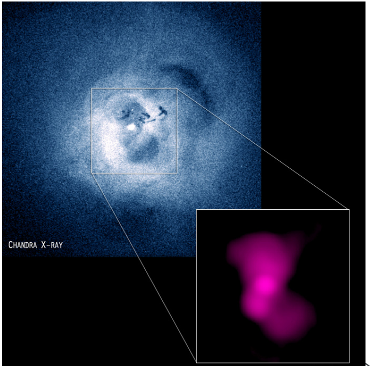
AGN jet feedback

Paradigm: super-massive black holes with $M \sim (10^9 \ldots 10^{10}) M_\odot$ co-evolve with their hosting cD galaxies at the centers of galaxy clusters. They launch relativistic jets that blow bubbles, potentially providing energetic feedback to balance cooling. Key points:



Perseus cluster (NRAO/VLA/G. Taylor)

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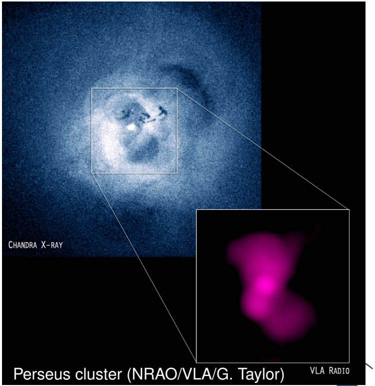
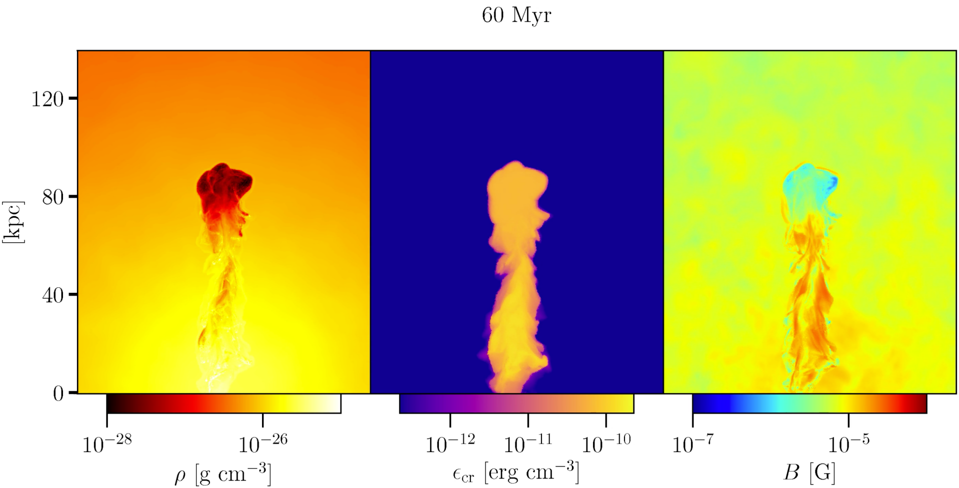
- energy source: release of non-gravitational energy due to accretion on a black hole and its spin
- jet interaction with magnetized cluster medium $\rightarrow$ turbulence
- jet accelerates relativistic particles (cosmic rays, CRs) $\rightarrow$ release from bubbles provides source of heat
- self-regulated heating mechanism to avoid overcooling



Perseus cluster (NRAO/VLA/G. Taylor)

Jet simulation: gas density, CR energy density, B field



Ehlert, Weinberger, CP+ (2018)

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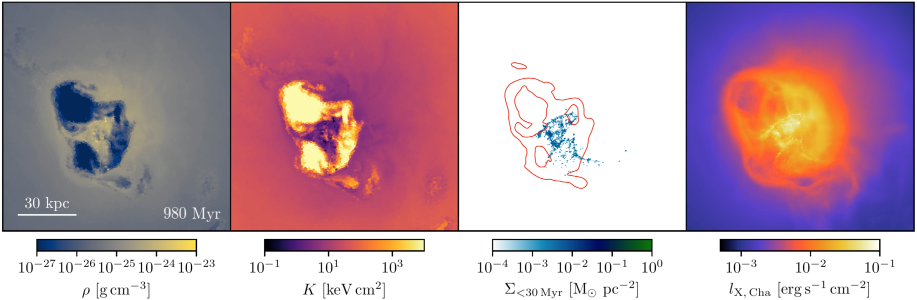
Simulating self-regulated AGN jet feedback

Jet simulation: gas density, entropy, cold gas ($t_{\rm cool} < 30$ Myr), X-ray surface brightness



Ehlert, Weinberger, CP+ (2022), movie by Jlassi

AGN feedback – energetics

- The mass of the stellar bulge and the SMBH obey the correlation

$$M_{\rm SMBH} \sim 0.005 M_{\rm bulge},$$

so that we obtain typical SMBH masses at the centers of clusters according to

$$M_{\star,\rm BCG} \sim 10^{12}\,\mathrm{M}_\odot \quad \Rightarrow \quad M_{\rm SMBH} \sim 5 \times 10^9 \mathrm{M}_\odot,$$

upon identifying $M_{\star,\rm BCG}$ with the bulge mass. This compares well with the latest mass measurement of the SMBH in M87 of $6 \times 10^9\,\mathrm{M}_\odot$ (M87 is the BCG in Virgo, our closest galaxy cluster with $D_{\rm Virgo} \sim 17$ Mpc).

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- The accretion power onto the SMBH can be estimated by the release of gravitational energy with a radiative efficiency of $\eta \sim 0.1$,

$$E_{\rm AGN} \sim \eta M_{\rm SMBH} c^2 \sim 10^{63} \left(\frac{M_{\rm SMBH}}{5\times 10^9\,\mathrm{M}_\odot}\right) \mathrm{erg}$$

$$\frac{E_{\rm AGN}\, m_{\rm p}}{M_{\rm gas}} \sim \frac{10^{63}\,\mathrm{erg}\, m_{\rm p}}{10^{14}\,\mathrm{M}_\odot} \sim \frac{10^{63-14-24-33}}{2}\, \frac{\mathrm{erg}}{\mathrm{nucleon}} \sim 5\, \frac{\mathrm{keV}}{\mathrm{nucleon}}.$$

From the energetic viewpoint, this is a much more promising heating source in comparison to supernova feedback.

AGN feedback – thermodynamics

- Relativistic jets displace the ICM at the location of the cavities, i.e. they do *pdV* work against the ICM, as well as supply internal energy to the cavities.
- The total energy required to create the cavity equals its enthalpy:

$$H = U + PV = \frac{1}{\gamma_b - 1} PV + PV = \frac{\gamma_b}{\gamma_b - 1} PV = 4PV, \text{ with } \gamma_b = 4/3$$

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of which only $1PV$ is directly available for mechanical work on the surroundings ($3PV$ is stored as internal energy).

- The work done on the ambient ICM by 2 bubbles in one outburst is

$$W = PV = 2\,\frac{4}{3}\pi r^3\, n_{\mathrm{ICM}} kT \sim 10^{59}\ \mathrm{erg}$$

where $r \sim 20\,\mathrm{kpc}$, $n_{\mathrm{ICM}} \sim 10^{-2}\,\mathrm{cm}^{-3}$, and $kT \sim 3$ keV.

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- To obtain a heating rate, we estimate the heating time scale which corresponds to the bubble's rise time. There are three different ways, using 1. the sound crossing time, 2. the buoyant rise time, and 3. the time required for the ambient medium to refill the displaced volume as the bubble rises upward.

AGN feedback: sound crossing and refill time

- The **sound crossing time** of the distance from the cavity center to the SMBH (using $\gamma_{\mathrm{a}} = 5/3$ for the ambient ICM) is given by

$$t_{\mathrm{s}} = \frac{R}{c_{\mathrm{s}}} = R\sqrt{\frac{\mu m_{\mathrm{p}}}{\gamma_{\mathrm{a}} k_{\mathrm{B}} T}} \approx 3.5 \times 10^{7} \left(\frac{R}{40\,\mathrm{kpc}}\right) \left(\frac{k_{\mathrm{B}} T}{3\,\mathrm{keV}}\right)^{-1/2} \mathrm{yr}.$$

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- The **time required to refill the volume** as the bubble rises upward is

$$t_\mathrm{refill} \approx 2R\sqrt{\frac{r}{GM(R)}} \approx t_\mathrm{s}\sqrt{\frac{2\gamma_\mathrm{a} r}{R}} \approx 1.3 t_\mathrm{s}\left(\frac{2r}{R}\right)^{1/2}.$$

In the third step, we used the potential of the SIS, $\Phi_\mathrm{SIS} = GM/R = 2k_\mathrm{B}T/(\mu m_\mathrm{p})$.

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AGN feedback: buoyancy time scale

- The **buoyancy time** is estimated by computing the buoyancy force acting upon the bubble
$$\boldsymbol{F}_{\rm buoy} = -\boldsymbol{g} V (\rho_{\rm a} - \rho_{\rm b}),$$
where $\boldsymbol{g}$ is the gravitational acceleration (assuming hydrostatic equilibrium of the ambient gas), V is the bubble volume, $\rho_{\rm a}$ and $\rho_{\rm b}$ denote the mass density of the ambient gas and the bubble, respectively.

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- The ram pressure exerts a drag force on the bubble, oppositely directed to the rise velocity,
$$\boldsymbol{F}_{\mathrm{drag}} = -\frac{C}{2}\sigma\rho_{\mathrm{a}}v^2\frac{\boldsymbol{v}}{v},$$
where σ is the cross section of the bubble, C is the drag coefficient that depends on bubble geometry and Reynolds number (i.e., whether the flow is turbulent or laminar): $C \approx 0.6$ for a Mach number $\mathcal{M} \approx 0.7$.

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- In equilibrium, the terminal velocity is obtained via $|\boldsymbol{F}_{\mathrm{buoy}}| = |\boldsymbol{F}_{\mathrm{drag}}|$, yielding
$$v = \sqrt{\frac{2gV}{\sigma C}\frac{\rho_{\mathrm{a}} - \rho_{\mathrm{b}}}{\rho_{\mathrm{a}}}} \approx \sqrt{\frac{2gV}{\sigma C}},$$
where we assumed $\rho_{\mathrm{b}} \ll \rho_{\mathrm{a}}$ in the last step. For a singular isothermal sphere (SIS), we can write down $g \approx v_{\mathrm{c}}^2/R = 2\sigma^2/R = 2k_{\mathrm{B}}T/(\mu m_{\mathrm{p}} R)$. With $\sigma = \pi r^2$ and $V = 4\pi r^3/3$, we obtain
$$t_{\mathrm{buoy}} \approx \frac{R}{v} = R\sqrt{\frac{\sigma C}{2gV}} \approx t_{\mathrm{s}}\sqrt{\frac{3C\gamma_{\mathrm{a}}}{16}\frac{R}{r}} \approx 0.6 t_{\mathrm{s}}\left(\frac{R}{2r}\right)^{1/2}.$$

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AGN feedback – luminosity

- We found that the energy release time scale is of order the sound crossing time $\sim$ buoyant rise time $\sim$ refill time of displaced bubble volume $\sim 3 \times 10^7$ yr.

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- We found that the energy release time scale is of order the sound crossing time $\sim$ buoyant rise time $\sim$ refill time of displaced bubble volume $\sim 3 \times 10^7$ yr.
- The AGN heating rate is

$$L_{\mathrm{AGN}} \sim \frac{PV}{t_{\mathrm{buoy}}} \sim \frac{10^{59}\,\mathrm{erg}}{10^{15}\,\mathrm{s}} \sim 10^{44}\,\frac{\mathrm{erg}}{\mathrm{s}} \sim L_X$$

i.e., the heating rate is comparable to the X-ray luminosity $\sim$ cooling rate.

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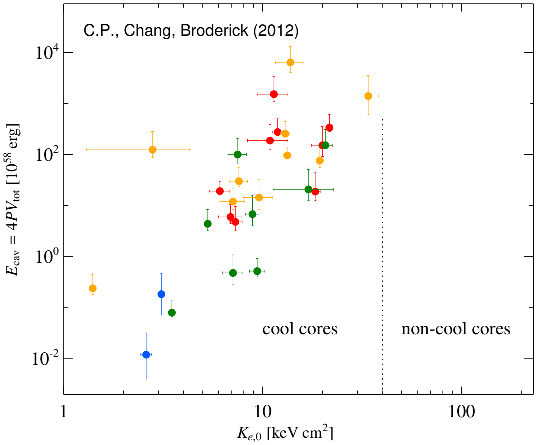
AGN feedback – luminosity

- We found that the energy release time scale is of order the sound crossing time $\sim$ buoyant rise time $\sim$ refill time of displaced bubble volume $\sim 3 \times 10^7$ yr.
- The AGN heating rate is

$$L_{\mathrm{AGN}} \sim \frac{PV}{t_{\mathrm{buoy}}} \sim \frac{10^{59}\,\mathrm{erg}}{10^{15}\,\mathrm{s}} \sim 10^{44}\,\frac{\mathrm{erg}}{\mathrm{s}} \sim L_X$$

  i.e., the heating rate is comparable to the X-ray luminosity $\sim$ cooling rate.
- This is a necessary condition for balancing X-ray cooling losses and increasing the core entropy $K_e = kT/n_e^{2/3}$ of the ambient ICM!

How efficient is heating by AGN feedback?



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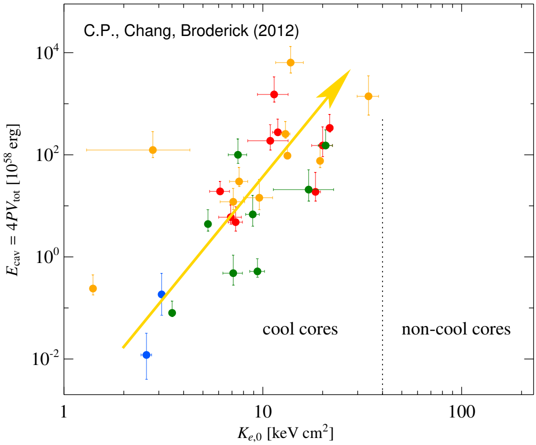

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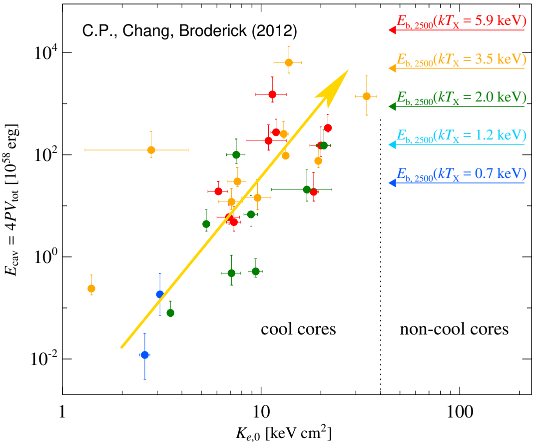

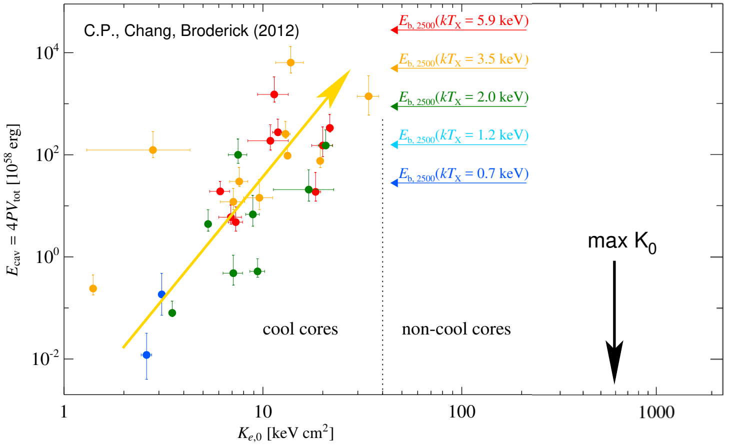

AGNs cannot transform CC to NCC clusters (on a buoyancy timescale)

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- The AGN jets can reach across these scales, but the heating mechanism also needs to provide self-regulation across these scales!

AIP

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  * Cooling fastest in the cluster centers
  * Cool core (CC) cluster: central cooling time $t_{\rm cool} < 1$ Gyr and central entropy $K_0 < 30$ keV cm^2
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- **AGN jet feedback**
  * promising mechanisms for self-regulated feedback
  * energetics and heating rate sufficient for balancing cooling losses, but not for transforming CC to NCC clusters
  * many open questions regarding the specific heating mechanism and tuning of self-regulation across 8 orders of magnitude