The Physics of Galaxy Clusters

13th Lecture

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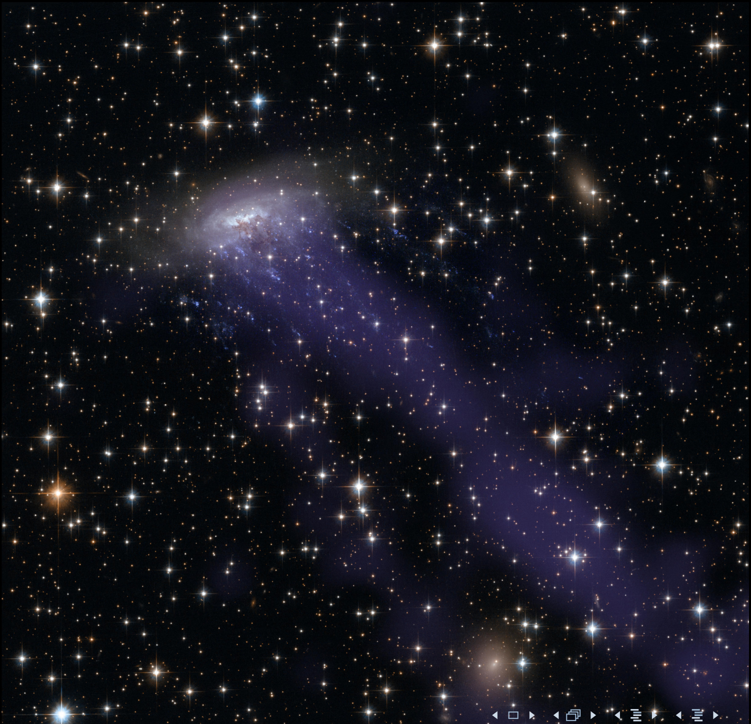
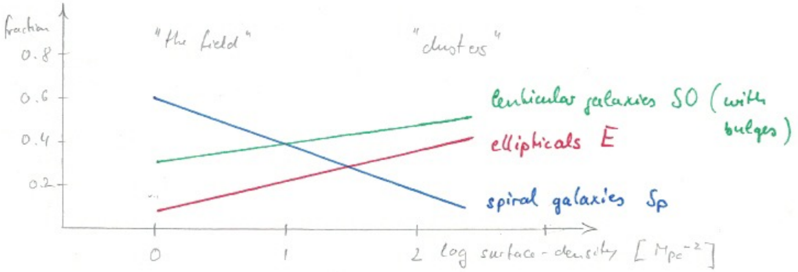
Lectures in the International Astrophysics Masters Program at Potsdam University



Morphology-density relation



- plot shows fraction of galaxy types versus environmental density

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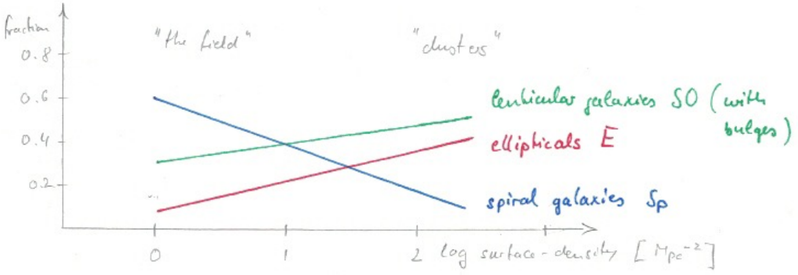

- plot shows fraction of galaxy types versus environmental density
- ***morphology-density relation:*** decreasing fraction of spirals and increasing fraction of ellipticals and galaxies with bulges at higher densities ($\rightarrow$ clusters)

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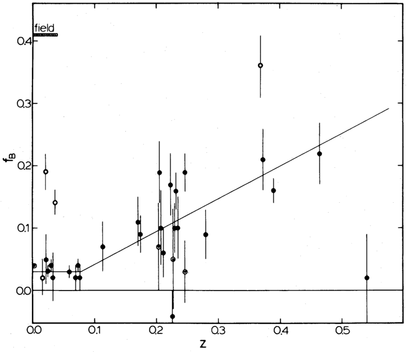
Butcher-Oemler effect



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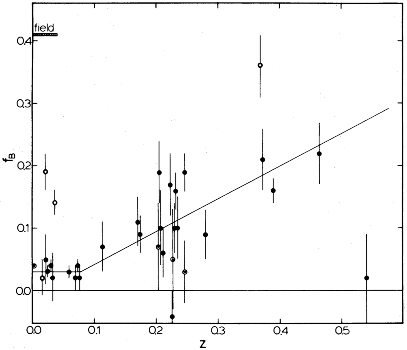

- plot shows fraction of blue galaxies versus redshift
- ***Butcher-Oemler effect*** (1987, 1984):
  cluster galaxies at intermediate redshifts ($0.3 \lesssim z \lesssim 0.5$) have increased fraction of blue galaxies in comparison to present-day clusters

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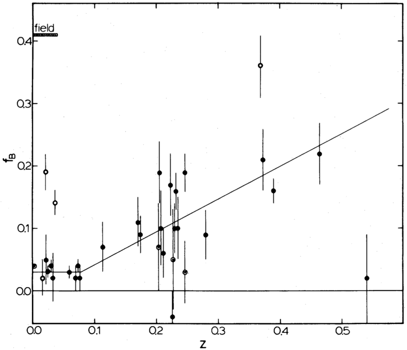

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  cluster galaxies at intermediate redshifts ($0.3 \lesssim z \lesssim 0.5$) have increased fraction of blue galaxies in comparison to present-day clusters
- effect is associated with an increasing spiral and star forming fraction with increasing redshift $\Rightarrow$ strength of morphology-density relation redshift dependent

Galaxy transformations in clusters

Galaxy transformations in clusters

- ***tidal interactions*** with other cluster galaxies or with the cluster potential can torque the stellar orbits and the gas distribution and cause a disturbed non-equilibrium morphology or even transform its type
- ***dynamical friction*** can cause the galaxy to slowly migrate to the cluster center and eventually to merge with the *brightest cluster galaxy (BCG)* that resides in the cluster center
- ***ram-pressure interactions*** of a galaxy with the hot, X-ray emitting ICM severely impacts the interstellar medium of a galaxy

Re-ignition of a passive elliptical

- where in a galaxy cluster is this happening and why is this condition not fulfilled for typical cluster ellipticals?

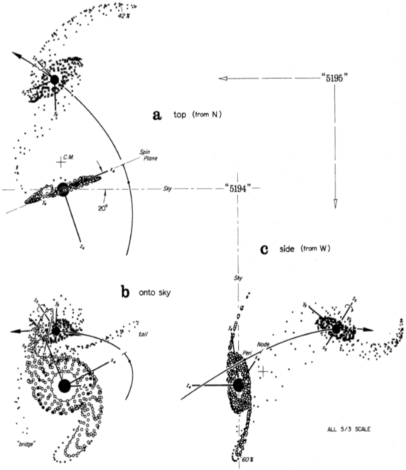
Re-ignition of a passive elliptical

- where in a galaxy cluster is this happening and why is this condition not fulfilled for typical cluster ellipticals?
- this needs a sufficient amount of cold gas: small $t_{\rm cool}$, low galaxy velocity relative to the ICM $\Rightarrow$ cD galaxies in the centers of cool core clusters


Antennae Galaxies

The merger hypothesis

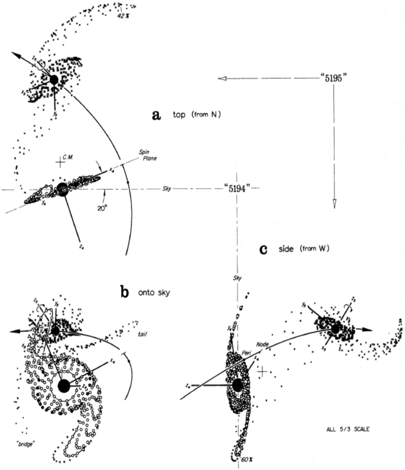
- the ***famous “merger hypothesis”*** suggests that strangely deformed pairs of galaxies result from galaxy mergers, which eventually leads to elliptical galaxies (Toomre & Toomre 1972)



A model for the interaction of M51 and NGC 5195

The merger hypothesis

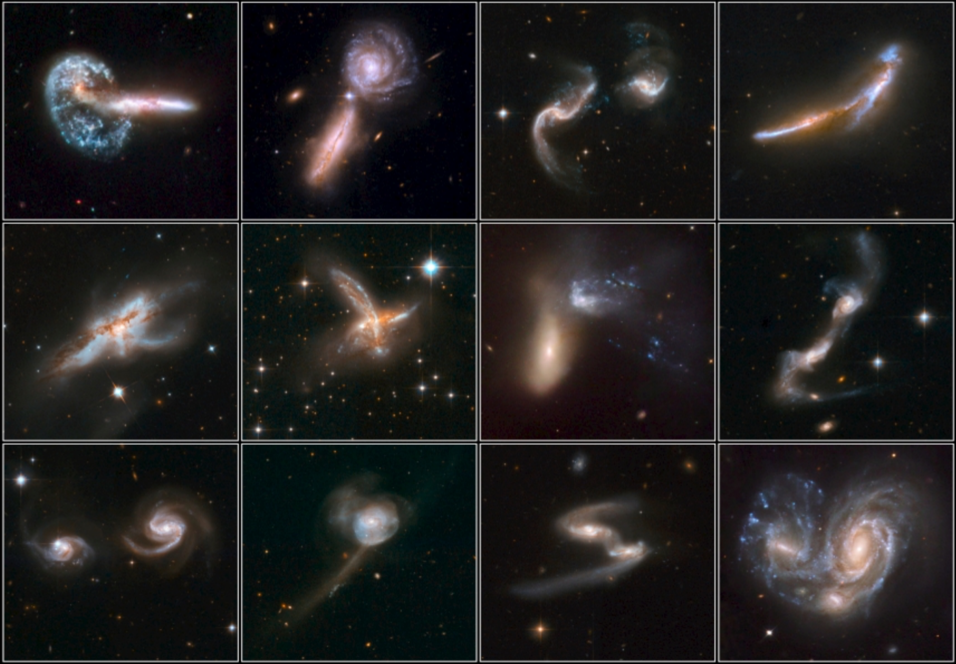
- the ***famous "merger hypothesis"*** suggests that strangely deformed pairs of galaxies result from galaxy mergers, which eventually leads to elliptical galaxies (Toomre & Toomre 1972)
- ***tidal interactions*** between group/cluster galaxies are mainly responsible for this transition
- other pathways (***ram-pressure stripping, dynamical friction***) can also transform spirals to passive lenticulars/ellipticals



A model for the interaction of M51 and NGC 5195

Interacting Galaxies

Hubble Space Telescope • ACS/WFC • WFPC2

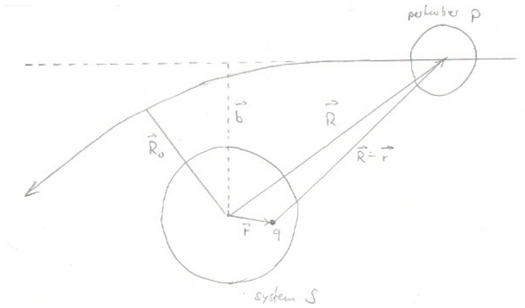


NASA, ESA, the Hubble Heritage (AURA/STScI)-ESA/Hubble Collaboration, and A. Evans (University of Virginia, Charlottesville/NRAO/Stony Brook University)

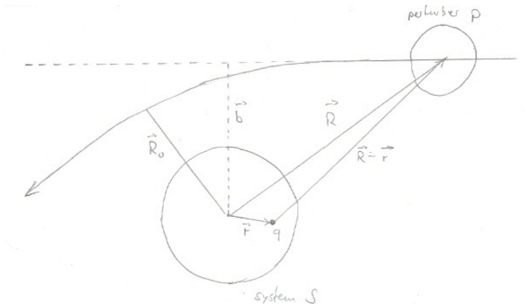
STScI-PRC08-16a

movie by Frank Summers (STScI)

Tidal interactions – 1



- ***tidal interactions between two systems:*** a perturber *P* modifies the orbit of a particle *q* in system *S* as a result of the gravitational interaction



- ***tidal interactions between two systems:*** a perturber P modifies the orbit of a particle q in system S as a result of the gravitational interaction
- the gravitational force due to P is not uniform over the body $S \Rightarrow$ particle q experiences a tidal force per unit mass,

$$\tilde{\boldsymbol{F}}_{\text{tide}}(\boldsymbol{r}) = -\boldsymbol{\nabla}\phi_p(|\boldsymbol{R}-\boldsymbol{r}|) + \boldsymbol{\nabla}\phi_p(\boldsymbol{R})$$

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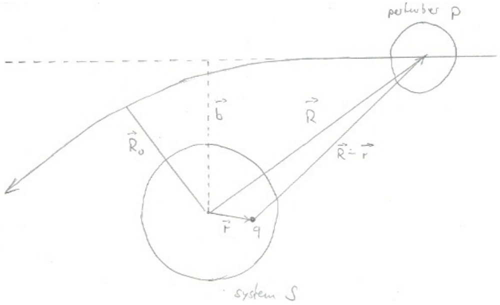

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$$\tilde{\boldsymbol{F}}_{\mathrm{tide}}(\boldsymbol{r}) = -\boldsymbol{\nabla}\phi_P(|\boldsymbol{R}-\boldsymbol{r}|) + \boldsymbol{\nabla}\phi_P(\boldsymbol{R})$$

- as a result, the particle q gains energy at a rate per unit mass

$$\frac{\mathrm{d}\epsilon_q}{\mathrm{d}t} = \boldsymbol{v}\cdot\tilde{\boldsymbol{F}}_{\mathrm{tide}}(\boldsymbol{r}),$$

where $\boldsymbol{v}$ is the velocity of q with respect to the center of S.

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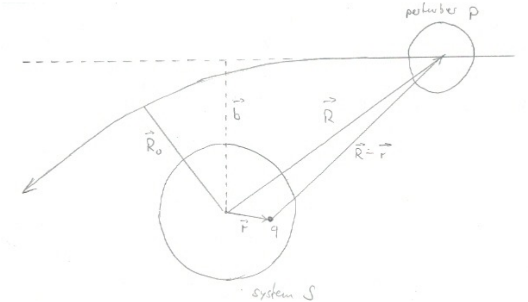
Tidal interactions – 2



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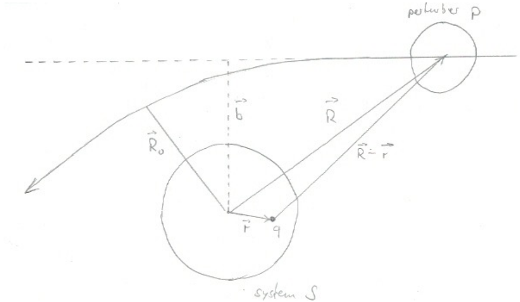

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- let t_tide be the time for the tide to rise and $t_\mathrm{enc} \approx R_\mathrm{max}/\Delta v_{SP}$ is the time of the encounter where Δv_{SP} is the relative velocity between S and P, $R_\mathrm{max} = \max(R_0, R_S, R_P)$, and R_0 is the minimum distance of the encounter

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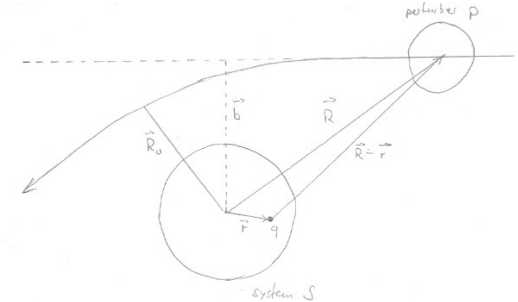

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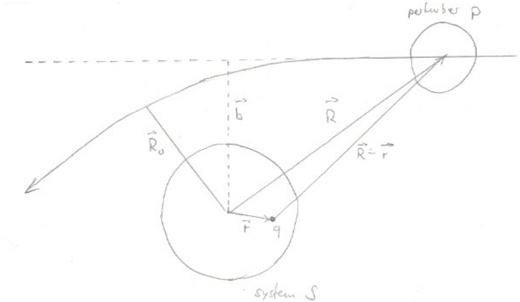

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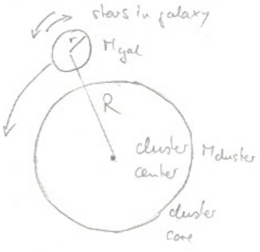
- $t_{\text{enc}} \gg t_{\text{tide}}$: internal structure of deformable bodies can adiabatically adjust to perturbation $\Rightarrow$ ***no transfer of energy***
- $t_{\text{enc}} < t_{\text{tide}}$: response of the bodies lags behind the instantaneous force $\Rightarrow$ ***transfer of orbital energy to internal energy of the two bodies,*** which increases their mutual binding energy

Tidal stripping – 1

- ***Tidal radius – the Roche problem:*** we consider a ***slow encounter*** and work out the "tidal radius" outside of which material can get stripped
- for simplification, we assume a circular galaxy orbit in a cluster ⇒ what is the fate of the stars inside this galaxy?



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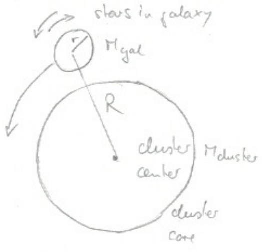
- the differential tidal force per unit mass between a star and the galaxy center is (by magnitude)

$$\tilde{F}_{\text{tide}}(R) = r\,\frac{\mathrm{d}}{\mathrm{d}R}\left(\frac{GM_{\text{cluster}}(R)}{R^2}\right)$$

- the restoring force from the galaxy is

$$\tilde{F}_{\text{gal}}(r) = -\frac{GM_{\text{gal}}(r)}{r^2}$$



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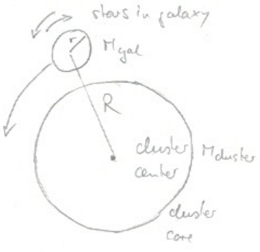

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- the tidal radius in the galaxy is defined by equating both forces, $\tilde{F}_{\mathrm{tide}} = \tilde{F}_{\mathrm{gal}}$:

$$\frac{M_{\mathrm{gal}}(r_{\mathrm{tide}})}{r_{\mathrm{tide}}^3} = -\frac{\mathrm{d}}{\mathrm{d}R}\left(\frac{M_{\mathrm{cluster}}(R)}{R^2}\right) = \left(2 - \frac{\mathrm{d}\ln M_{\mathrm{cluster}}}{\mathrm{d}\ln R}\right)\frac{M_{\mathrm{cluster}}}{R^3}$$

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Tidal stripping – 2

- let's recall the equation for the tidal radius on the previous slide:

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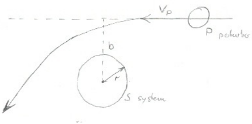
  $\Rightarrow$ internal (stellar) distributions can adiabatically adjust to the perturbation because the dynamical time of these stars is smaller
- ***outside*** r_{tide}***, stars are only loosely bound so that they can be "stripped off"*** by tidal forces exerted by the cluster core on the stars in the galaxy
  $\Rightarrow$ resonance condition where the period of the stellar orbit in the galaxy at r_{tide} matches the period of the galaxy orbit inside the clusters:

$$P_\star(r_{\mathrm{tide}}) \approx P_{\mathrm{gal}}(R)$$

Tidal shocks

- now, we treat a ***rapid encounter*** in the impulse approximation and assume that the time of passing t_{enc} is much smaller than the internal dynamical time t_{tide}
- what is the relative change in energy due to an impulsive encounter?



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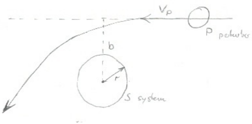

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where we used $\bar\rho_P(b) = \frac{M_P(b)}{\frac{4}{3}\pi b^3}, \quad \bar\rho_S(r) = \frac{M_S(r)}{\frac{4}{3}\pi r^3}, \quad v_{\mathrm{parabolic}}^2 = \frac{GM_P}{b}$

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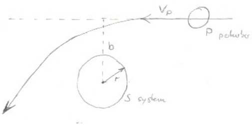

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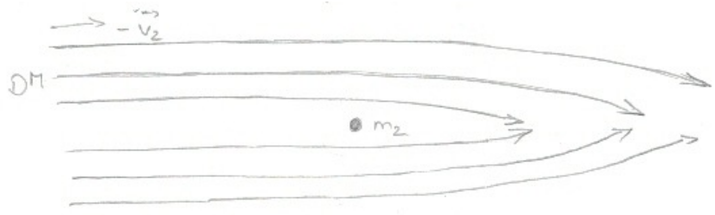
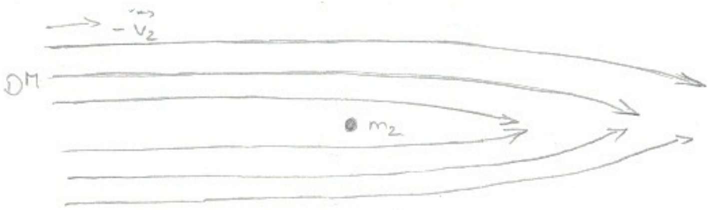
- discussion:
  - if $v_P \sim v_{\rm parabolic}$, we obtain $\Delta\epsilon \sim \epsilon$ for $\rho_P \sim \rho_S$, i.e., we recover the Roche criterion
  - the formalism only applies to fast encounters, i.e., down to $r_{\rm tide}$, below which the adiabatic case applies
  - the cumulative effect of many encounters can strip stars to radii $r \ll r_{\rm tide}$

Dynamical friction – 1



- when a heavy object of mass m_2 (e.g., a galaxy) moves through a large collisionless system that consists of particles of mass $m_1 \ll m_2$ (e.g., dark matter particles), it experiences a drag force that is called dynamical friction
  $\Rightarrow$ transfer of energy and momentum from the galaxy to the dark matter particles



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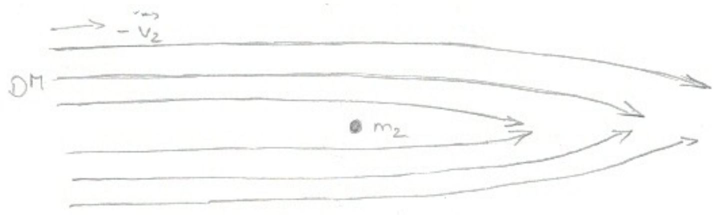
- ***reason: system evolves towards thermodynamic equilibrium*** through energy exchange by means of two-body encounters
- if particles have different masses, thermodynamical equilibrium implies $m_1 \langle v_1^2 \rangle = m_2 \langle v_2^2 \rangle$; because $m_2 \gg m_1$ and dark matter particles at the same radius have similar orbital velocities, the galaxy has usually a larger kinetic energy than the dark matter particles it encounters
  $\Rightarrow$ ***galaxy loses net energy and momentum***

Dynamical friction – 2



- an alternative way to understand dynamical friction is to move in the rest system of the galaxy
- in this frame, dark matter particle are deflected through the gravity of the galaxy to form an overdensity of dark matter in the wake (a so-called “trailing enhancement”)
  $\Rightarrow$ ***the gravitational pull of this wake on the galaxy slows it down***

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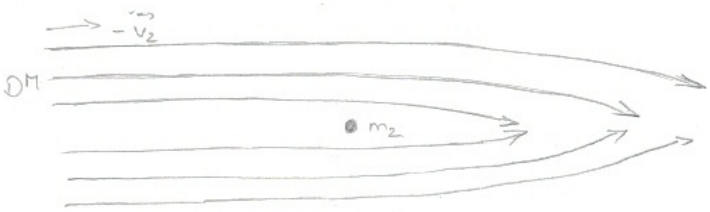

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- in real systems with a gravitational potential, dynamical friction produces mass segregation and not equipartition: consider a massive galaxy on a circular orbit within a cluster that experiences dynamical friction
  $\Rightarrow$ ***its energy loss causes it to transition inward to a tighter bound orbit and to move faster***

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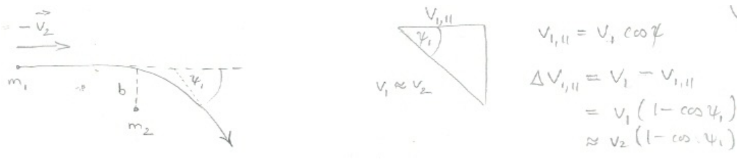
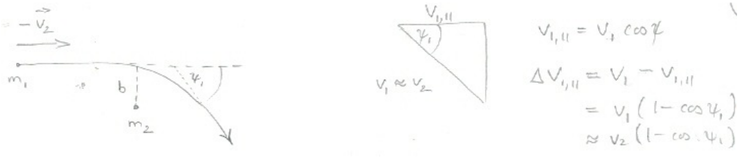
The Chandrasekhar formula – 1



- in a single encounter, the deflection angle due to gravity is (in the rest system of the galaxy with mass m_2)

$$\psi_1 = \frac{\Delta v_\perp}{v_2} \approx \underbrace{\frac{G m_2}{b^2}}_{|\tilde{\boldsymbol{F}}_\perp|} \underbrace{\frac{2b}{v_2}}_{t} \frac{1}{v_2} = \frac{2 G m_2}{b v_2^2}. \qquad (2)$$

The Chandrasekhar formula – 1



- in a single encounter, the deflection angle due to gravity is (in the rest system of the galaxy with mass m_2)

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- momentum balance of the deflection yields (with $m_1 \ll m_2$, see figure):

$$\Delta v_{2,\parallel} = -\frac{m_1}{m_2} \Delta v_{1,\parallel} = -\frac{m_1}{m_2} v_2 (1 - \cos\psi_1) \approx -\frac{2 G^2 m_1 m_2}{b^2 v_2^3}, \tag{3}$$

upon Taylor expanding $\cos\psi_1 = 1 - \psi_1^2/2 + \mathcal{O}(\psi_1^4)$ and substituting Eq. (2), and we assume small-angle scatterings that dominate the total scattering rate

The Chandrasekhar formula – 2

- only particles with speed $v_1 < v_2$ contribute to dynamical friction and are deflected by the galaxy with a differential rate that is given by

$$d\Gamma = n(< v_2) v_2 d\sigma$$

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AIP

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- using Eq. (3), $\Delta v_{2,\parallel} \approx -\frac{2G^2 m_1 m_2}{b^2 v_2^3}$, we obtain for the rate of change of the galaxy velocity

$$\begin{aligned}\frac{\mathrm{d}\boldsymbol{v}_2}{\mathrm{d}t} &= \int_{b_{\mathrm{min}}}^{b_{\mathrm{max}}} \Delta v_{2,\parallel} n(< v_2) v_2 2\pi b \mathrm{d}b \times \frac{\boldsymbol{v}_2}{|\boldsymbol{v}_2|} \\ &= -4\pi G^2 m_2 \rho(< v_2) \ln \Lambda \frac{\boldsymbol{v}_2}{|\boldsymbol{v}_2|^3},\end{aligned}$$

where $\Lambda = b_{\mathrm{max}}/b_{\mathrm{min}}$, $\rho(< v_2) = m_1 n(< v_2)$ is the mass density of light particles moving slower than v_2

AIP

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- symmetry causes the galaxy only to slow down and not to change direction

The Chandrasekhar formula – discussion

Discussion of the Chandrasekhar formula between galaxies and dark matter (DM):

$$\frac{\mathrm{d}\boldsymbol{v}_{\mathrm{gal}}}{\mathrm{d}t} = -4\pi G^2 m_{\mathrm{gal}} \rho(< v_{\mathrm{gal}}) \ln\Lambda \frac{\boldsymbol{v}_{\mathrm{gal}}}{|\boldsymbol{v}_{\mathrm{gal}}|^3} \tag{4}$$

- the rate of change of the galaxy velocity, $\dot{v}_{\mathrm{gal}}$, is ***independent of the DM mass*** and only depends on the mass density ρ

AIP

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AIP

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- we assume $v_{\rm gal} \ll \sigma_{\rm DM}$, where $\sigma_{\rm DM}$ is the DM velocity dispersion and a Boltzmann distribution for DM, $f(r,\boldsymbol{v}) = n(r)(2\pi\sigma_{\rm DM}^2)^{-3/2}\exp(-\boldsymbol{v}^2/2\sigma_{\rm DM}^2)$:

$$f \propto \exp\left(-\frac{v^2}{2\sigma_{\rm DM}^2}\right) \to 1, \quad \text{for} \quad v_{\rm gal} \ll \sigma_{\rm DM}$$

$$\Longrightarrow \quad \rho(< v_{\rm gal}) = 4\pi m_{\rm DM} \int_0^{v_{\rm gal}} f(r,v) v^2 \mathrm{d}v \propto \frac{v_{\rm gal}^3}{\sigma_{\rm DM}^3} \quad (5)$$

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- substituting Eq. (5) into Eq. (4) yields

$$\dot{v}_{\rm gal} \propto v_{\rm gal} \quad \text{for} \quad v_{\rm gal} \ll \sigma_{\rm DM},$$

  $\Rightarrow$ ***this is also called Stokes friction***

Orbit decay through dynamical friction – 1

- we assume a singular isothermal sphere model for DM

$$\rho(r) = \frac{\sigma_1^2}{2\pi G r^2}, \quad M(r) = \frac{2\sigma_1^2}{G} r,$$

and adopt a galaxy circular orbit in the cluster $\forall r$:

$$\frac{v_c^2}{r} = \frac{GM(r)}{r^2} \quad \Rightarrow \quad v_c = \sqrt{2}\sigma_1$$

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- if we assume that dark matter locally follows a Maxwellian velocity distribution, the phase space distribution is given by

$$f(r, v) = \frac{n(r)}{\left(2\pi\sigma_1^2\right)^{3/2}} \exp\left(-\frac{v^2}{2\sigma_1^2}\right)$$

so that we obtain

$$\begin{aligned} n(r, < v_2) &= 4\pi \int_0^{v_2} f(r, v) v^2 \mathrm{d}v \\ &= n(r) \left[\mathrm{erf}\left(\frac{v_2}{v_c}\right) - \frac{2}{\sqrt{\pi}} \frac{v_2}{v_c} \exp\left(-\frac{v_2^2}{v_c^2}\right)\right] = 0.43\, n(r), \end{aligned}$$

because on circular orbits in a singular isothermal sphere $v_2 = v_c$.

AIP

Orbit decay through dynamical friction – 2

- the galaxy's binding energy per unit mass is

$$\frac{\Phi(r)}{M(r)} = \frac{G m_{\mathrm{shell}}(r)}{r} = 4\pi G \int_{r_0}^{r} \frac{r'^2 \rho(r') \mathrm{d}r'}{r'} = 2\sigma_1^2 \int_{r_0}^{r} \frac{\mathrm{d}r'}{r'} = 2\sigma_1^2 \ln \frac{r}{r_0}$$

so that the galaxy looses energy per unit mass at a rate

$$\dot{\epsilon} = \frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\Phi(r)}{M(r)}\right) = \frac{2\sigma_1^2}{r}\frac{\mathrm{d}r}{\mathrm{d}t} \tag{6}$$

AIP

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$$\dot{\epsilon} = \boldsymbol{v}_2 \cdot \tilde{\boldsymbol{F}}_2 = \boldsymbol{v}_2 \cdot \frac{\mathrm{d}\boldsymbol{v}_2}{\mathrm{d}t} = -4\pi G m_2 \ln\Lambda \, \frac{0.43\sigma_1^2}{2\pi G r^2} \frac{1}{\sqrt{2}\sigma_1} = -0.61 \frac{G m_2 \sigma_1 \ln\Lambda}{r^2} \tag{7}$$

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- equating Eqs. (6) and (7) yields

$$\frac{2\sigma_1^2}{r}\frac{\mathrm{d}r}{\mathrm{d}t} = -0.61 \frac{G m_2 \sigma_1 \ln\Lambda}{r^2}, \quad \text{or}$$

$$\frac{\mathrm{d}r^2}{\mathrm{d}t} = 2r\frac{\mathrm{d}r}{\mathrm{d}t} = -0.61 \frac{G m_2 \ln\Lambda}{\sigma_1} = -\frac{G m_2 \ln\Lambda}{1.64\sigma_1 r_0^2} r_0^2 \equiv -\frac{r_0^2}{t_{\rm dec}}$$

Orbit decay through dynamical friction – 3

- recall the last equation on the previous slide:

$$\frac{\mathrm{d}r^2}{\mathrm{d}t} = -\frac{r_0^2}{t_\mathrm{dec}}, \quad \text{where} \quad t_\mathrm{dec} = 1.64\,\frac{\sigma_1 r_0^2}{G m_\mathrm{gal} \ln\Lambda} = \frac{0.185}{\ln\Lambda}\,\frac{M(r_0)}{m_\mathrm{gal}}\,t_\text{orbit, init}$$

and $t_\text{orbit, init} = 2\pi r_0 / v_2 = \sqrt{2}\pi r_0/\sigma_1$ and $m_2 = m_\mathrm{gal}$

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  which means that larger galaxies migrate faster to the center

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AIP

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  $\Rightarrow$ ***only the most massive galaxies are expected to sink to the center*** and to segregate by mass due to dynamical friction

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  ⇒ ***only the most massive galaxies are expected to sink to the center*** and to segregate by mass due to dynamical friction

  ⇒ ***this causes them to merge with the brightest cluster galaxy*** in the center: "galactic cannibalism"

Ram pressure stripping – 1

- consider a ***gas-rich spiral galaxy*** with a disk of radius r_d moving through the ICM of mass density ρ_ICM with velocity $\boldsymbol{v}$
- for simplicity, we consider a face-on moving disk (i.e., $\boldsymbol{v}$ is parallel to the disk normal)

Ram pressure stripping in the galaxies NGC 4522 and NGC 4402

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$$\dot{p} = \dot{M}_\mathrm{ICM} v = \pi r_\mathrm{d}^2 \rho_\mathrm{ICM} v^2 = \pi r_\mathrm{d}^2 P_\mathrm{ram}$$

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Ram pressure stripping in the galaxies NGC 4522 and NGC 4402

$\Rightarrow$ ***the ICM wind exerts a ram pressure*** $P_\mathrm{ram} = \rho_\mathrm{ICM} v^2$ ***onto the disk***

Ram pressure stripping – 2

- if the ram pressure $P_{\rm ram} = \rho_{\rm ICM} v^2$ exceeds the force per unit area that binds the ISM to the disk, then the gas gets stripped off

Ram pressure stripping in the galaxies NGC 4522 and NGC 4402

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- ram-pressure stripping of gas occurs if $P_{\rm ram} > \dfrac{F_{\rm grav}}{A}$, or $\rho_{\rm ICM} > \dfrac{2\pi G \Sigma_\star \Sigma_{\rm ISM}}{v^2}$

AIP

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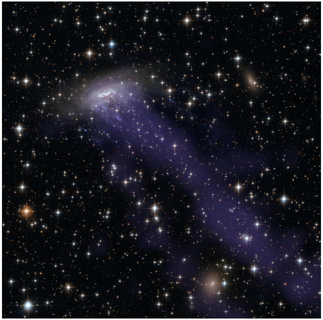
Ram pressure stripping in the galaxies NGC 4522 and NGC 4402

- ram-pressure stripping of gas occurs if $P_{\rm ram} > \dfrac{F_{\rm grav}}{A}$, or $\rho_{\rm ICM} > \dfrac{2\pi G \Sigma_\star \Sigma_{\rm ISM}}{v^2}$
- clearly this defines a minimum wind density of $n \gtrsim 3 \times 10^{-3}\ {\rm cm}^{-3}$ which is only reached towards cluster cores and not in cosmic filaments

Ram pressure stripping – 3

- $P_{\rm ram}$ will change as a function of time due to the eccentric orbit of a galaxy
- since $\Sigma_\star$ and $\Sigma_{\rm ISM}$ decrease to the disk periphery, there will be critical radius beyond which ram pressure stripping will be effective



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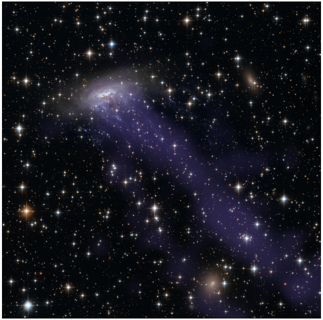

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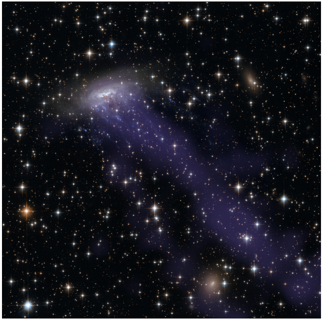

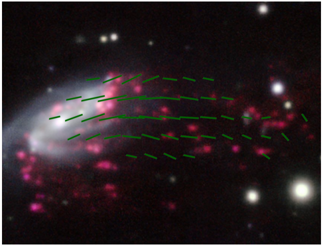

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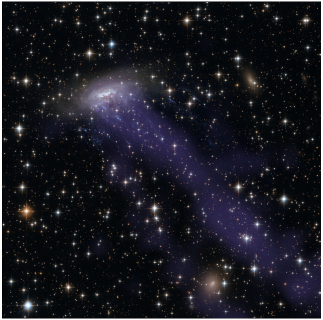
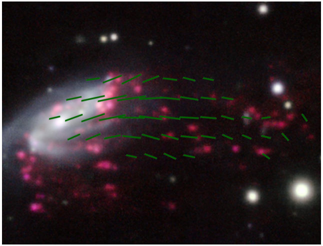
  $\Rightarrow$ ram-pressure stripping is considered important to explain the ***quenching of star formation in clusters***





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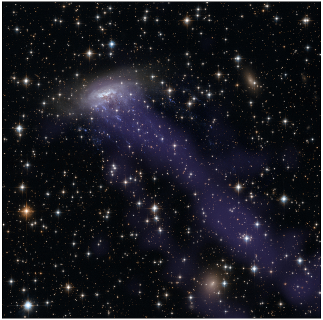
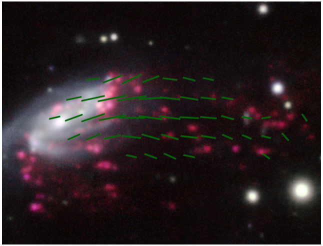
- if only parts of the outer disk gets stripped, star formation can continue until it runs out of its fuel after about a few Gyrs
  $\Rightarrow$ ***"strangulation" of a galaxy***





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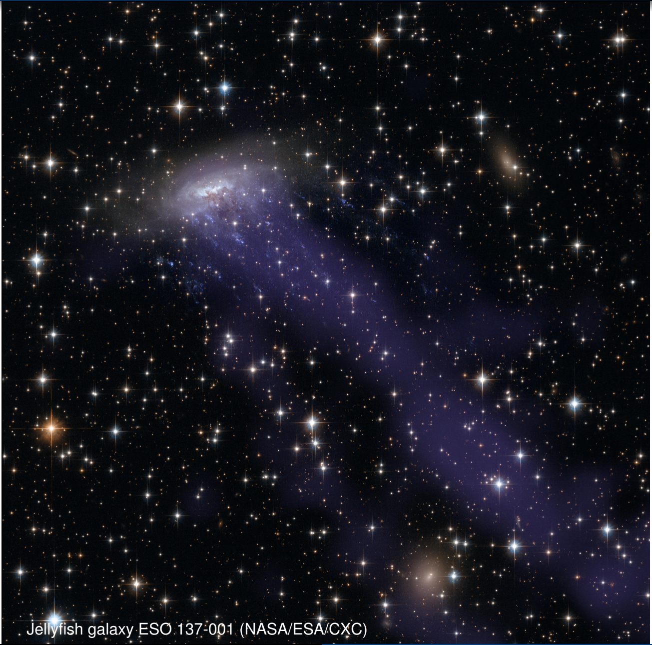
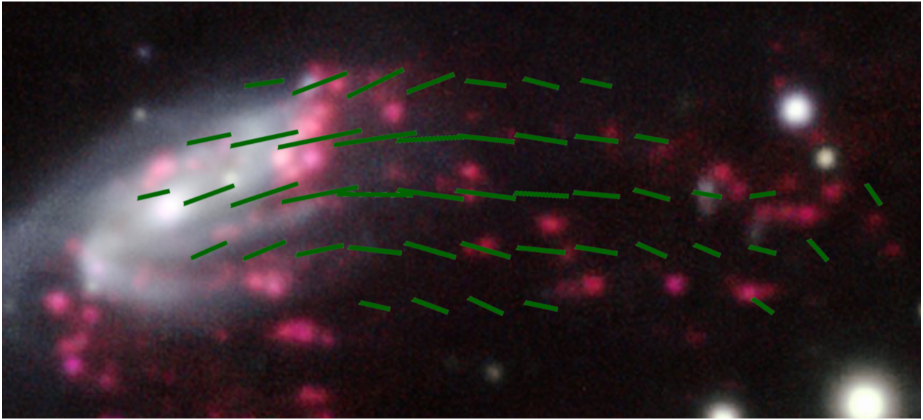
Jellyfish galaxies in clusters



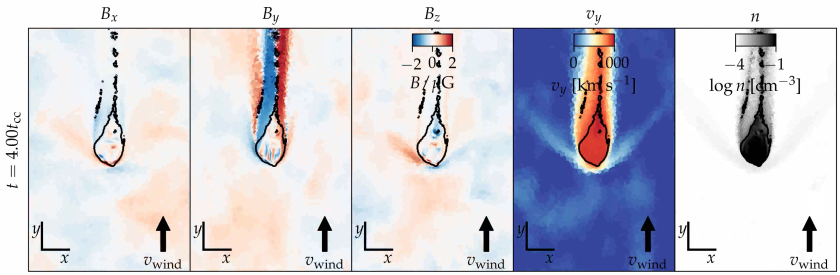
Jellyfish galaxy ESO 137-001 (NASA/ESA/CXC)

Protective layer: magnetic field of a jellyfish galaxy

Observations of aligned *B* polarization vectors along the tail of galaxy JO206



Müller+ (2021, Nature Astronomy)

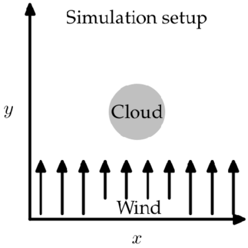


Müller+ (2021, Nature Astronomy), Sparre, CP+ (2020)

- a jellyfish galaxy experiences ram-pressure stripping as a result of its fast motion in the intracluster medium
- the turbulent wind magnetic field is wrapped around the galaxy and stretched in the wake by shear motions as well as cooling of thermally unstable mixed wind material
- the magnetic field facilitates the formation of long gaseous filaments

Interaction of a cold cloud with a hot wind

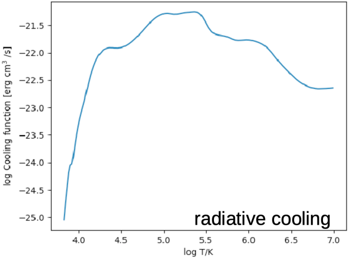
Magnetic tension and pressure modify the dynamics of the interaction



$T_{cloud} = 10^4$ K
$n_{cloud} = 0.1$ cm^{-3}
$R = 25$ pc

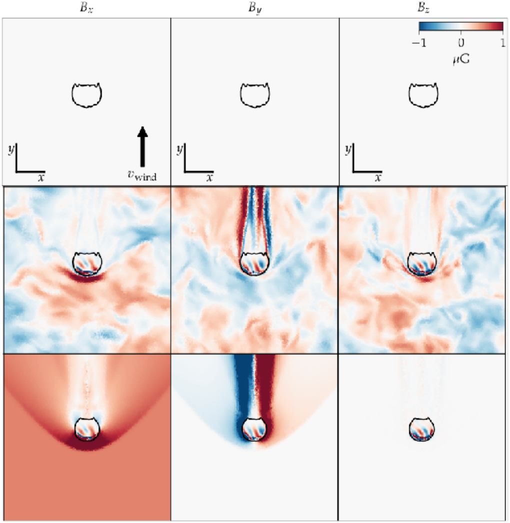
$T_{wind} = 5.0 \times 10^6$ K
$n_{wind} = 2 \times 10^{-4}$ cm^{-3}
$M = 1.5$

$\beta = 10$ or ∞ ($\beta \equiv P_{th}/P_B$)



Sparre, CP, Ehlert (2020)

Magnetic field configurations



no magnetic field

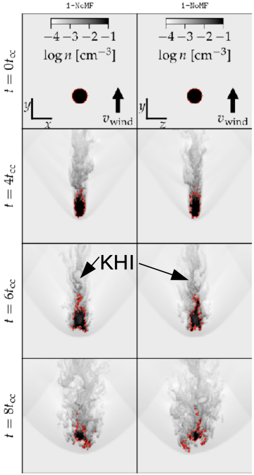
turbulent ***B***

uniform ***B***

Sparre, CP, Ehlert (2020)

Magnetic field alters dynamics of cloud shattering

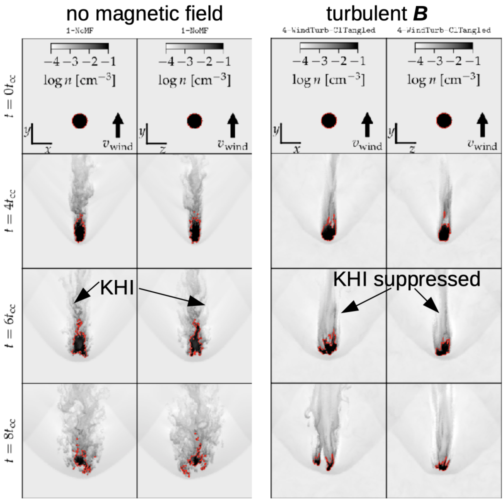
no magnetic field



Sparre, CP, Ehlert (2020)

KHI = Kelvin Helmholtz instability

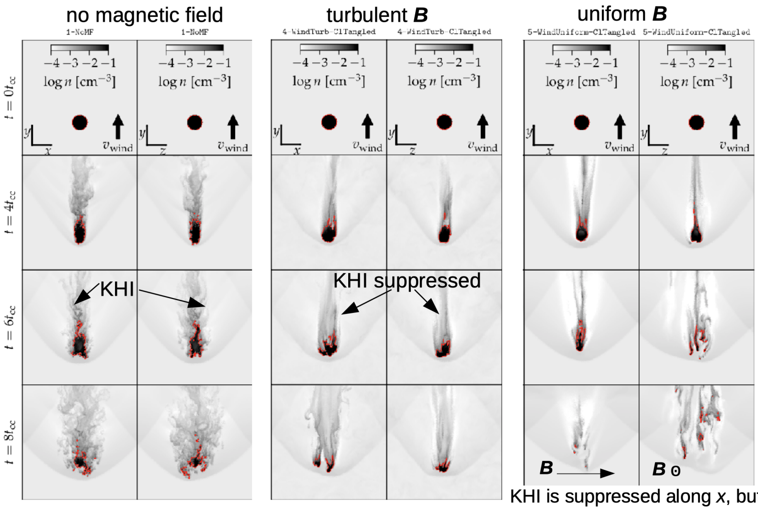
Magnetic field alters dynamics of cloud shattering



Sparre, CP, Ehlert (2020)

A magnetic field suppresses the Kelvin-Helmholtz instability (KHI) in the wake of the cloud

Magnetic field alters dynamics of cloud shattering

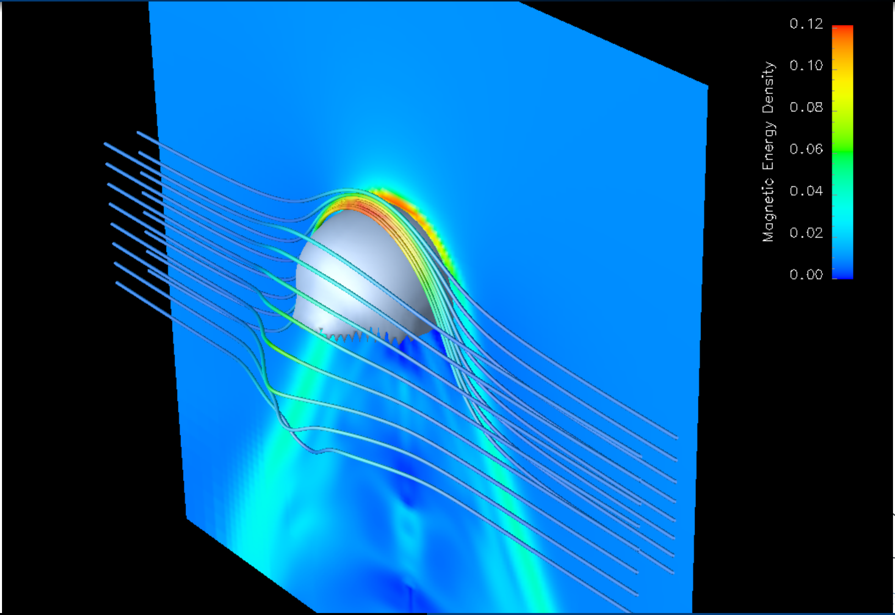


KHI is suppressed along x, but not z

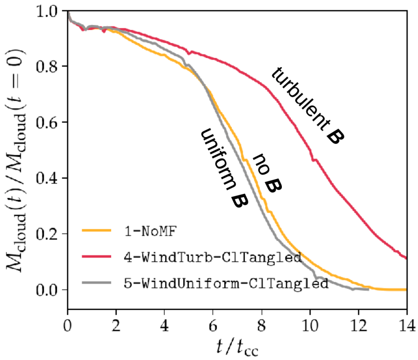
Sparre, CP, Ehlert (2020)

A magnetic draping layer protects against instabilities

Magnetic pressure and tension forces alter the dynamics of the interaction



A turbulent $\boldsymbol{B}$ field extends cloud's lifetime



$$t_{\rm cc} \equiv \frac{R_{\rm cloud}}{v_{\rm wind}} \sqrt{\frac{\rho_{\rm cloud}}{\rho_{\rm wind}}}.$$

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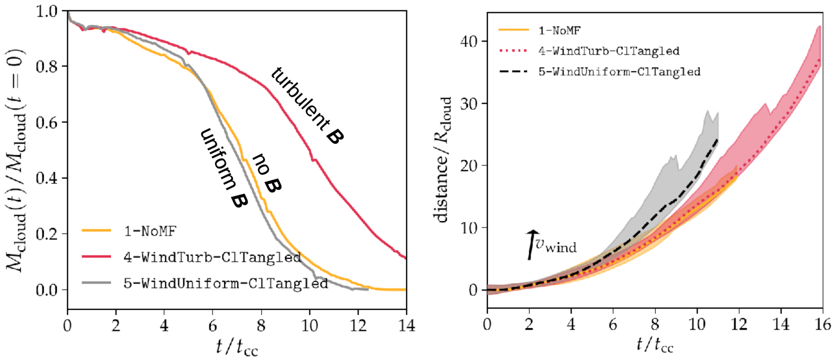
A uniform *B* field initially accelerates cloud more



- KHI instability shatters a **small cloud** into small pieces that mix with and dissolve into the hot wind
- magnetic field protects against instabilities and increases survival time by 30%, but does not halter the cloud's destruction

(Sparre, CP, Ehlert 2020)

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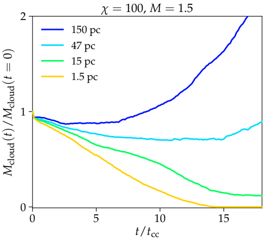
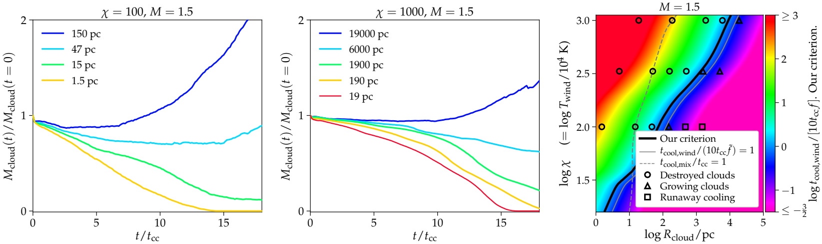
The growth regime



- ram-pressure stripped gas from a **large cloud** mixes with the hot wind to intermediate temperatures
- thermal instability causes further cooling and **net accretion of hot gas to the cold tail** (Armillotta+ 2017, Gronke & Oh 2018, 2019, Li+ 2019, Sparre+ 2020, Kanjilal+ 2020)
- **momentum transfer from hot wind to cooling accreting material** implies formation of long gaseous tail of the jellyfish galaxy!

The growth regime



- **hot-wind cooling time sets transition radius** and not the mixed-phase cooling time $\Rightarrow$ cloud growth criterion (Sparre+ 2020):

$$\frac{t_{\rm cool,wind}}{t_{\rm cc}} < 10 f(M, R_{\rm cloud}, n_{\rm wind}, v_{\rm wind})$$

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Conclusions on magnetic fields dynamics

Interaction of a cold cloud with a hot wind:

- magnetic field provides tension on a moving object and decelerates it
- magnetic field protects against instabilities and increases the survival time
- destruction regime: transport of dense gas to several kpcs hard to explain because cloud shatters and dissolves in the wind
- growth regime: momentum transfer from hot wind to the cooling and accreting material implies formation of long gaseous tail of the jellyfish galaxy

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Virial theorem

- in the lecture notes, we derived the virial theorem,

$$\frac{1}{2}\frac{\mathrm{d}^2}{\mathrm{d}t^2}\left(\sum_{i=1}^{N} m_i|\boldsymbol{r}_i|^2\right) - \sum_{i=1}^{N} m_i|\dot{\boldsymbol{r}}_i|^2 = -\sum_{\substack{i,j=1\\ i>j}}^{N} \frac{Gm_im_j}{|\boldsymbol{r}_i - \boldsymbol{r}_j|},$$

$$\frac{1}{2}\ddot{I} = 2E_{\mathrm{kin}} + E_{\mathrm{pot}} = 0$$

  in equilibrium

- here, we introduced the moment of inertia, I, the total kinetic energy, E_{kin}, and the total gravitational energy, E_{pot}, via

$$I = \sum_{i=1}^{N} m_i\boldsymbol{r}_i^2$$

$$E_{\mathrm{kin}} = \frac{1}{2}\sum_{i=1}^{N} m_i\dot{\boldsymbol{r}}_i^2$$

$$E_{\mathrm{pot}} = -\sum_{\substack{i,j=1\\ i>j}}^{N} \frac{Gm_im_j}{|\boldsymbol{r}_i - \boldsymbol{r}_j|}$$

Virial theorem – assumptions

- **assumption 1:** the system is close to equilibrium and not forming at the present time

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  $\Rightarrow$ ***unbiased velocity estimator*** well behaved:

$$\left\langle v_i^2 \right\rangle = 3 \left\langle v_{\mathrm{los},\,i}^2 \right\rangle \qquad \text{with} \qquad 2E_{\mathrm{kin}} = 3 \left\langle \sum_{i=1}^{N} m_i v_{\mathrm{los},\,i}^2 \right\rangle$$

  $\Rightarrow$ ***unbiased estimator for the angular separation*** of two galaxies unreliable:

$$\frac{1}{r_{ij}} = \left\langle \frac{2}{\pi} \frac{1}{D\theta_{ij}} \right\rangle$$

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  - no objects are excluded by selection, and
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- **assumption 4:** positions and velocities of objects are representative

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Virial theorem – application

- with the caveats following from these considerations we can weigh a cluster with galaxies to order of magnitude:

  adopting a cluster radius $R_{\rm cl} \sim 1$ Mpc, a galaxy velocity dispersion in a cluster, $v_{\rm cl} \sim 1000$ km s^{-1}, $N_{\rm gal} \sim 200$ cluster galaxies with radius $R_{\rm gal} \sim 3$ kpc and rotation velocities $v_{\rm gal} \sim 150$ km s^{-1}, we estimate the ratio of the mass in visible galaxies to the total cluster mass,

$$\frac{\sum_i m_{{\rm gal},\,i}}{M_{\rm cl}} \sim \frac{N_{\rm gal}\, m_{\rm gal}}{M_{\rm cl}} \sim \frac{N_{\rm gal}\, R_{\rm gal}\, v_{\rm gal}^2}{R_{\rm cl}\, v_{\rm cl}^2} \sim N_{\rm gal}\, \frac{R_{\rm gal}}{R_{\rm cl}} \left(\frac{v_{\rm gal}}{v_{\rm cl}}\right)^2 \sim 0.0135$$

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- hence, to get $\sum_i m_{\rm gal} \sim M_{\rm cl}$ you would need $R_{\rm gal} \sim 0.2 R_{\rm cl}$: there are several theoretical solutions to this problem:
  - most mass is not attached to galaxies,
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- clearly, the first solution is right, but to see this we need to include gravitational lensing and hydrostatic mass estimates from the X-rays

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The Physics of Galaxy Clusters - recap

- **Observational facts:**
  * Morphology-density relation: increasing fraction of ellipticals in clusters
  * Butcher-Oemler effect: decreasing fraction of blue cluster galaxies with time

  $\Rightarrow$ galaxies seem to have transformed in clusters

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  * gravitational interaction between two deformable bodies enhances the mutual gravitational multipole moments $\Rightarrow$ backreaction on their orbit

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- **Ram-pressure stripping:**
  * a gas-rich spiral moving through the ICM feels a ram pressure wind that can unbind the ISM, strip it off and form spectacular tails: "jellyfish galaxies"
  * magnetic draping forms a protective layer around the galaxy, shielding the stripped ISM from the hot ICM wind and enables some modest star formation
  * when spirals loose their gas, the potential for future star formation is reduced and their star formation is eventually quenched

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