The Physics of Galaxy Clusters

Lecture 14 - X-ray Cluster Astrophysics and the Sunyaev-Zel'dovich Effect

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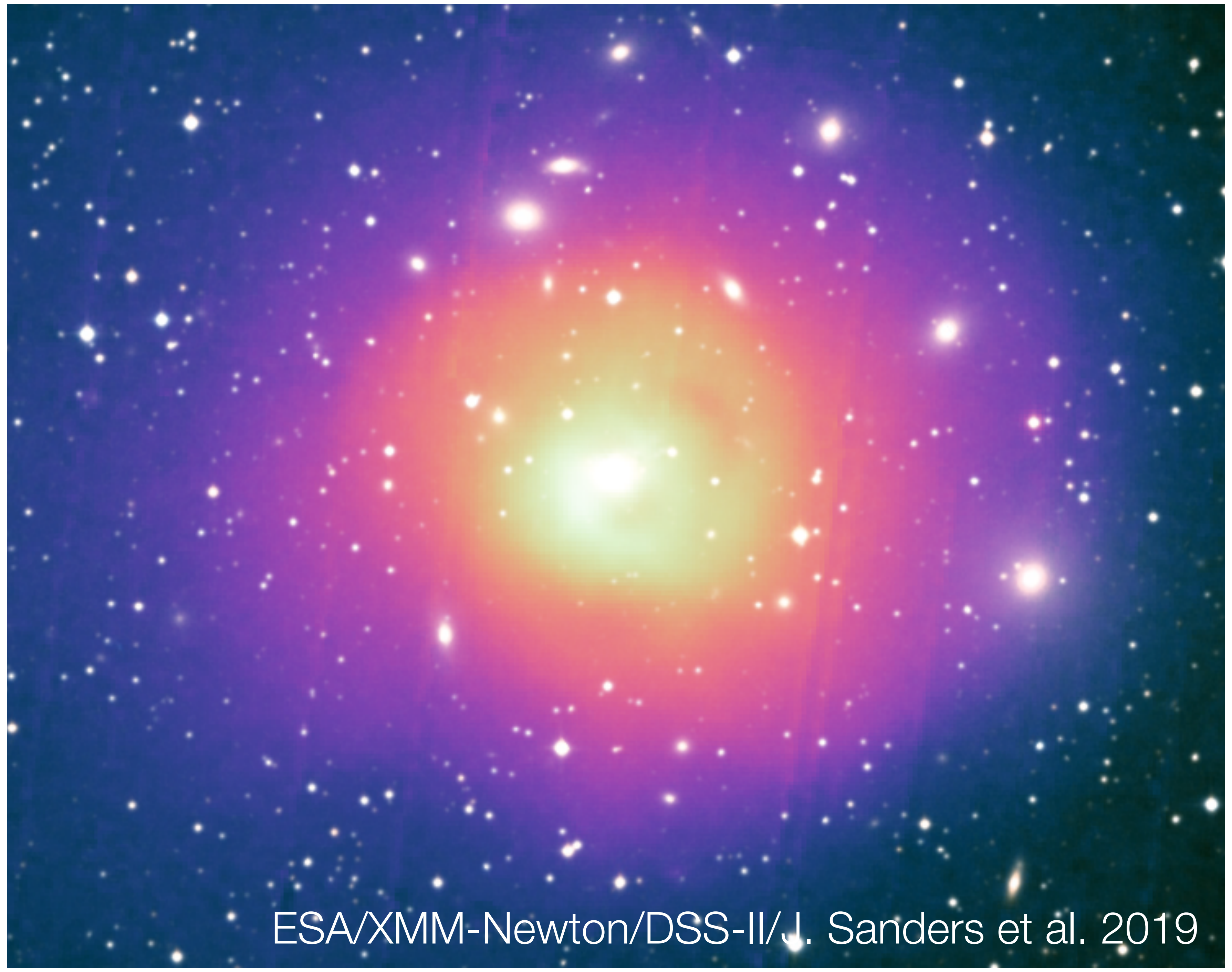
(based on slides by Ewald Puchwein)

Cluster X-ray emission

- probes hot intracluster gas ($T \sim 10^7$ - 10^8 K)
  - at high temperatures ($\gtrsim$ 2 keV) mostly free-free emission (electron-ion collisions)
  - at lower temperatures metal lines (e.g., Fe)
- allows measuring gas density and temperature
  - emissivity ~ square of density
  - spectral shape depends on temperature



Hydrostatic equilibrium

- Unless the intracluster gas gets continuously disturbed by (major) mergers, we expect the ICM to relax on a few sound crossing time scales:

$$t_{\rm s} \equiv \frac{D}{c_{\rm s}} \approx 7 \times 10^8 \left(\frac{T}{10^8\,{\rm K}}\right)^{-1/2} \left(\frac{D}{1\,{\rm Mpc}}\right)\ {\rm yr}$$

- What are typical values for the speed of sound in a galaxy cluster? How do they compare to typical galaxy velocities? What is the reason for this?

Hydrostatic mass estimates

- if the gas is in hydrostatic equilibrium in the gravitational potential
  - ➡ allows cluster mass measurement

$$\boldsymbol{\nabla} P = -\rho_{\mathrm{gas}} \boldsymbol{\nabla} \Phi$$ force balance

$$\frac{1}{\rho_{\mathrm{gas}}} \frac{\mathrm{d}P}{\mathrm{d}r} = -\frac{GM(r)}{r^2}$$ for spherical symmetry

Consider gas parcel

Why can the hydrodynamic force per unit volume can be written as **∇**P?

$$F_x = P \times A$$ ->

$$\boxed{F_{\mathrm{grav},x} = \rho V \overrightarrow{\nabla} \Phi \bigg|_x}$$

<- $$F_x = \left(P + \frac{dP}{dx} l\right) \times A = P \times A + \frac{dP}{dx} V$$

to cluster center

$$\Delta F_x \approx \frac{dP}{dx} V$$

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force balance

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for spherical symmetry

-> $$M(r) = -\frac{r k_{\mathrm{B}} T}{G \bar{m}} \left(\frac{\mathrm{d} \ln \rho_{\mathrm{gas}}}{\mathrm{d} \ln r} + \frac{\mathrm{d} \ln T}{\mathrm{d} \ln r} \right)$$

Gas particle vs. galaxy velocities

$$M(r) = -\frac{rk_{\mathrm{B}}T}{G\bar{m}}\left(\frac{\mathrm{d}\ln\rho_{\mathrm{gas}}}{\mathrm{d}\ln r} + \frac{\mathrm{d}\ln T}{\mathrm{d}\ln r}\right)$$

$$= -\frac{r\sigma^2_{v,\mathrm{gas}}}{G}\left(\frac{\mathrm{d}\ln\rho_{\mathrm{gas}}}{\mathrm{d}\ln r} + \frac{\mathrm{d}\ln\sigma^2_{v,\mathrm{gas}}}{\mathrm{d}\ln r}\right)$$

<- as a function of (1D) gas particle velocity dispersion

galaxies have similar kinematics

$$M(r) = -\frac{r\sigma^2_{v,\mathrm{gal}}}{G}\left(\frac{\mathrm{d}\ln\rho_{\mathrm{gal}}}{\mathrm{d}\ln r} + \frac{\mathrm{d}\ln\sigma^2_{v,\mathrm{gal}}}{\mathrm{d}\ln r}\right)$$

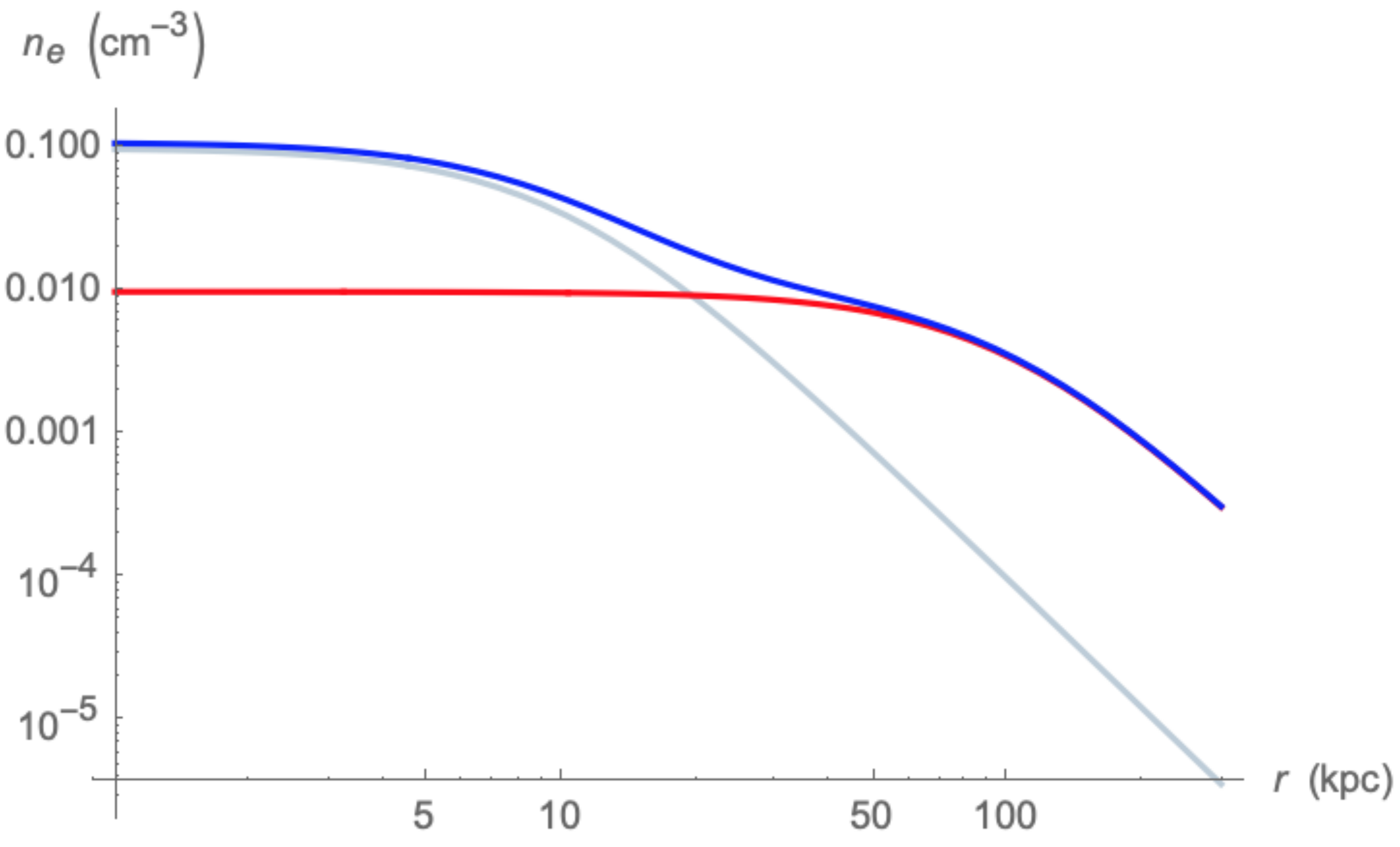
with $\sigma^2_{v,\mathrm{gal}} = \sigma^2_{v,\mathrm{gas}}\beta$

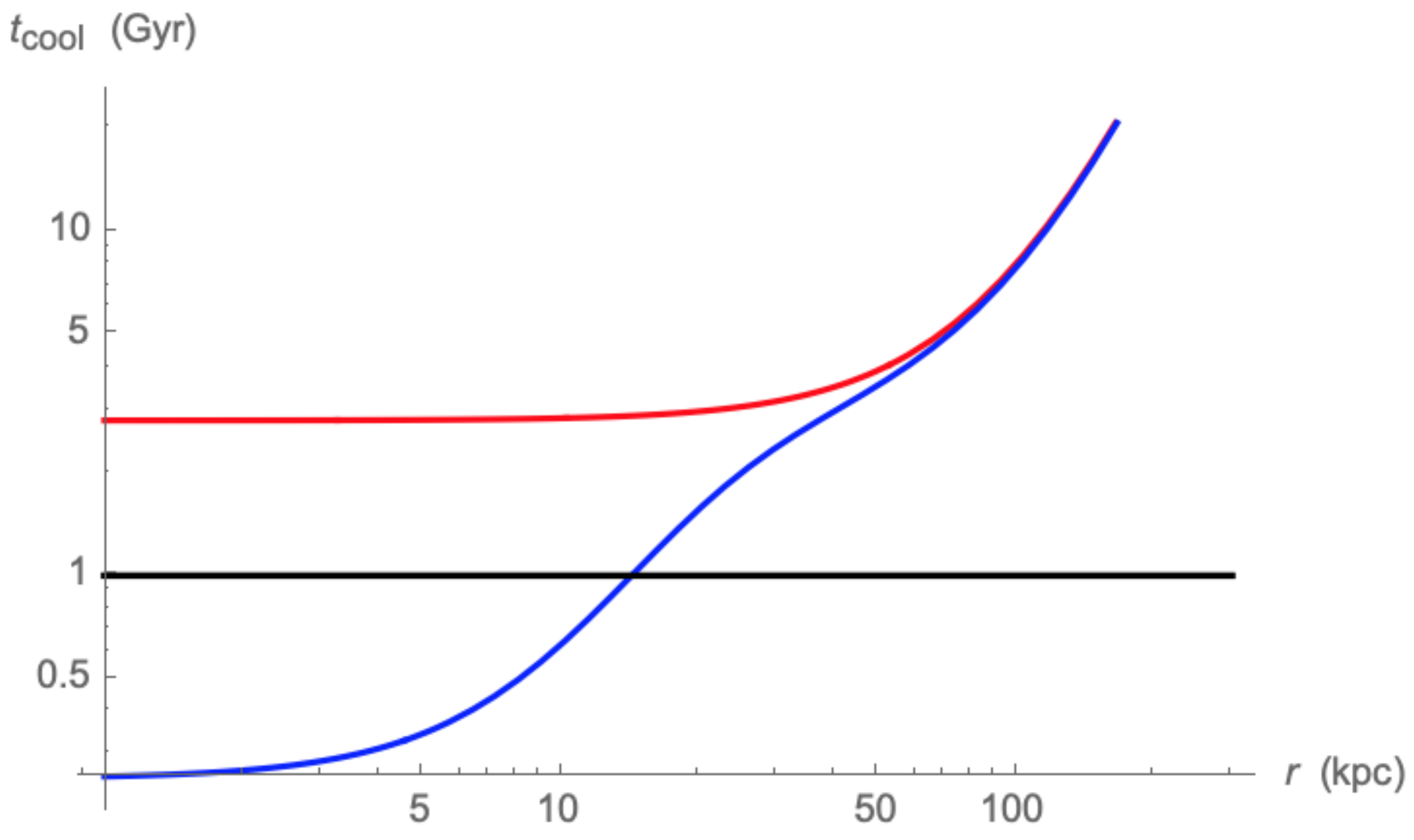
allows relating gas density to galaxy density profile (if isothermal & symmetric)

$$\rho_{\mathrm{gal}}(r) = \rho_0\left[1 + \left(\frac{r}{r_{\mathrm{c}}}\right)^2\right]^{-3/2} \longleftrightarrow \rho_{\mathrm{gas}}(r) = \rho_0\left[1 + \left(\frac{r}{r_{\mathrm{c}}}\right)^2\right]^{-3\beta/2}$$

Beta profile

Cool core vs. non-cool core clusters





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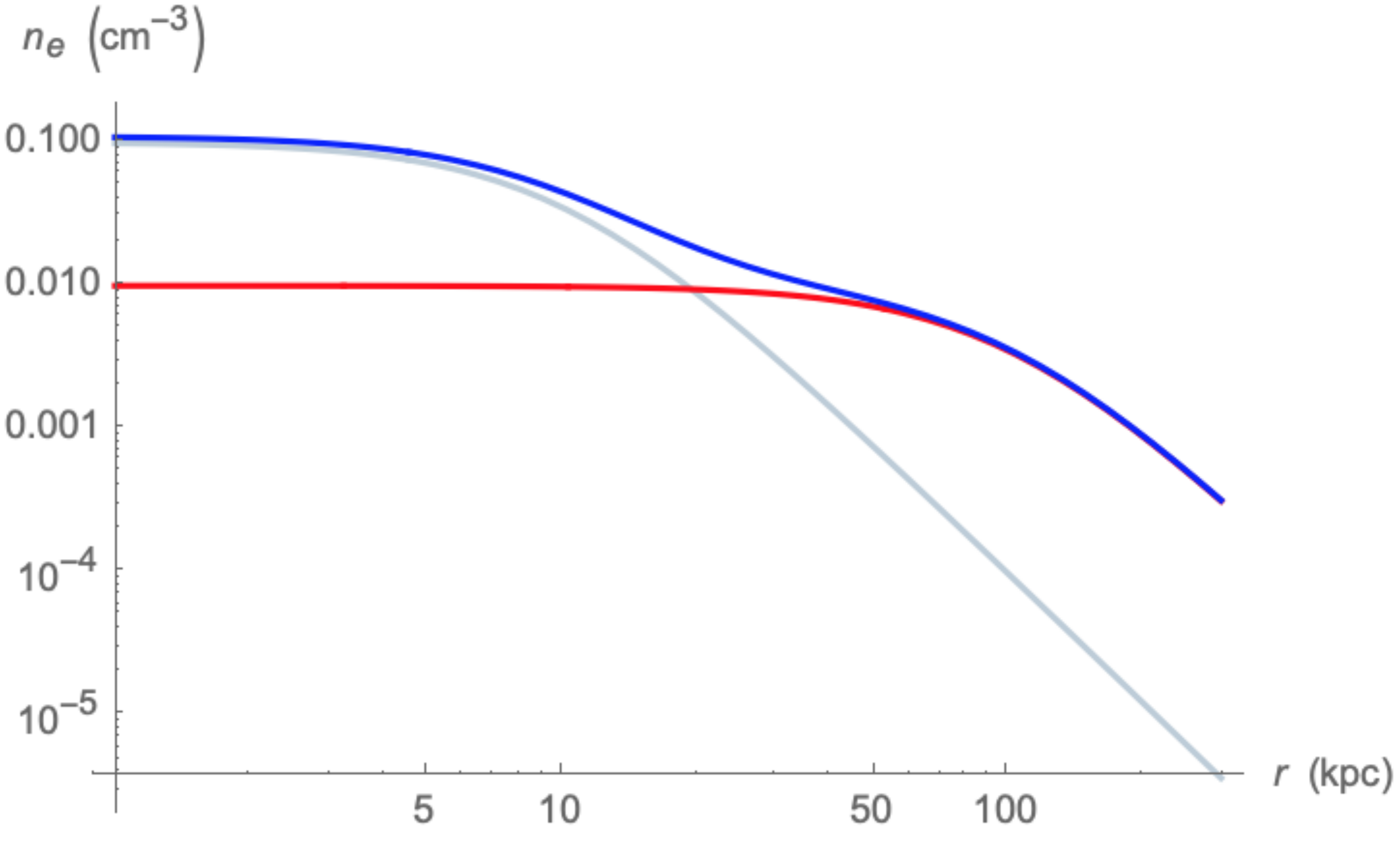
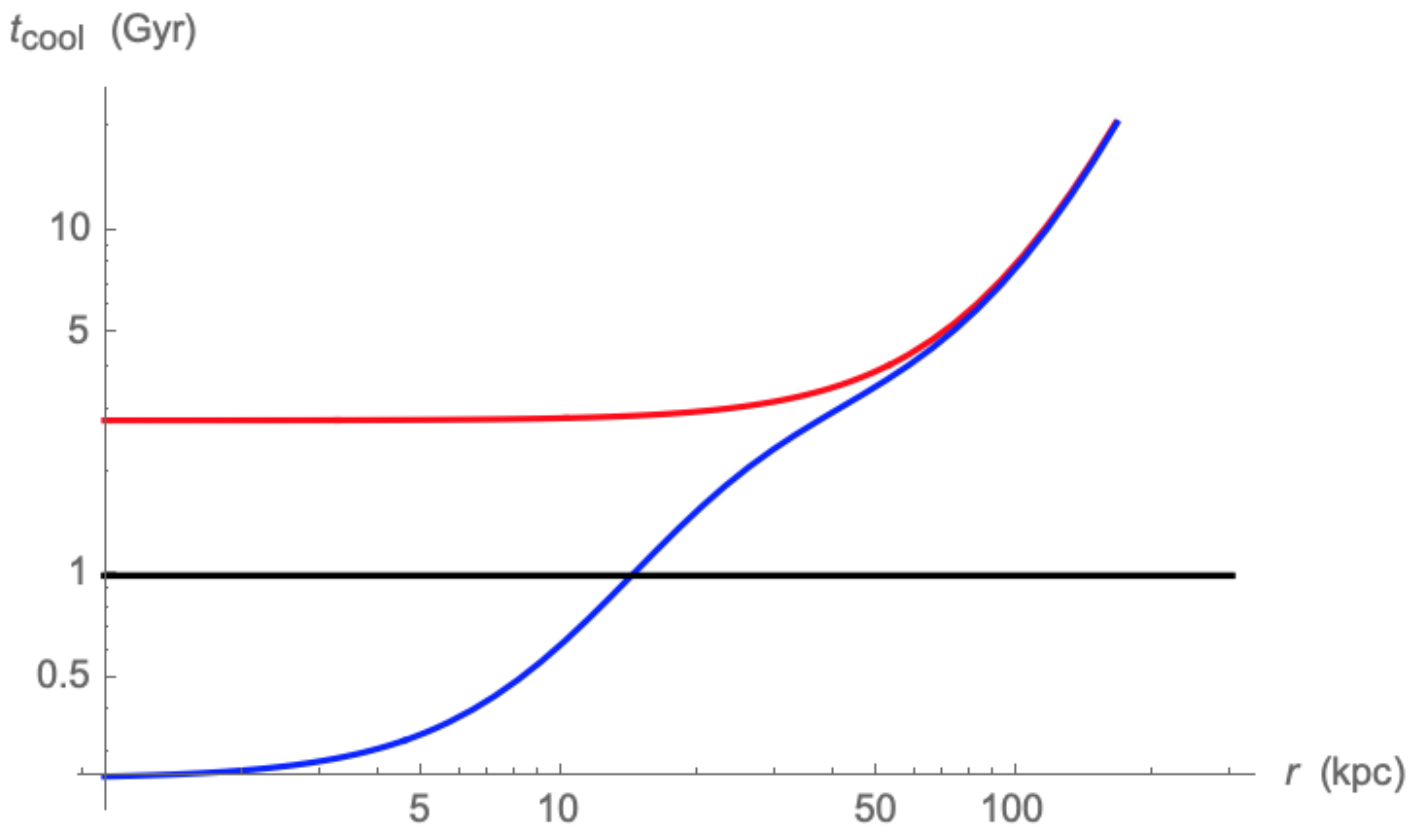
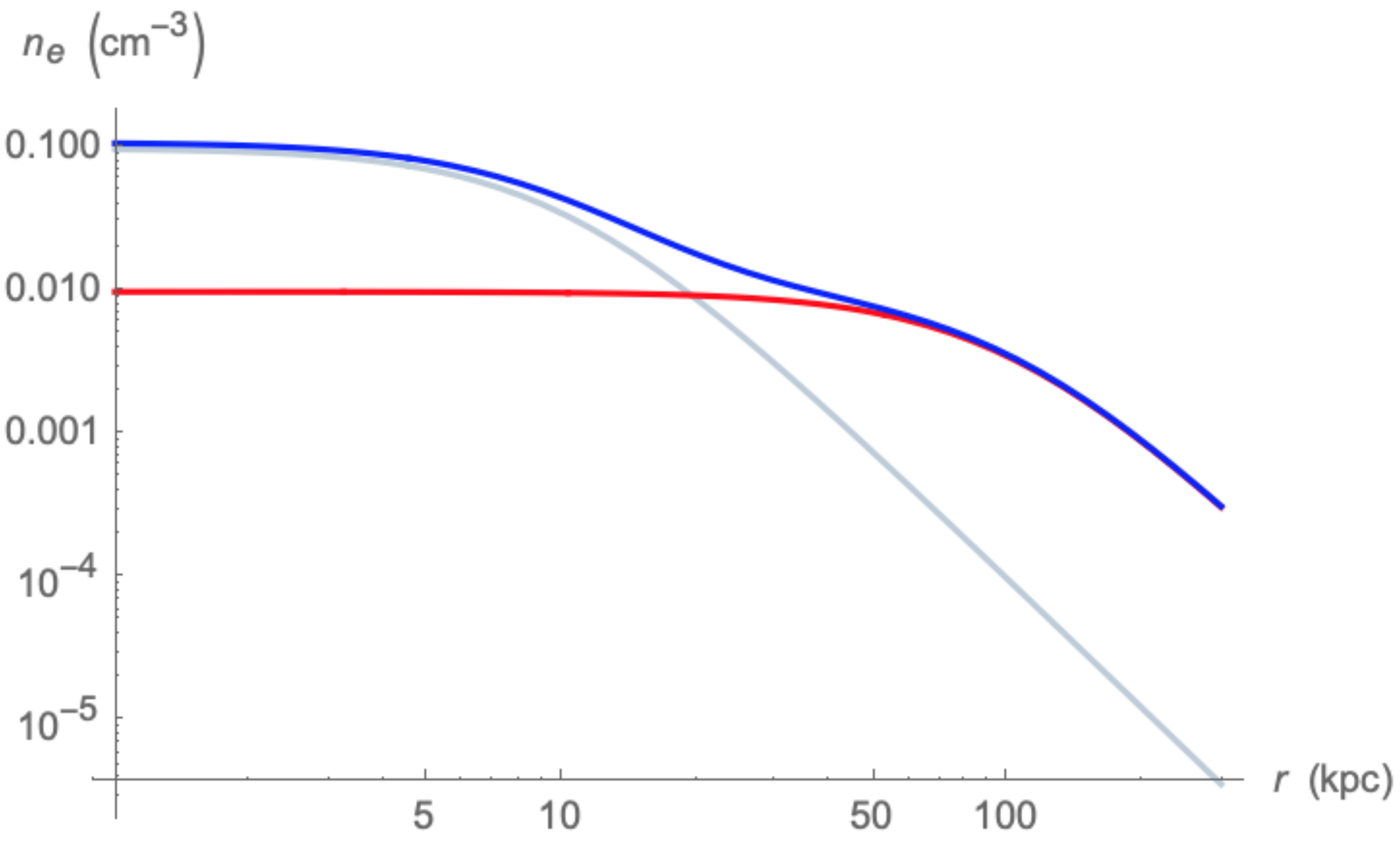


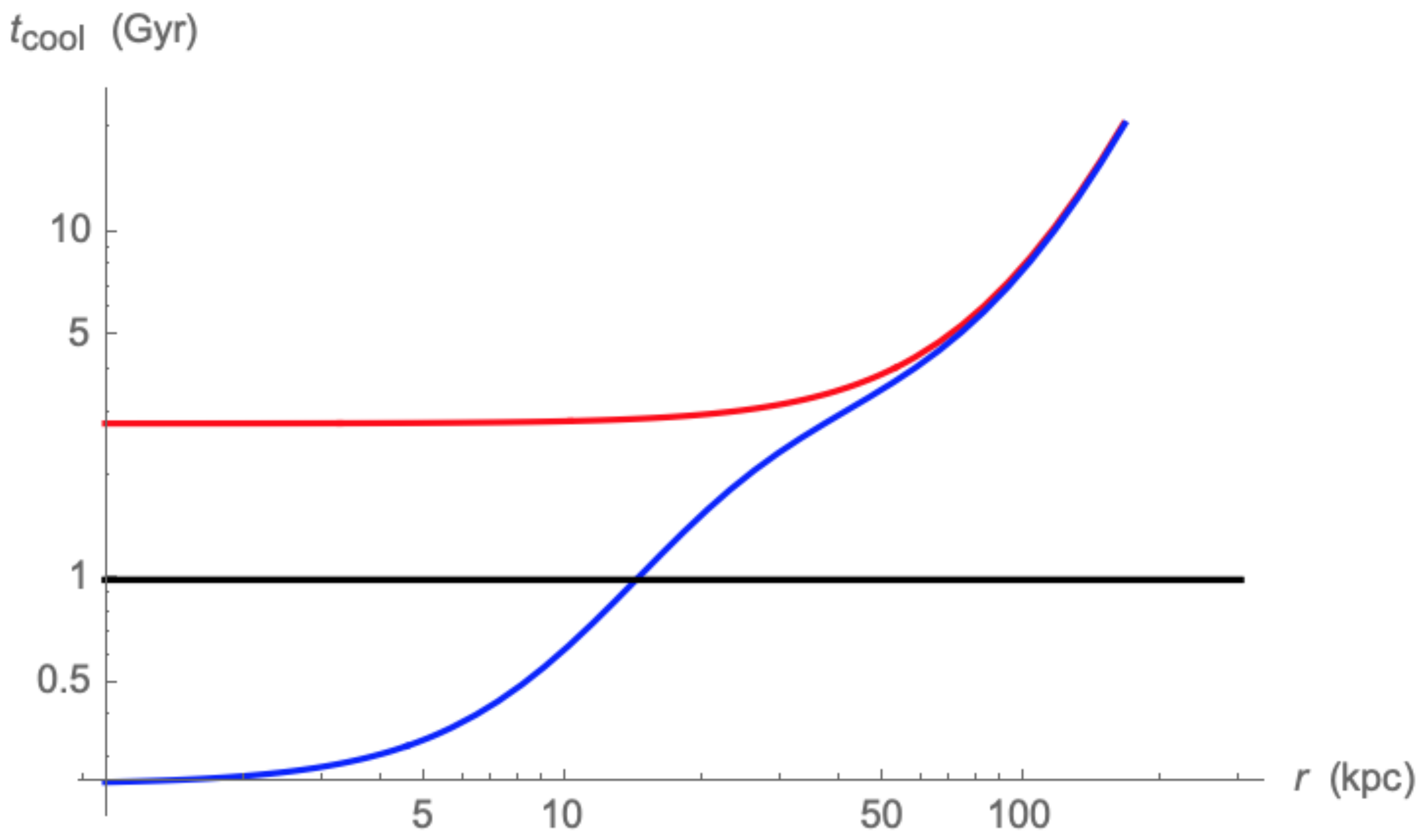


gas density profile:

$$n_{\rm e}(r) = \sum_{i=1,2} n_i \left[1 + \left(\frac{r}{r_{{\rm c},i}}\right)^2\right]^{-3\beta_i/2}$$

Cool core vs. non-cool core clusters





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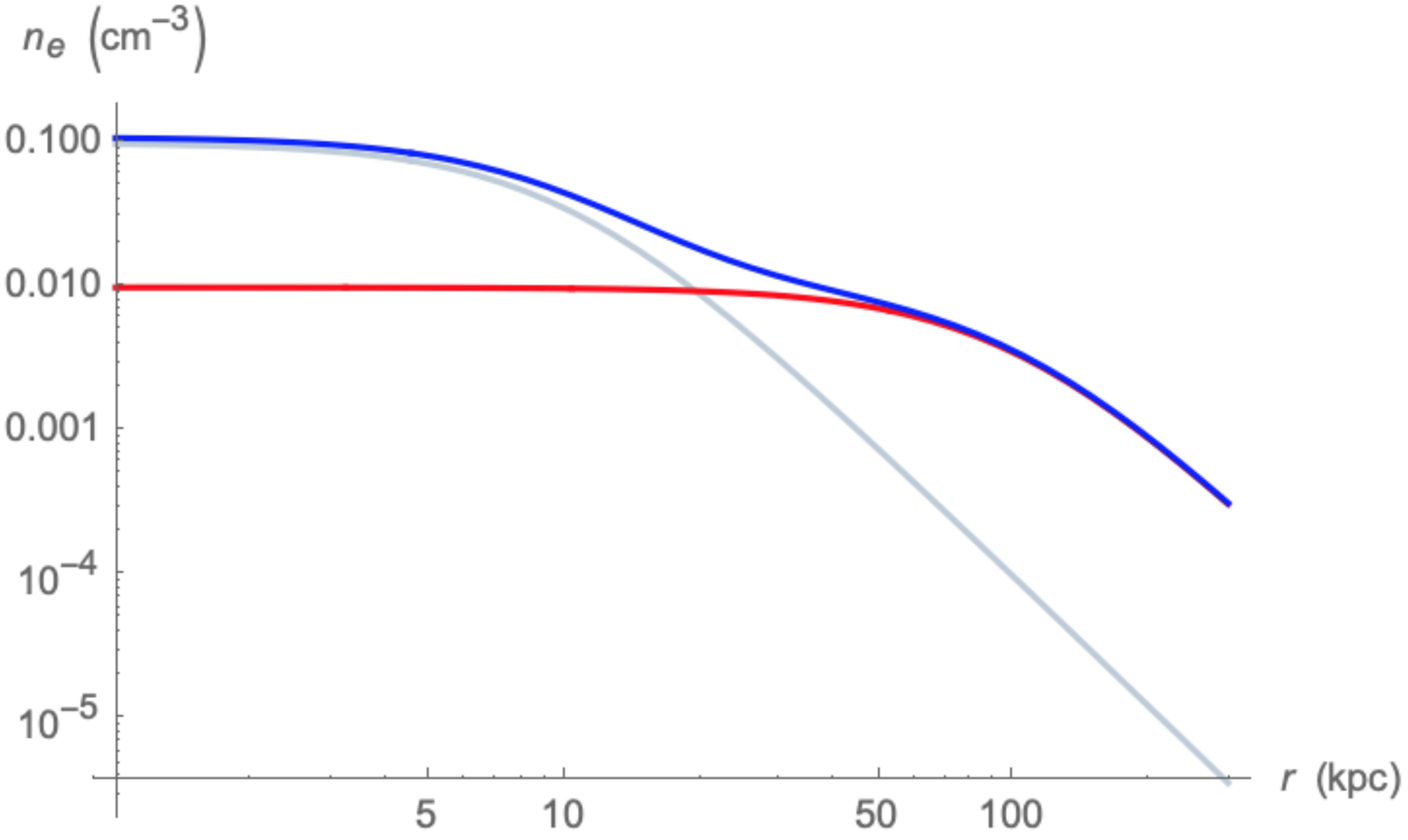
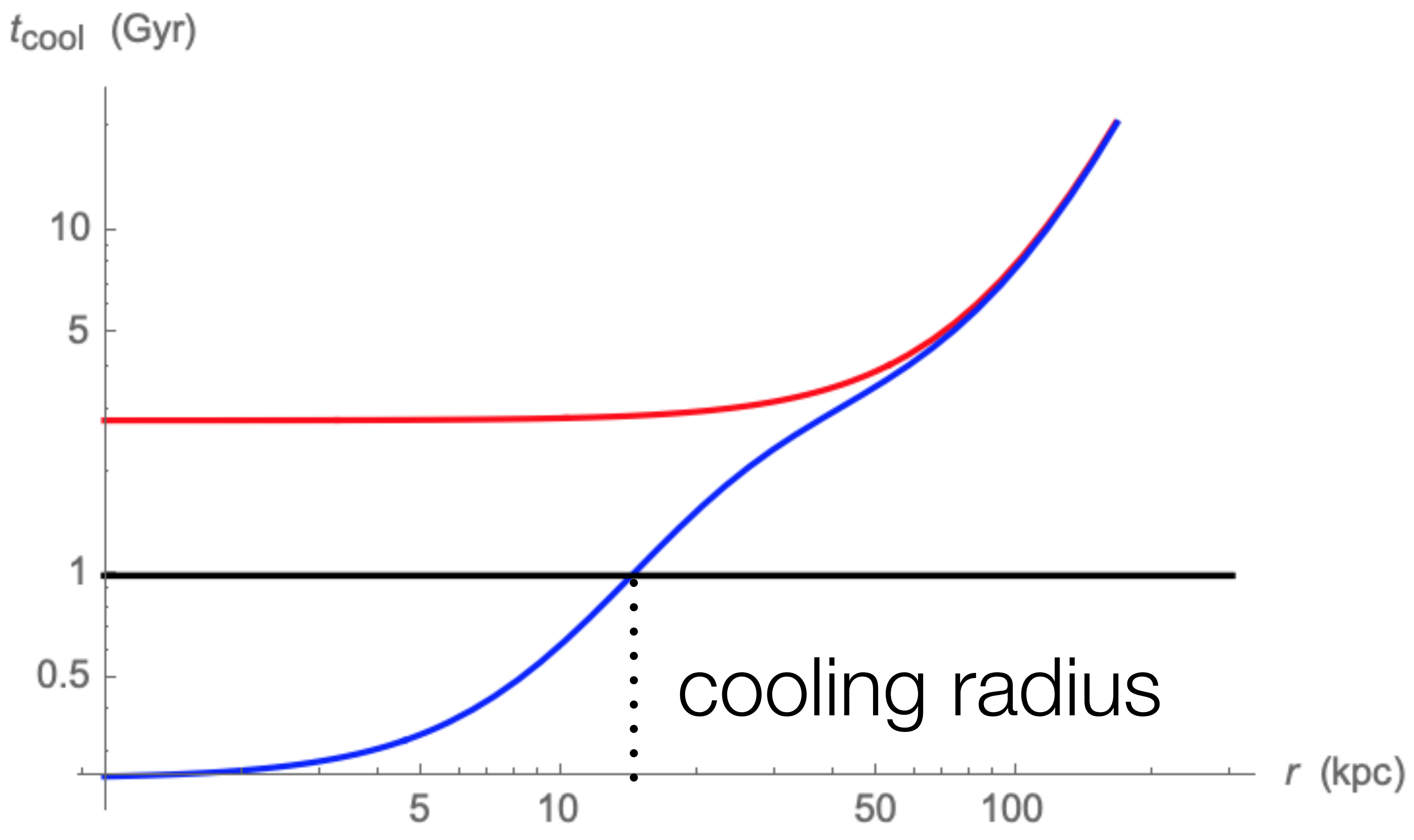
$$n_e(r) = \sum_{i=1,2} n_i \left[1 + \left(\frac{r}{r_{c,i}}\right)^2\right]^{-3\beta_i/2}$$

cooling time (assuming bremsstrahlung):

$$t_{cool} = \frac{\varepsilon_{th}}{\dot{\varepsilon}_{brems}} \approx 0.5 \left(\frac{k_B T}{1\ \mathrm{keV}}\right)^{1/2} \left(\frac{n_e}{4 \times 10^{-2}\ \mathrm{cm}^{-3}}\right)^{-1} \mathrm{Gyr}$$

Cool core vs. non-cool core clusters





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Entropy profiles

$$K_\mathrm{e} = \frac{kT}{n_e^{2/3}}$$ constant in adiabatic processes

in pressure equilibrium $P \propto nT \approx$ const. $\rightarrow K_e \propto n_e^{-5/3}$

-> higher entropy gas has lower density -> rises buoyantly

entropy profile typically well fit by

$$K_\mathrm{e}(r) = K_0 + K_{100}\left(\frac{r}{100\ \mathrm{kpc}}\right)^\alpha$$

Entropy profiles

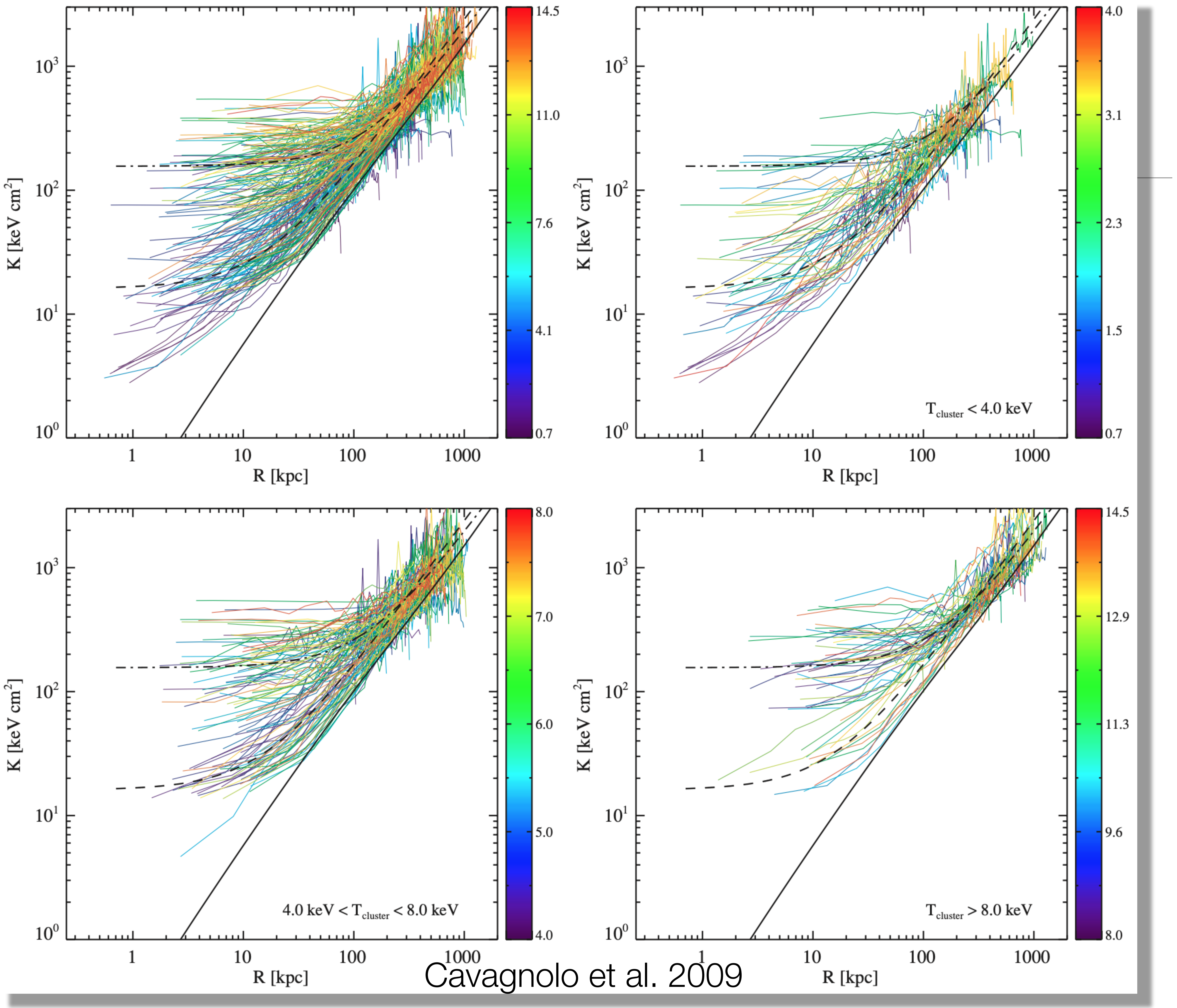
$$K_{\mathrm{e}} = \frac{kT}{n_e^{2/3}}$$ constant

in pressure equilibrium

-> higher ent

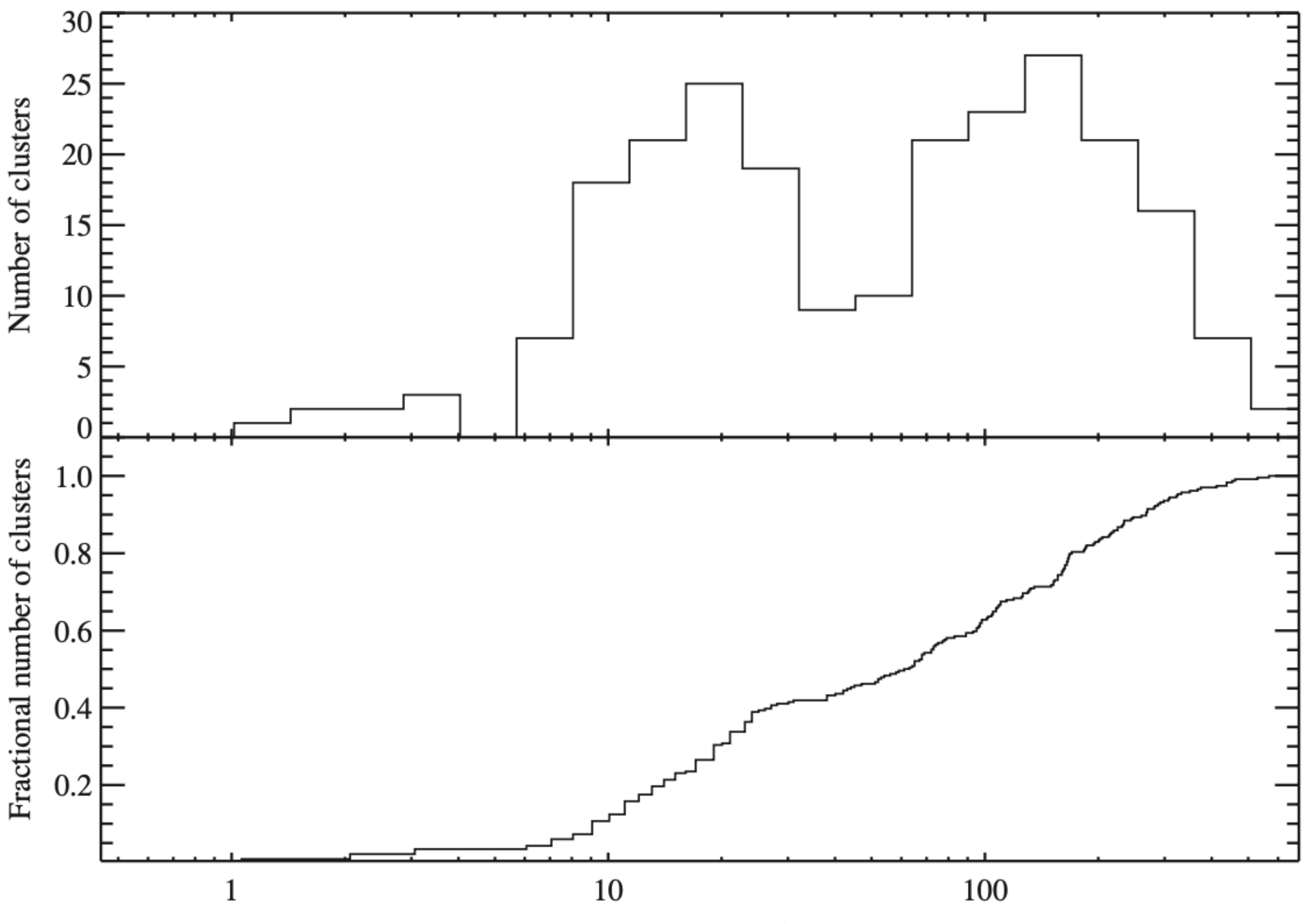
entropy profile typically

$$K_{\mathrm{e}}(r) = K_0 + K_{100}\left(\frac{}{10\ldots}\right.$$

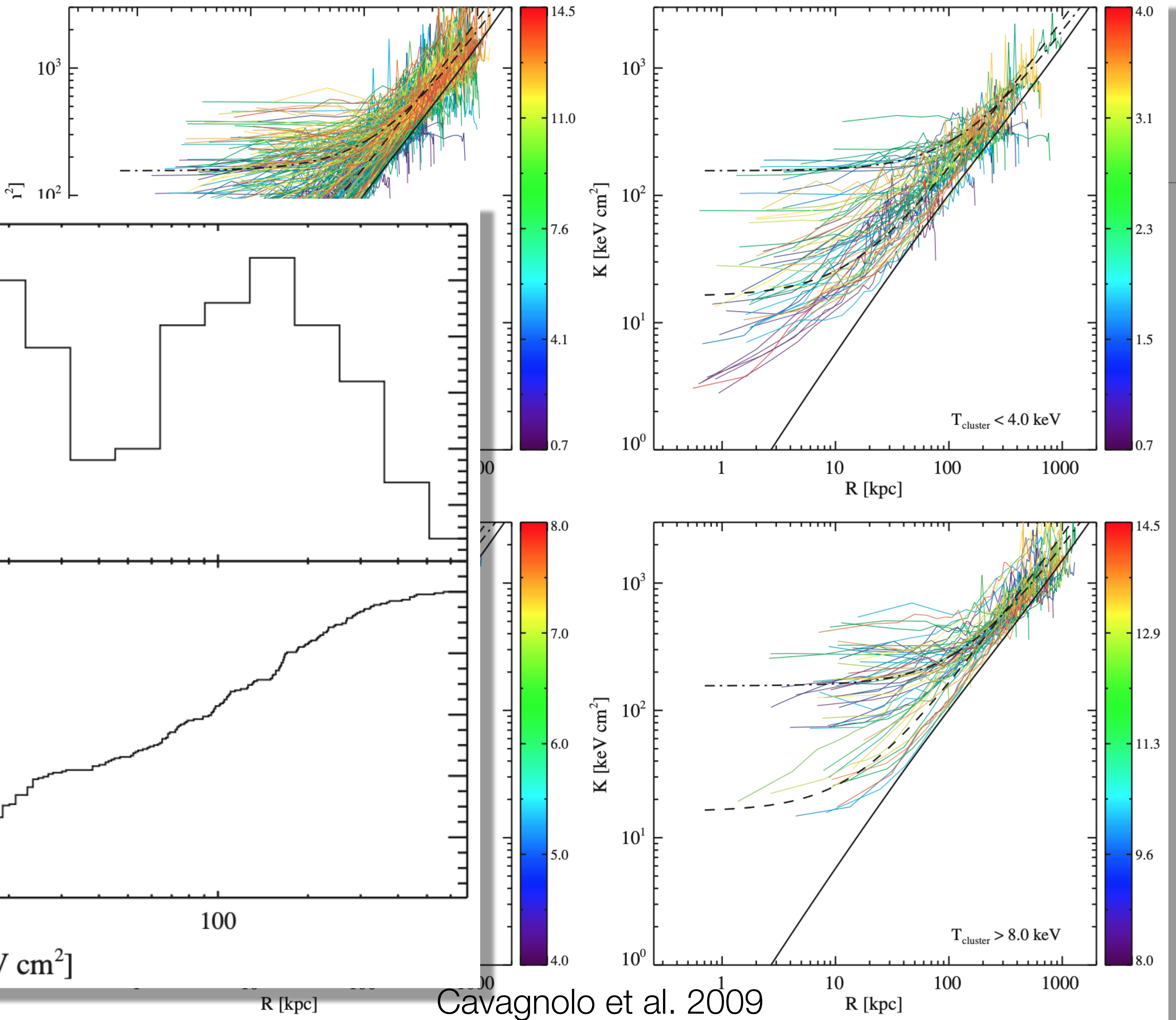


Cavagnolo et al. 2009

Entropy profiles



Cavagnolo et al. 2009



Cavagnolo et al. 2009

X-ray surface brightness

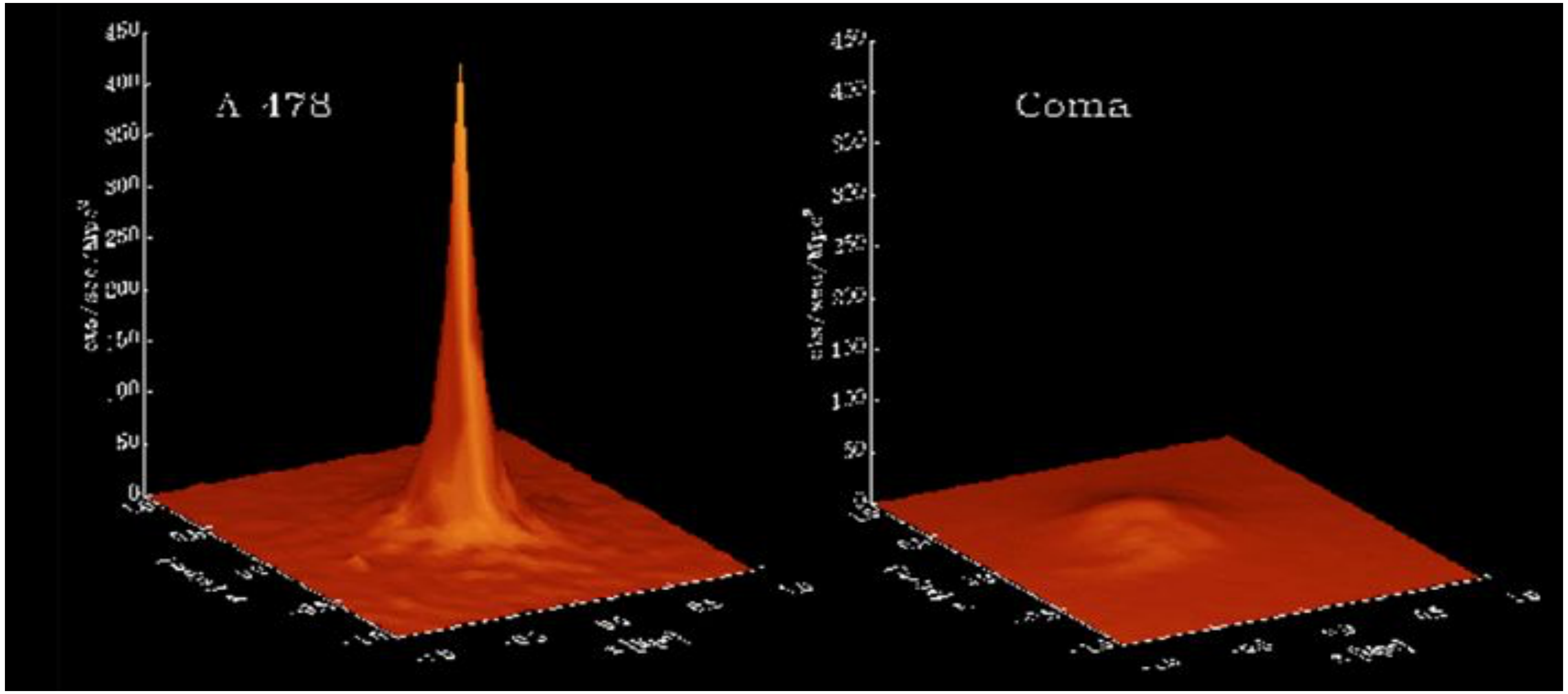
- emissivity ~ square of density:

$$j_X(r) = j_0\left[1 + \left(\frac{r}{r_c}\right)^2\right]^{-3\beta}$$

- X-ray surface brightness profile is obtained upon a line-of-sight integration:

$$\begin{aligned} S_X(r_\perp) &= \int_{-\infty}^{\infty} j_X[r(z)]\mathrm{d}z \\ &= 2\int_{r_\perp}^{\infty} \frac{j_X(r)r\,\mathrm{d}r}{\sqrt{r^2 - r_\perp^2}} = S_0\left[1 + \left(\frac{r_\perp}{r_c}\right)^2\right]^{-3\beta+1/2} \end{aligned}$$

X-ray surface brightness

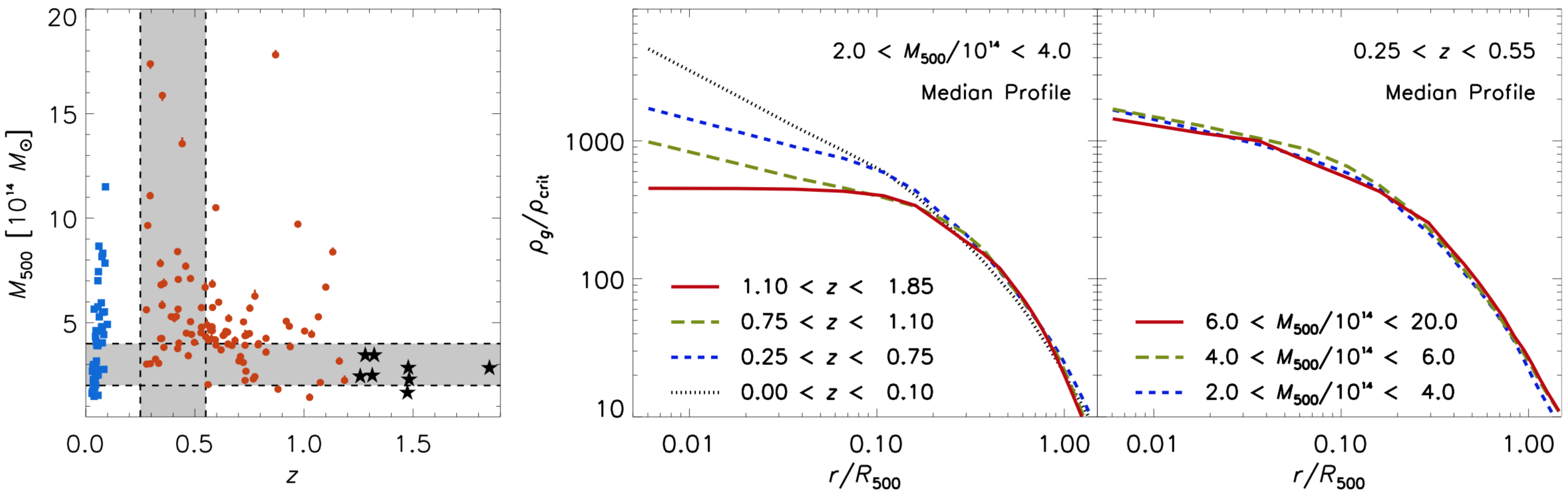


Allen & Ebeling

cool core

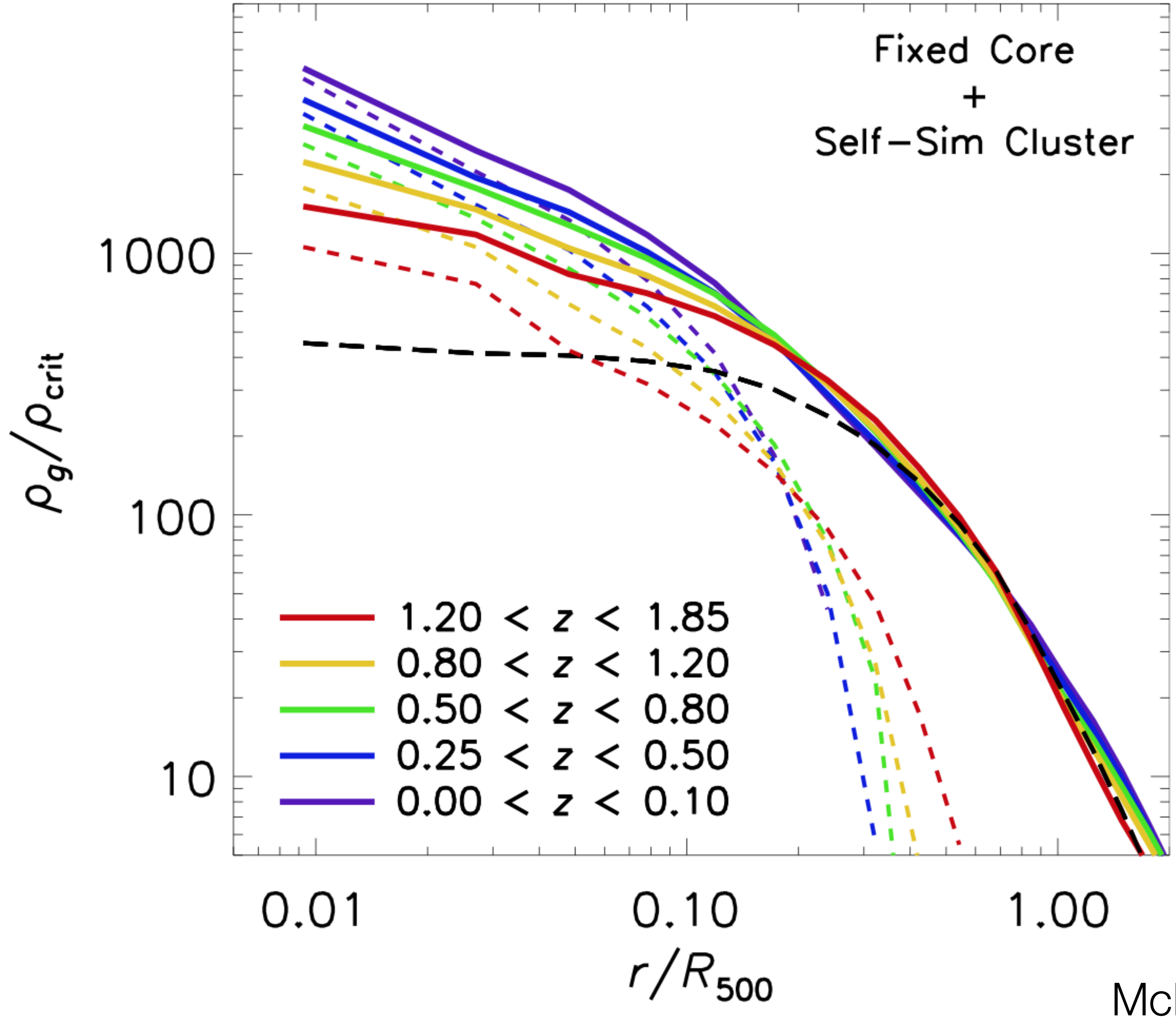
non-cool core

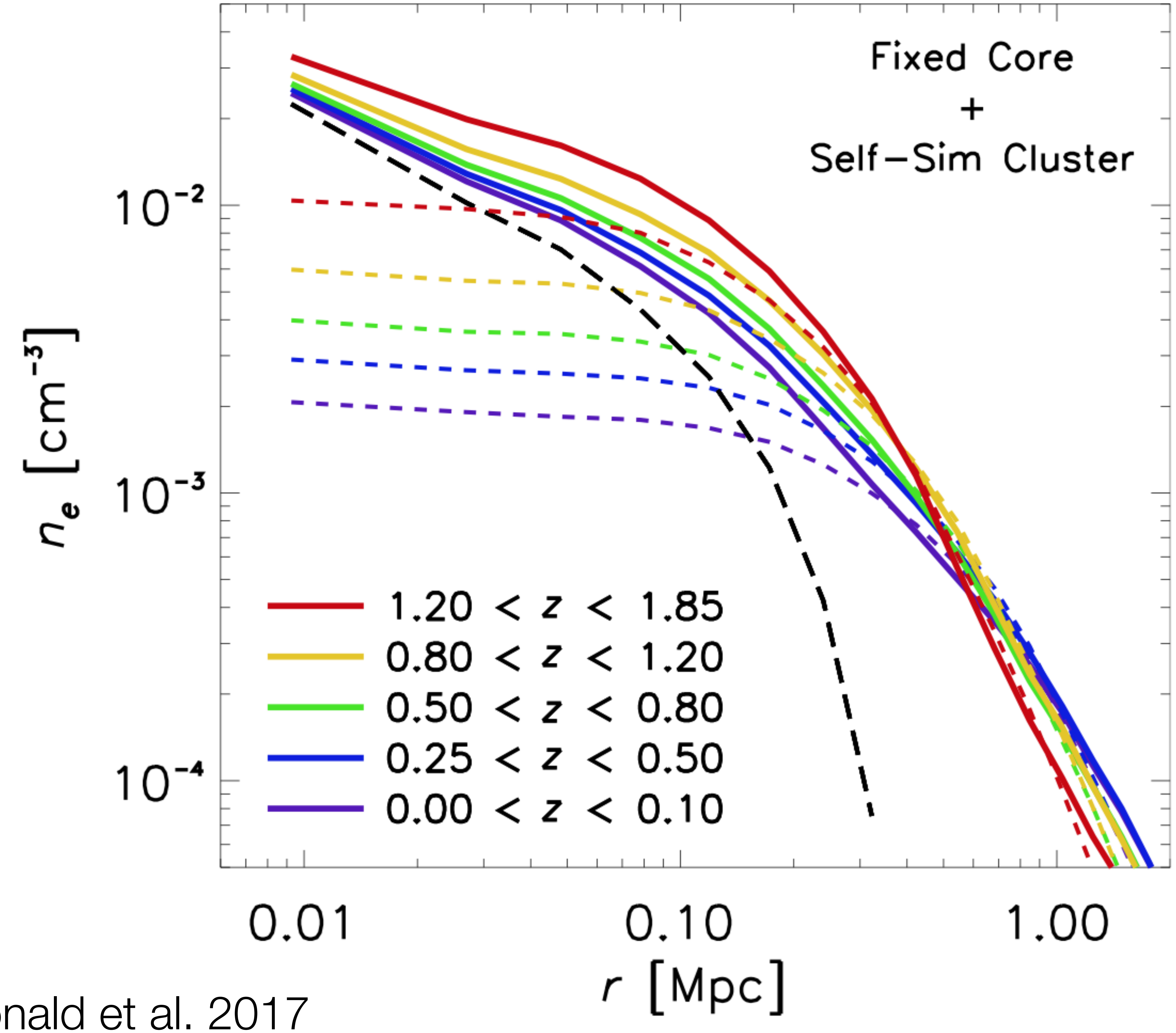
Cluster density profiles



McDonald et al. 2017

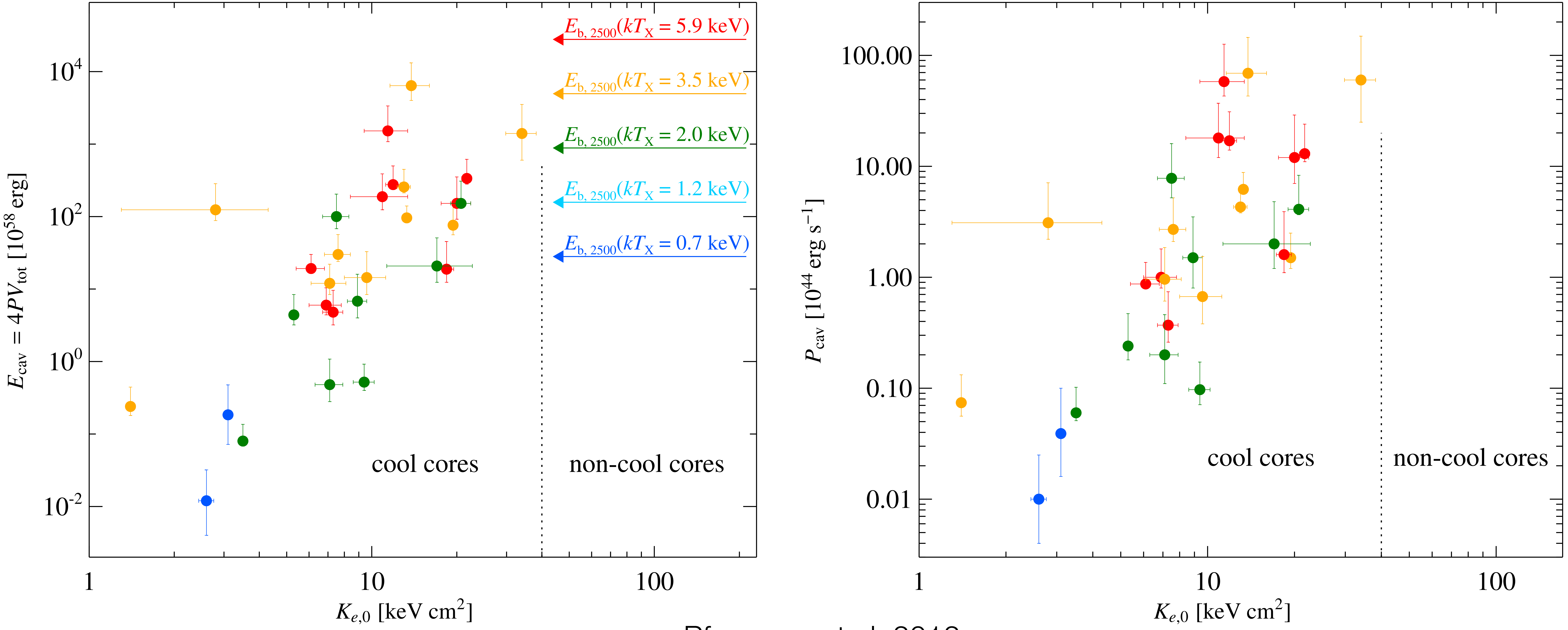
Cool core static / evolving?





McDonald et al. 2017

Can AGN outbursts destroy cool cores?



Pfrommer et al. 2012

Intracluster medium turbulence: sources of turbulence?

Reynolds number

$$\mathrm{Re} = \frac{t_{\mathrm{diss}}}{t_{\mathrm{adv}}} \approx \frac{3L^2}{\lambda_{\mathrm{mfp}} v_{\mathrm{th}}} \frac{v}{L} = \frac{3L}{\lambda_{\mathrm{mfp}}} \frac{v}{v_{\mathrm{th}}}$$

$$t_{\mathrm{adv}} \approx \frac{L}{v}$$

$$t_{\mathrm{diss}} \approx \frac{L^2}{\nu} \approx \frac{3L^2}{\lambda_{\mathrm{mfp}} v_{\mathrm{th}}}$$

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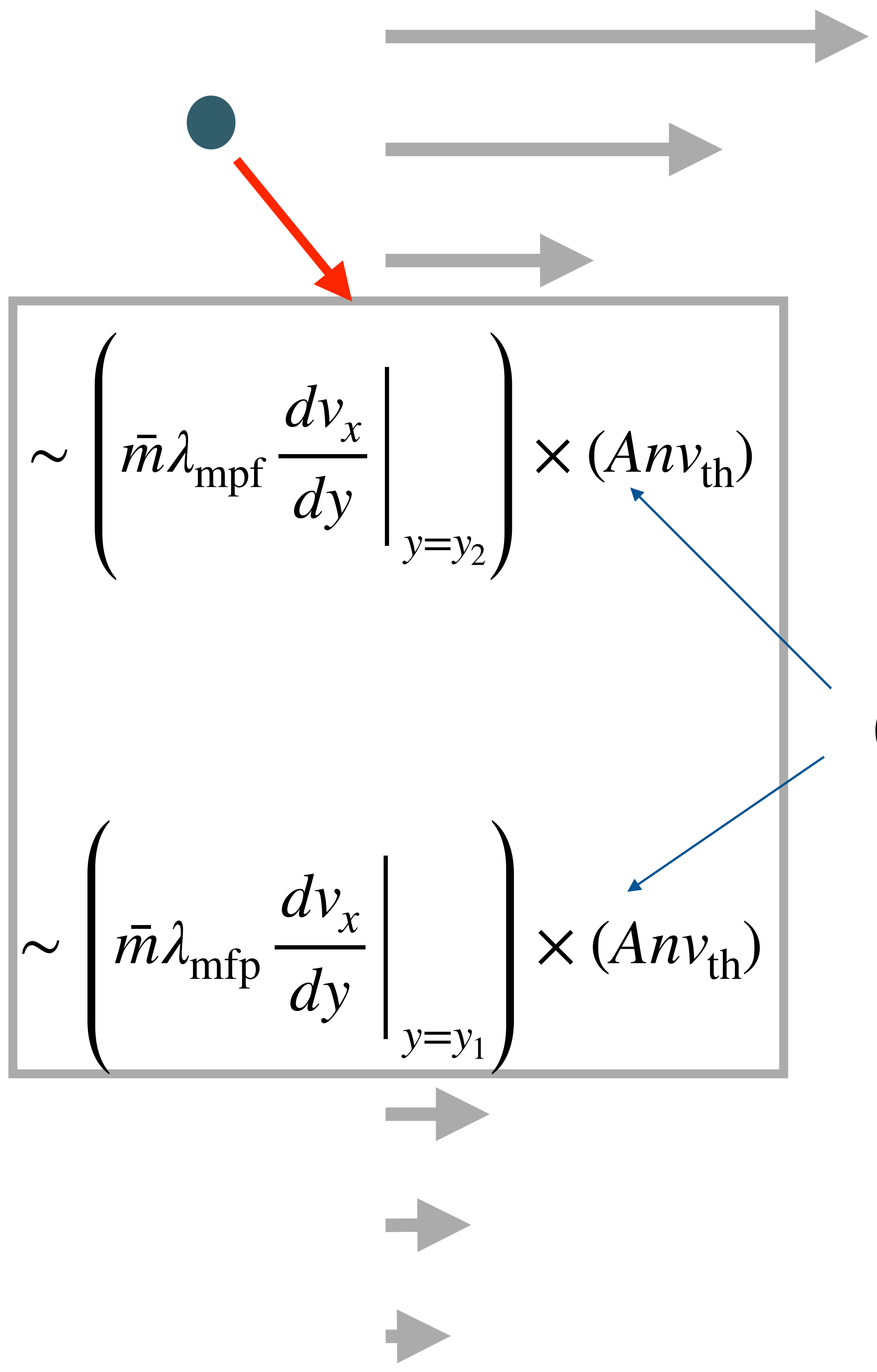

diffusion of momentum

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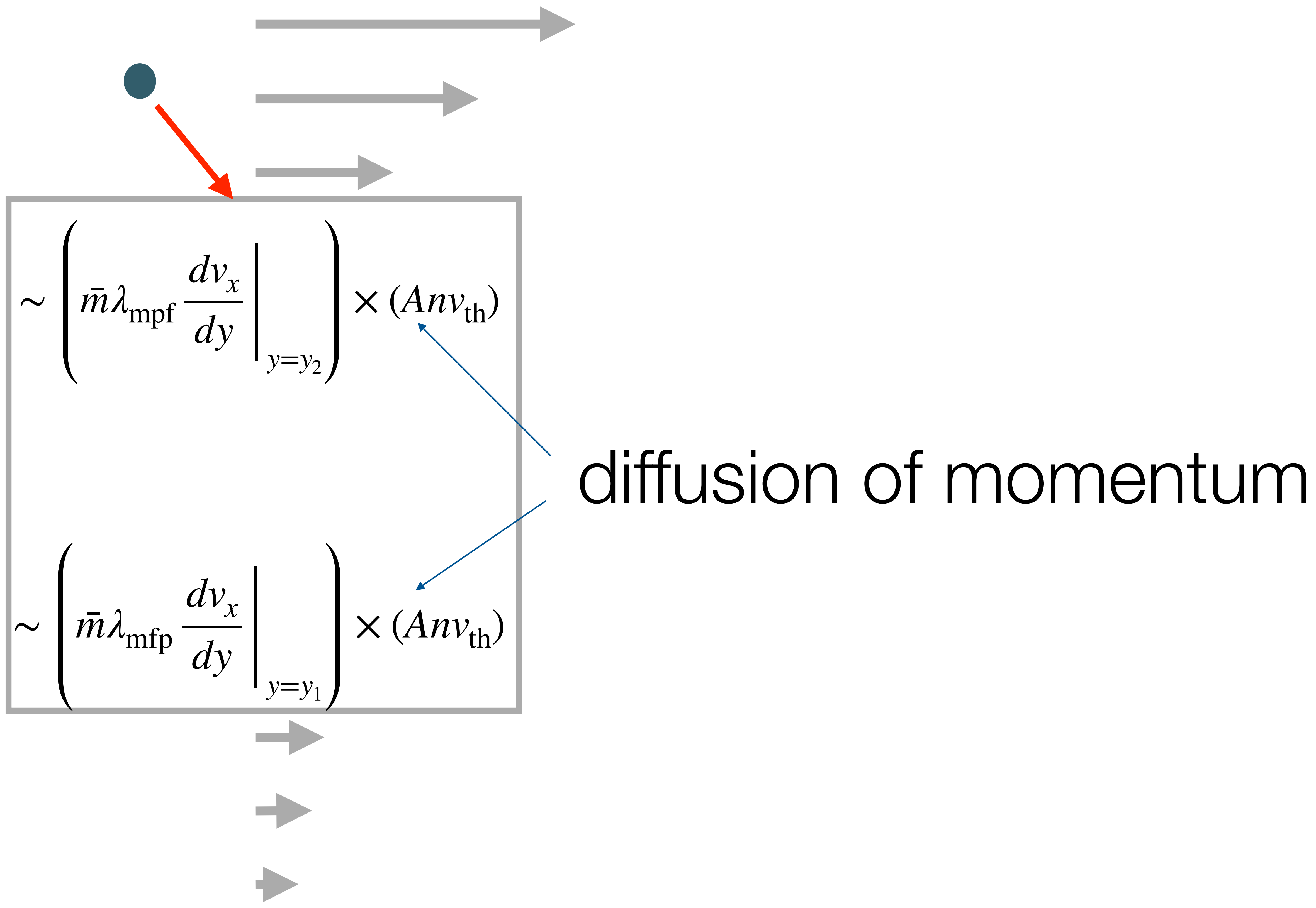

$$F_{\mathrm{viscous}} \sim \bar{m}\lambda_{\mathrm{mfp}} \frac{\mathrm{d}^2 v_x}{\mathrm{d}y^2} lAnv_{\mathrm{th}} = M \frac{\mathrm{d}^2 v_x}{\mathrm{d}y^2} \lambda_{\mathrm{mpf}} v_{\mathrm{th}} \sim M \frac{v}{L^2} \lambda_{\mathrm{mpf}} v_{\mathrm{th}} \quad \rightarrow \quad t_{\mathrm{diss}} \sim \frac{p}{F_{\mathrm{viscous}}} \sim \frac{L^2}{\lambda_{\mathrm{mfp}} v_{\mathrm{th}}}$$

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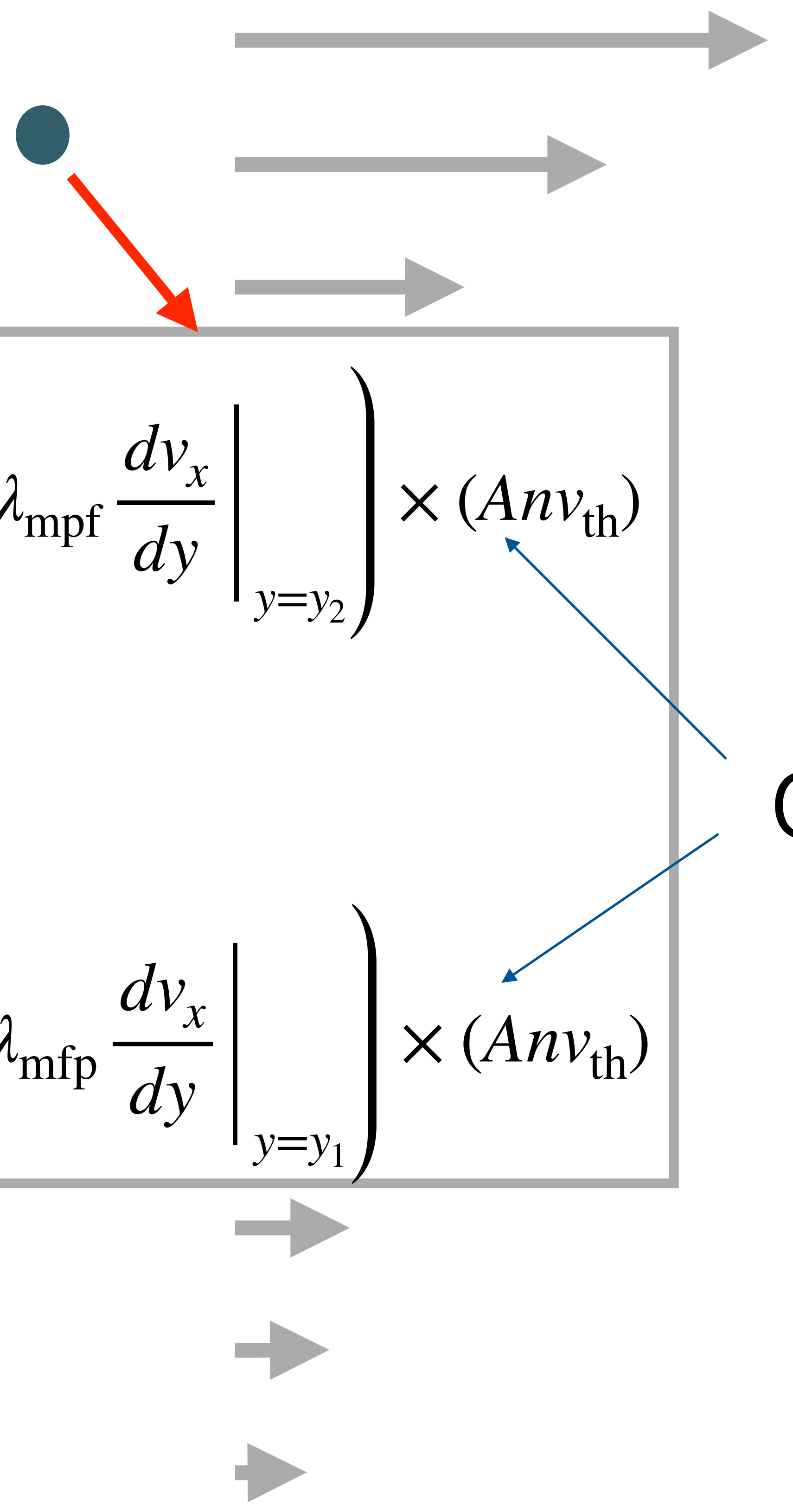

$$\sim \left(\bar{m} \lambda_{\mathrm{mpf}} \frac{dv_x}{dy} \bigg|_{y=y_2} \right) \times (A n v_{\mathrm{th}})$$

$$\sim \left(\bar{m} \lambda_{\mathrm{mfp}} \frac{dv_x}{dy} \bigg|_{y=y_1} \right) \times (A n v_{\mathrm{th}})$$

diffusion of momentum

Re > ~2300 -> turbulence
Re < ~2300 -> laminar flow

$$F_{\mathrm{viscous}} \sim \bar{m} \lambda_{\mathrm{mfp}} \frac{\mathrm{d}^2 v_x}{\mathrm{d}y^2} l A n v_{\mathrm{th}} = M \frac{\mathrm{d}^2 v_x}{\mathrm{d}y^2} \lambda_{\mathrm{mpf}} v_{\mathrm{th}} \sim M \frac{v}{L^2} \lambda_{\mathrm{mpf}} v_{\mathrm{th}} \quad \rightarrow \quad t_{\mathrm{diss}} \sim \frac{p}{F_{\mathrm{viscous}}} \sim \frac{L^2}{\lambda_{\mathrm{mfp}} v_{\mathrm{th}}}$$

Mean free path

$$\lambda_{\rm mfp} = \frac{1}{n\sigma \ln\Lambda} \sim \frac{1}{\pi n \ln\Lambda}\left(\frac{k_{\rm B}T_{\rm e}}{Ze^2}\right)^2$$

$$\sim 5\ {\rm kpc}\left(\frac{n}{10^{-3}\ {\rm cm}^{-3}}\right)^{-1}\left(\frac{k_{\rm B}T_{\rm e}}{6\ {\rm keV}}\right)^2$$

center CC cluster (~ 1 kpc) $\rightarrow n \sim 10^{-1}$ cm^{-3}, $k_{\rm B}T \sim 3$ keV $\rightarrow 0.01$ kpc

center NCC cluster (~ 1 kpc) $\rightarrow n \sim 10^{-2}$ cm^{-3}, $k_{\rm B}T \sim 6$ keV $\rightarrow 0.5$ kpc

outskirts (~ 1 Mpc) $\rightarrow n \sim 10^{-4}$ cm^{-3} $\rightarrow 50.0$ kpc

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can be large compared to X-ray resolution

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-> Re ~ 20 to 100
laminar flow?

can be large compared to X-ray resolution

Lamor radius

$$r_\mathrm{L} = \frac{m_\mathrm{p} v_\perp c}{ZeB} = 10^5\ \mathrm{km} \left(\frac{v_\perp}{10^3\ \mathrm{km\ s^{-1}}}\right)\left(\frac{B}{1\,\mu\mathrm{G}}\right)^{-1}$$

$\sim 3 \times 10^{-12}$ kpc

Reynolds number perpendicular to magnetic field:

$$\mathrm{Re}_\perp = \frac{L}{r_\mathrm{L}} \frac{v}{v_\mathrm{th}} = 10^{14} \frac{v}{v_\mathrm{th}}$$ -> very turbulent

Turbulence spectrum in ICM

$$E_v(k) = C_{\mathrm{K}} \dot{\epsilon}^{2/3} k^{-5/3}$$ energy spectrum of velocity field (Kolmogorov)

wave number

Turbulence spectrum in ICM

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$$\frac{\delta\rho_k}{\rho_0} \approx \eta_{\mathrm{turb}}\frac{v_k}{c_{\mathrm{s}}}$$ density fluctuations related to turbulent velocities

-> these effect X-ray surface brightness

with $$\frac{3}{2}v_k^2 = kE_v(k)$$

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density fluctuations related to turbulent velocities
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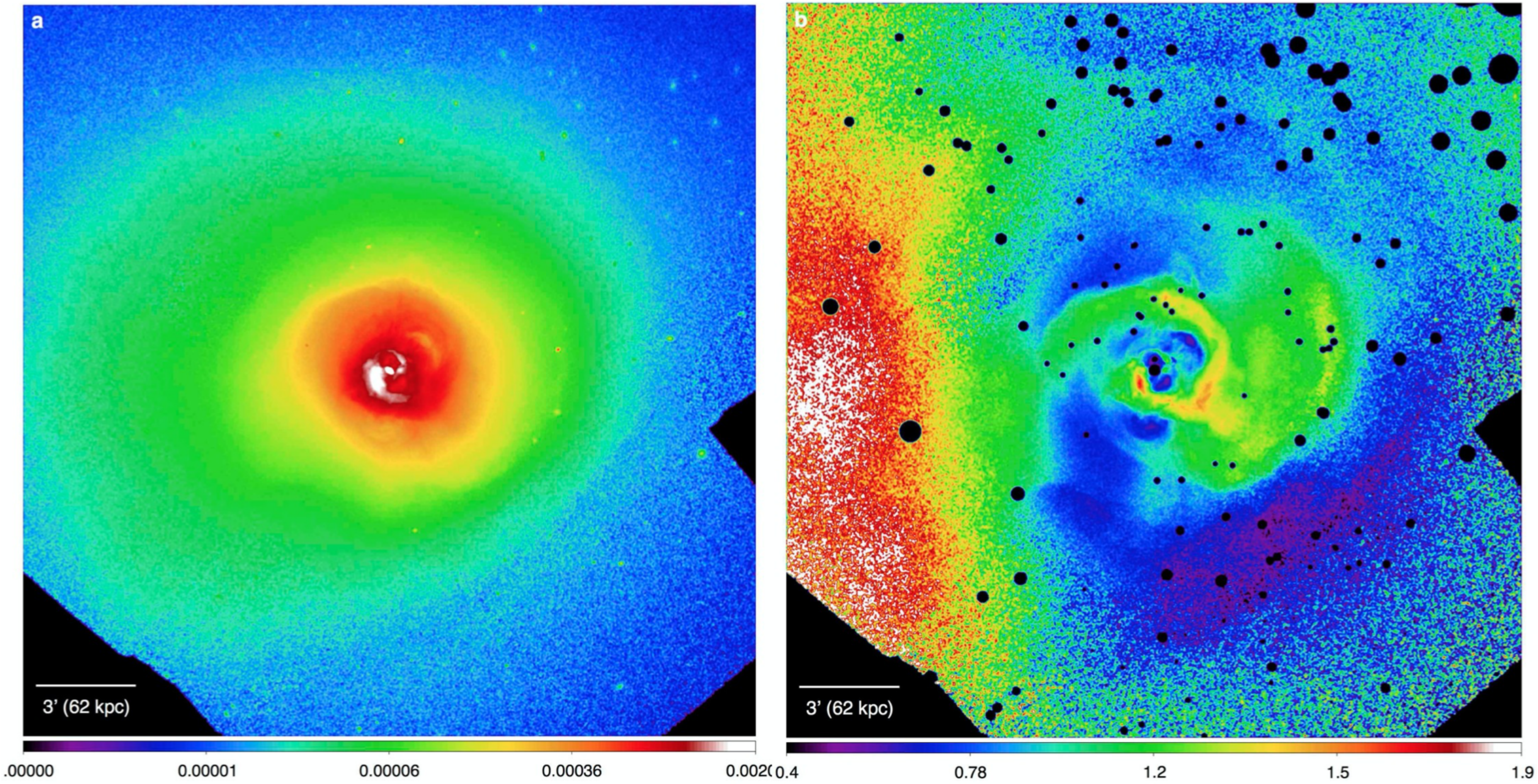
with $\frac{3}{2} v_k^2 = k E_v(k)$ -> $v_k \propto k^{-1/3}$

Measuring turbulence in the ICM from density fluctuations



Zhuravleva et al. 2014

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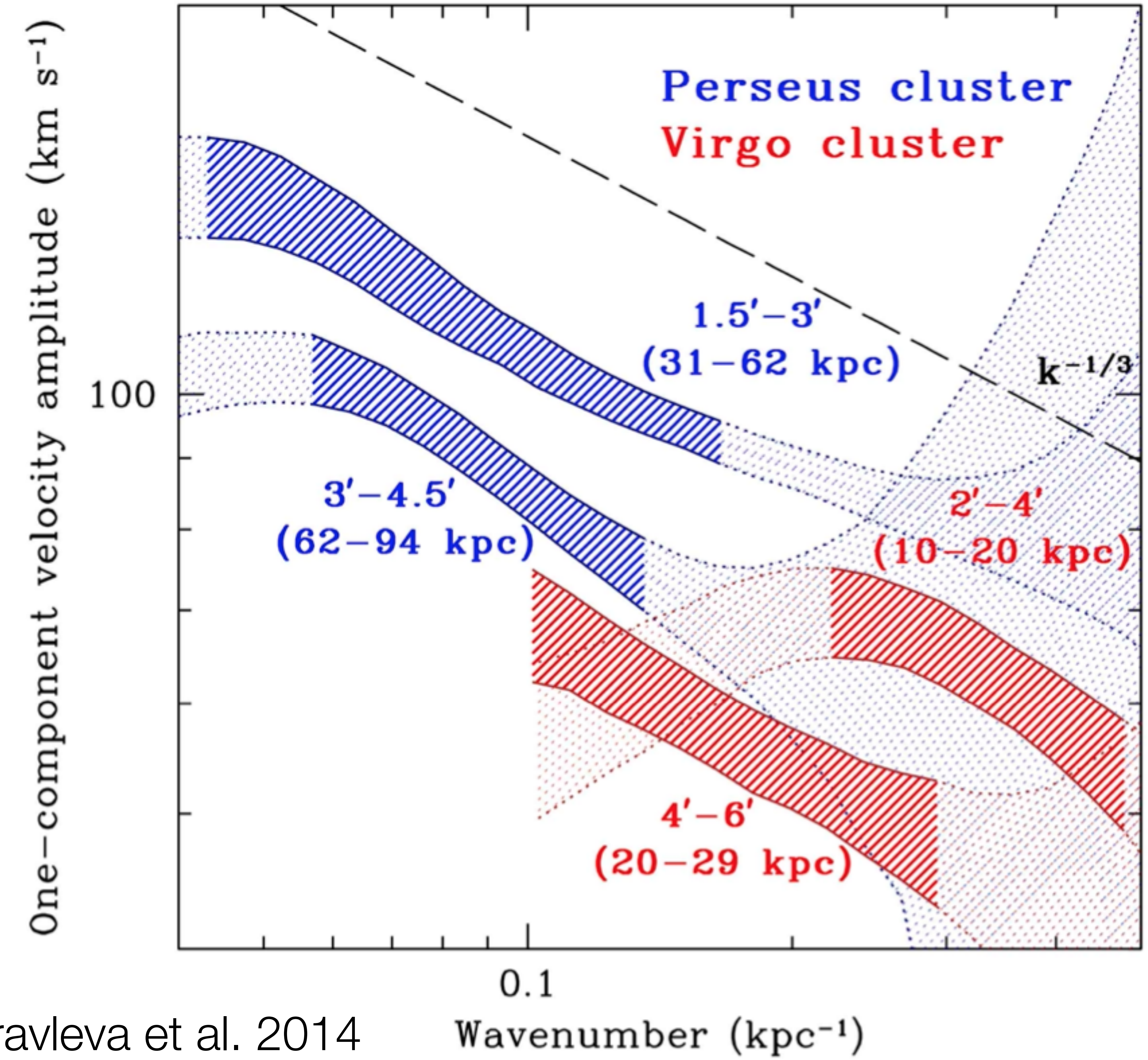
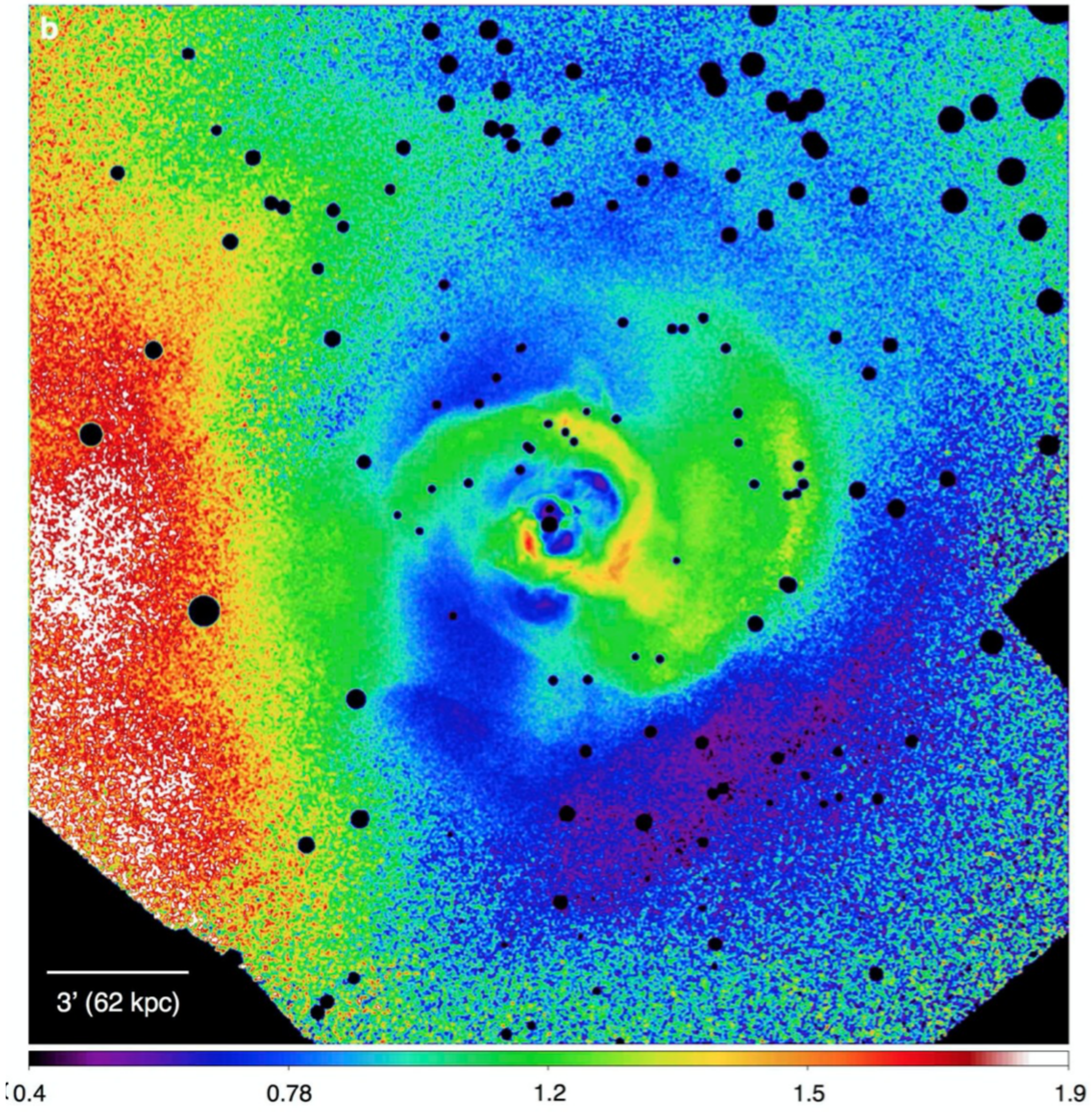




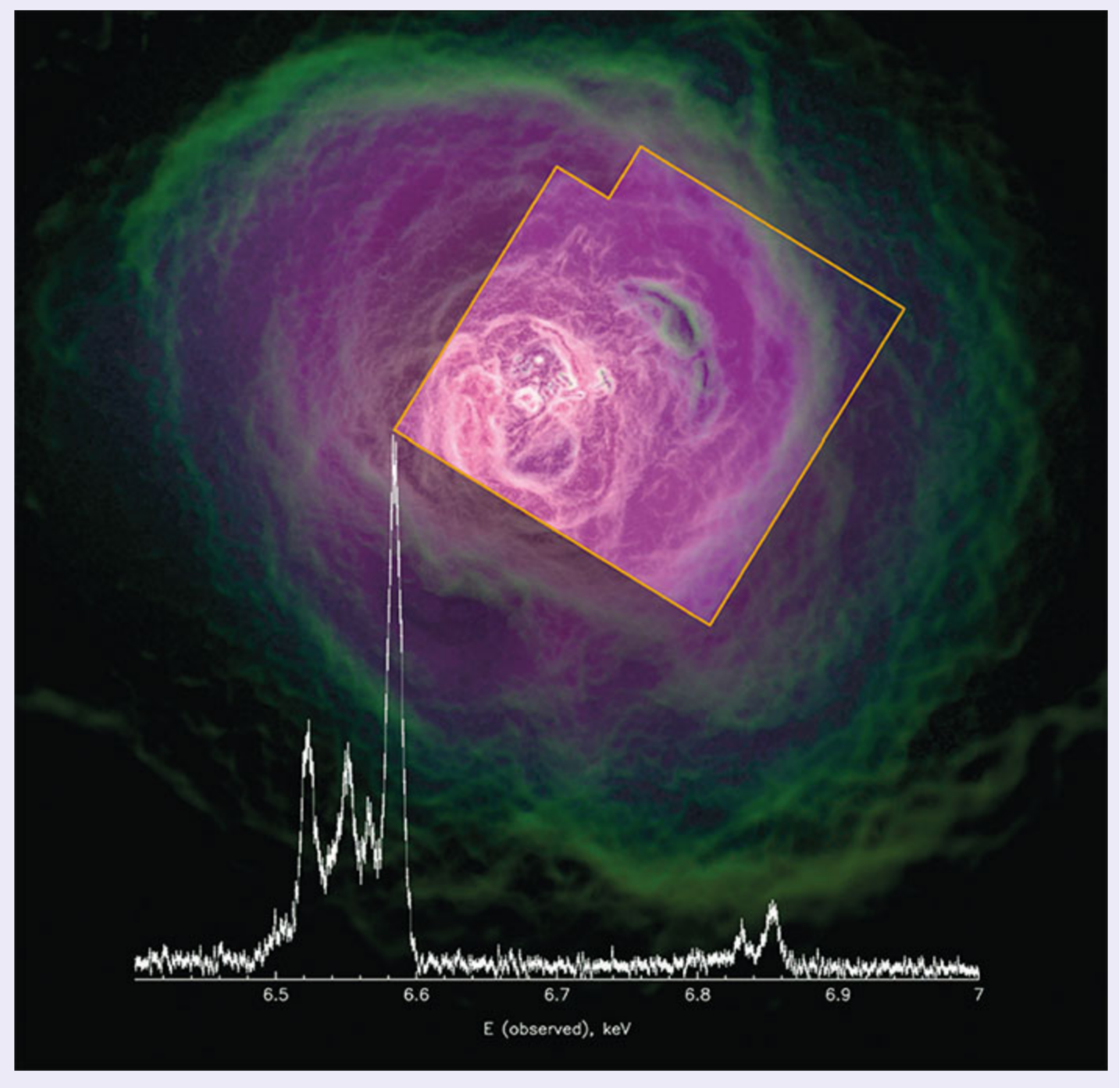
Zhuravleva et al. 2014

Measuring turbulence in the ICM from broadening of lines

Hitomi measured turbulent Doppler-broadening in the Perseus cluster

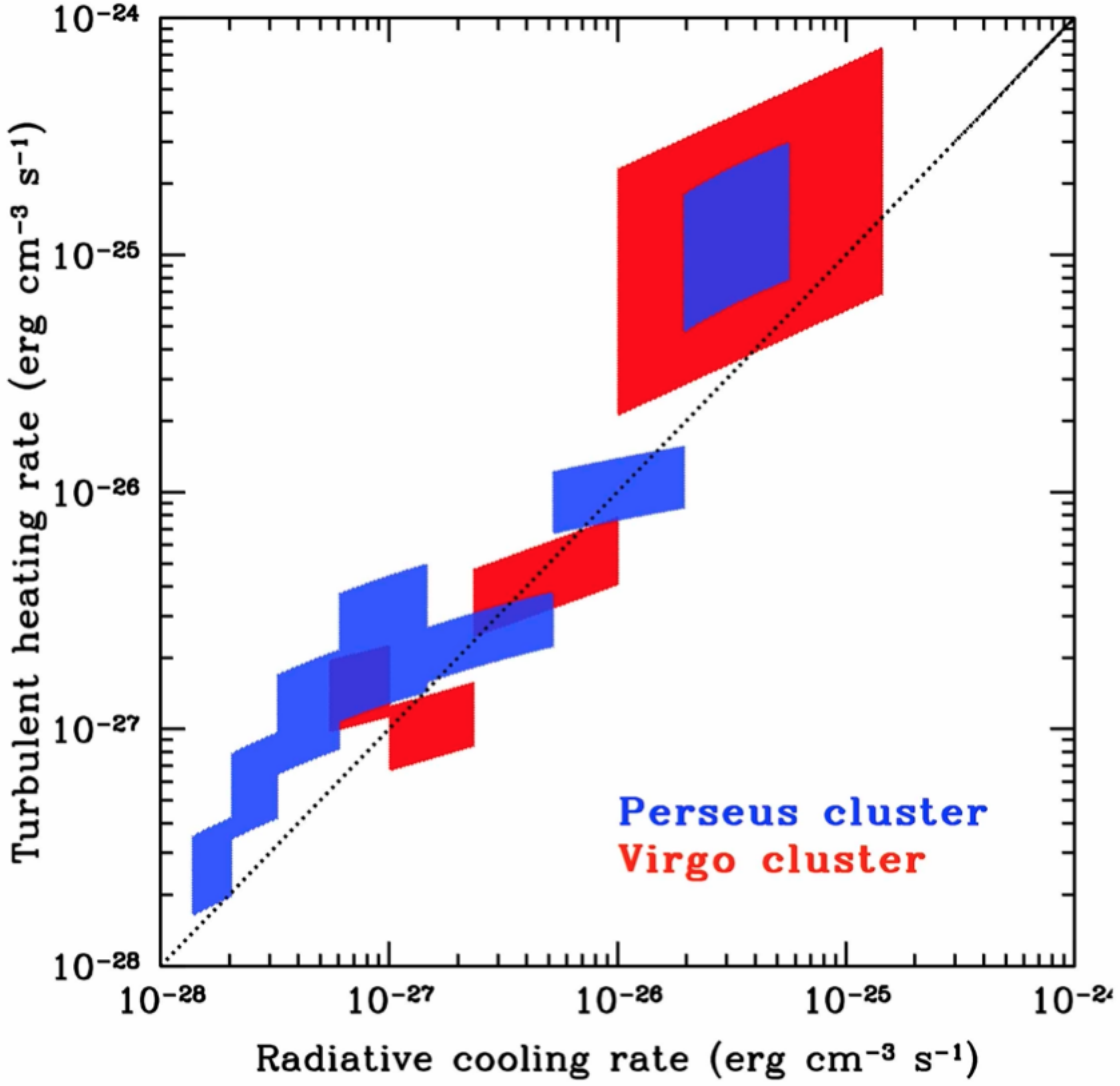
$$164 \pm 10\ \mathrm{km\ s^{-1}}$$

-> turbulent pressure only 4% of thermal pressure



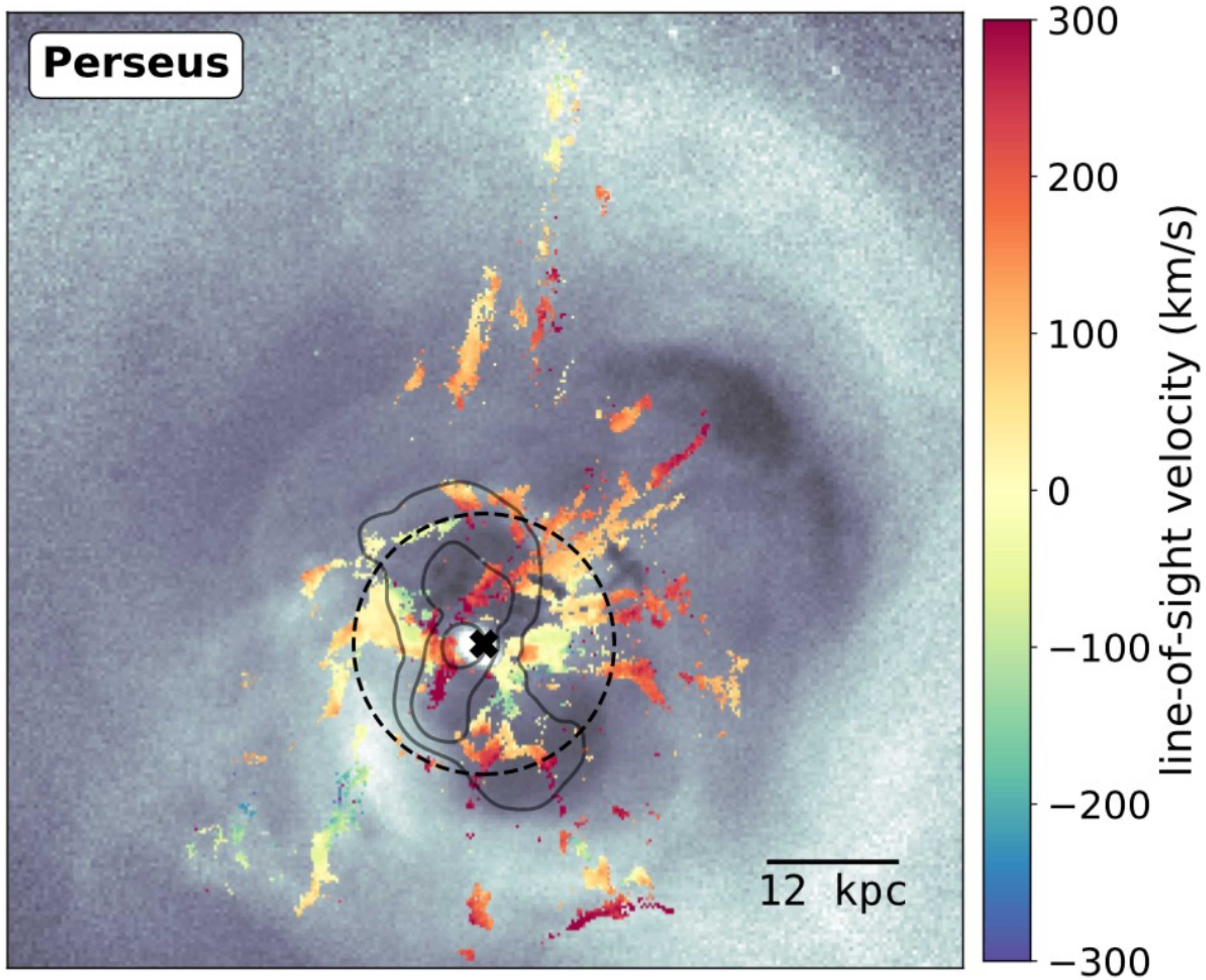
Hitomi Collaboration 2016

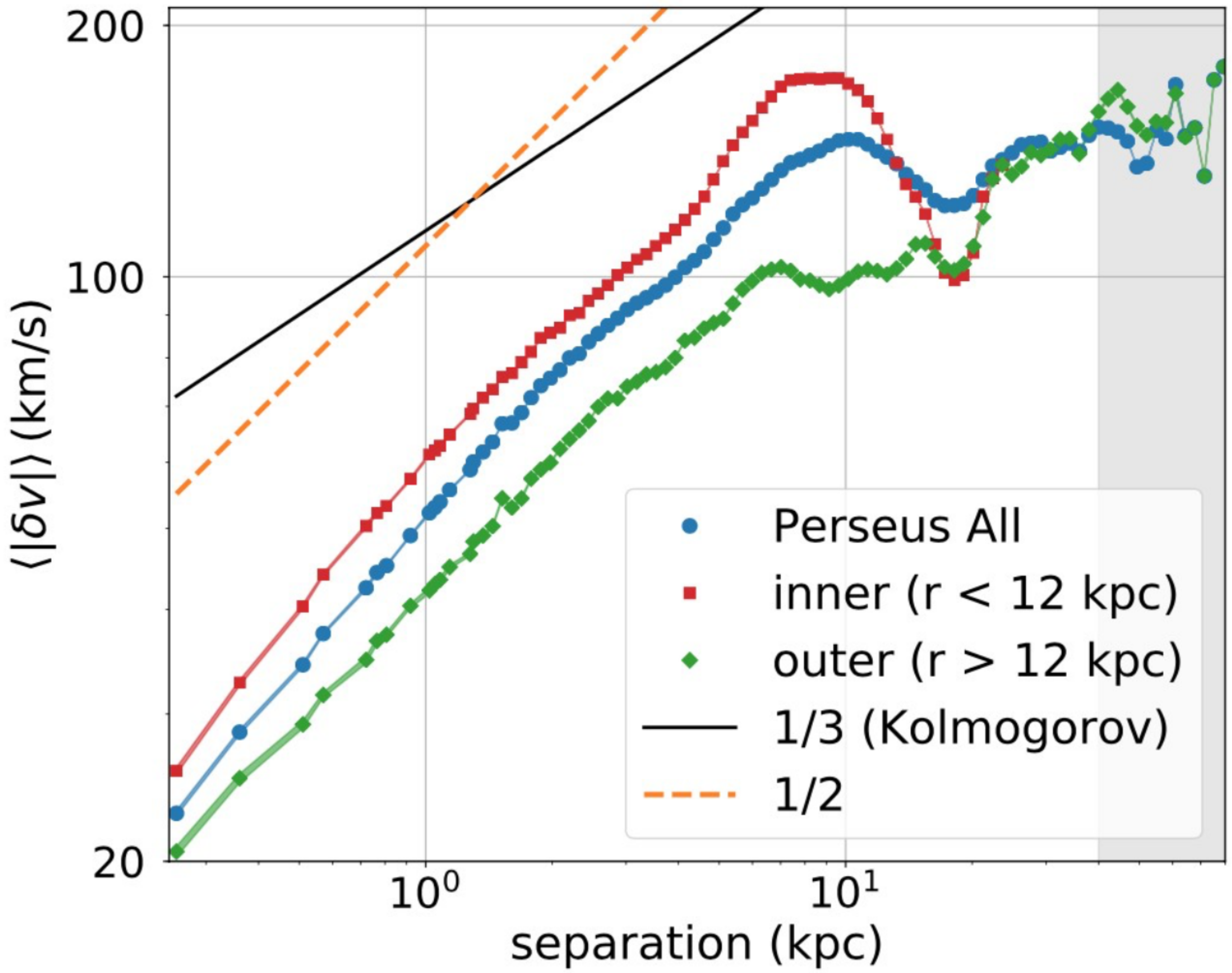
Heating by dissipation of turbulence



Zhuravleva et al. 2014

Measuring turbulence in the ICM from H-alpha velocities





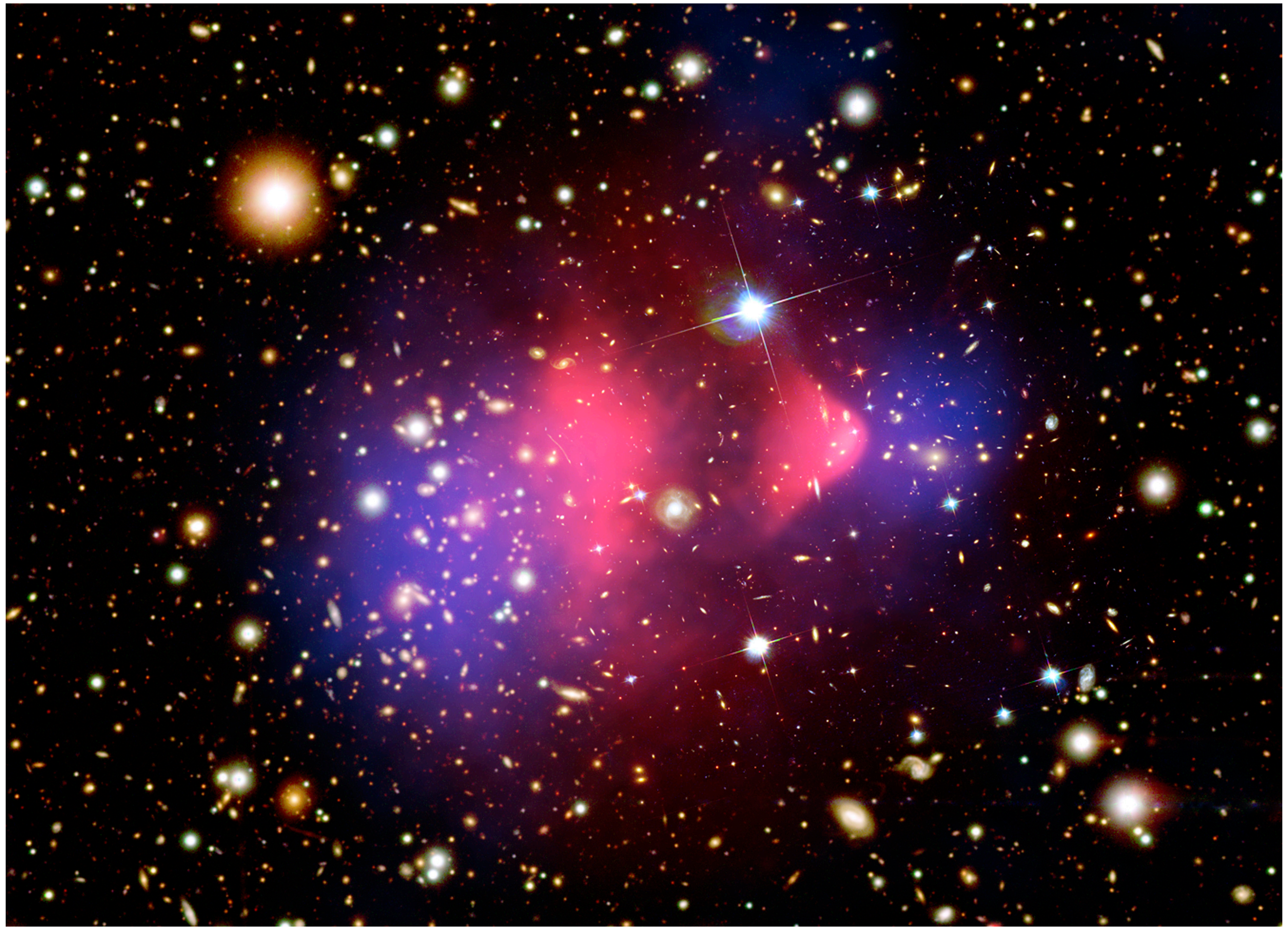
Li et al. 2020

Merger Shocks and Electron Equilibration

temperature boost by shock
(if no energy exchange):

$$\Delta(k_{\mathrm{B}}T_{\mathrm{i}}) \simeq m_{\mathrm{i}}(\Delta v)^2,$$
$$\Delta(k_{\mathrm{B}}T_{\mathrm{e}}) \simeq m_{\mathrm{e}}(\Delta v)^2,$$

The Bullet Cluster



X-ray: NASA/CXC/CfA/M.Markevitch, Optical/lensing: NASA/STScI, Magellan/U.Arizona/D.Clowe, Lensing: ESO

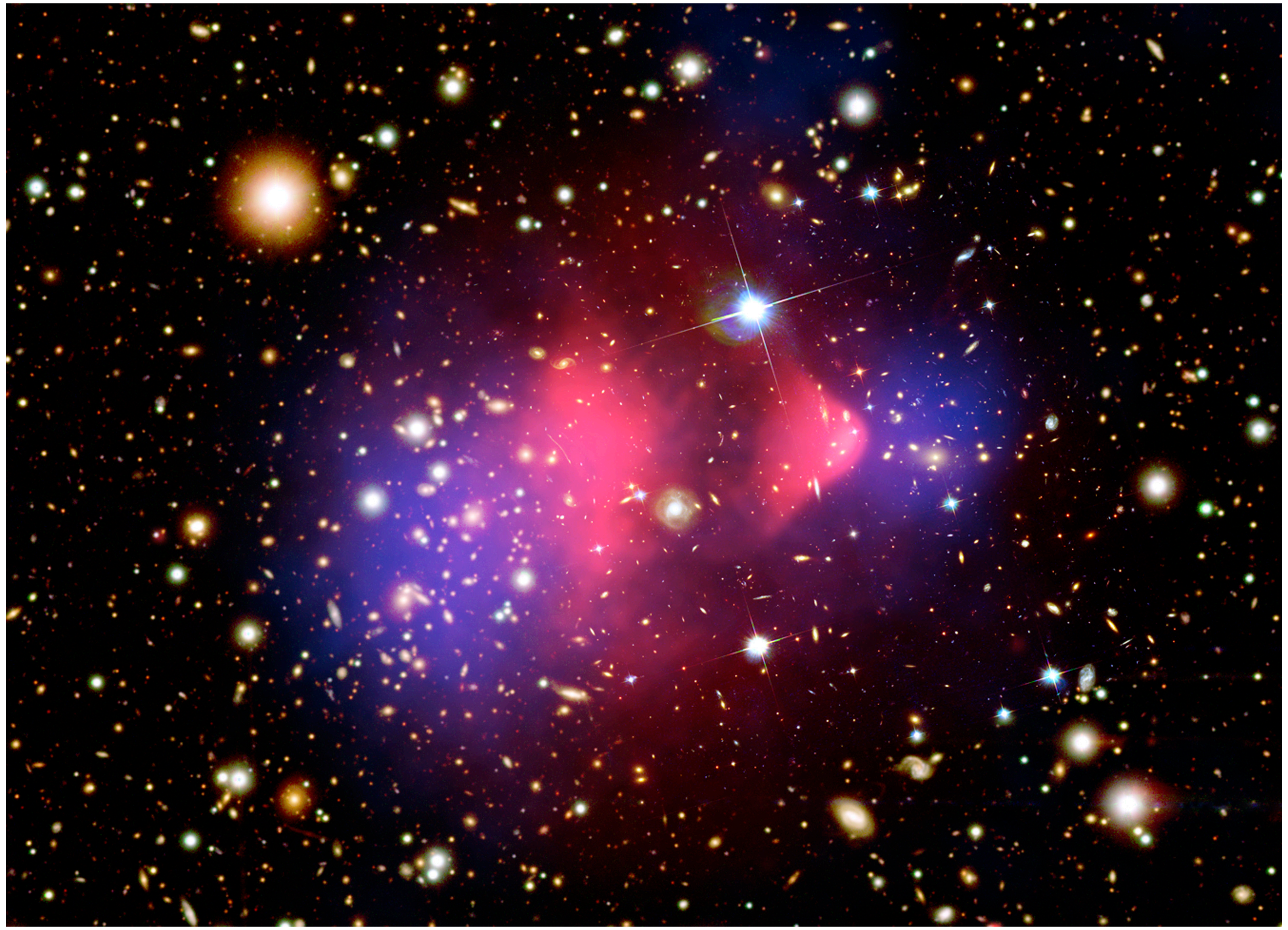
Merger Shocks and Electron Equilibration

temperature boost by shock
(if no energy exchange):

$$\Delta(k_B T_i) \simeq m_i (\Delta v)^2,$$
$$\Delta(k_B T_e) \simeq m_e (\Delta v)^2,$$

$$\Delta v = 1000\,\text{km/s} \rightarrow \Delta T_p \sim 4 \times 10^7\,\text{K}$$
$$\Delta T_e \sim 2 \times 10^4\,\text{K}$$

The Bullet Cluster



X-ray: NASA/CXC/CfA/M.Markevitch, Optical/lensing: NASA/STScI, Magellan/U.Arizona/D.Clowe, Lensing: ESO

What temperature do X-ray observations measure?

electron-ion
collision

electron-electron
collision

ion-ion
collision

collision
of different ions

high relative velocity
+ dipole moment

high relative velocity
no dipole moment

low relative velocity
no dipole moment

low relative velocity
dipole moment

What temperature do X-ray observations measure?

electron-ion collision	electron-electron collision	ion-ion collision	collision of different ions

high relative velocity + dipole moment	high relative velocity no dipole moment	low relative velocity no dipole moment	low relative velocity dipole moment

we measure this

Energy exchange between electrons and ions

temperature equilibration

$$\frac{\partial(T_\mathrm{e} - T_\mathrm{i})}{\partial t} = -\nu_\mathrm{ei}(T_\mathrm{e} - T_\mathrm{i}) \qquad \text{with} \qquad \nu_\mathrm{ei} \approx 4\frac{m_\mathrm{e}}{m_\mathrm{p}}\frac{v_\mathrm{the}}{\lambda_\mathrm{mfp}}$$

-> solved by:

$$T_\mathrm{e} - T_\mathrm{i} = \mathrm{e}^{-\nu_\mathrm{ei} t}$$

relevant scales:

$$\tau_\mathrm{ei} = \nu_\mathrm{ei}^{-1} \approx 95\ \mathrm{Myr}\left(\frac{k_\mathrm{B}T_\mathrm{e}}{10\ \mathrm{keV}}\right)^{3/2}\left(\frac{n_\mathrm{e}}{10^{-3}\ \mathrm{cm}^{-3}}\right)^{-1},$$

$$L_\mathrm{ei} = \frac{v_\mathrm{post}}{\tau_\mathrm{ei}} \approx 155\ \mathrm{kpc}\left(\frac{v_\mathrm{post}}{1600\ \mathrm{km\ s}^{-1}}\right),$$

Energy exchange between electrons and ions

temperature equilibration

$$\frac{\partial(T_\mathrm{e} - T_\mathrm{i})}{\partial t} = -\nu_\mathrm{ei}(T_\mathrm{e} - T_\mathrm{i}) \qquad \text{with} \qquad \nu_\mathrm{ei} \approx 4\frac{m_\mathrm{e}}{m_\mathrm{p}}\frac{\upsilon_\mathrm{the}}{\lambda_\mathrm{mfp}}$$

-> solved by:

$$T_\mathrm{e} - T_\mathrm{i} = \mathrm{e}^{-\nu_\mathrm{ei} t}$$

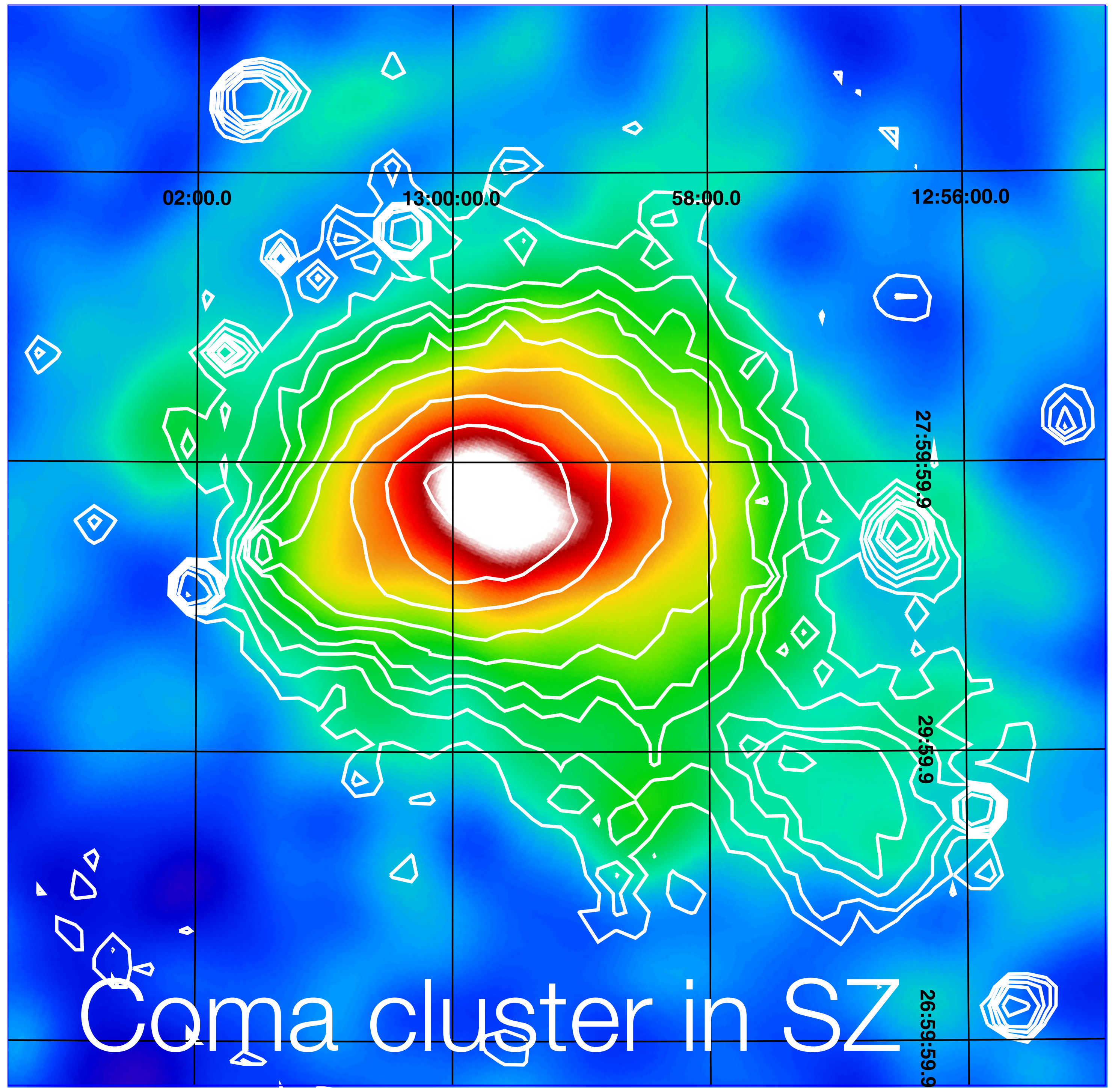
relevant scales:

but plasma physics effects missing in this simple picture
-> likely causing faster equilibration

The Sunyaev-Zel'dovich (SZ) Effect

- CMB photons can inverse Compton scatter on hot electrons in galaxy clusters
  - ➡ this results in deviations from the black body spectrum
  - ➡ can be used to probe the intracluster medium

Planck collaboration 2011



The Sunyaev-Zel’dovich (SZ) Effect

initial black body spectrum of the CMB:

$$I(x) = i_0 i(x) = i_0 \frac{x^3}{\mathrm{e}^x - 1}, \text{ where}$$

$$x = \frac{\hbar\omega}{k_\mathrm{B} T_\mathrm{cmb}},$$

$$i_0 = \frac{2(k_\mathrm{B} T_\mathrm{cmb})^3}{(hc)^2} = 22.8 \text{ Jy arcmin}^{-2}.$$

changes in spectrum described by the Kompaneets equation:

$$\frac{\partial n}{\partial t} = \frac{\sigma_\mathrm{T} n_\mathrm{e} \hbar}{m_\mathrm{e} c} \frac{1}{\omega^2} \frac{\partial}{\partial \omega} \left[\omega^4 \left(\frac{k_\mathrm{B} T_\mathrm{e}}{\hbar} \frac{\partial n}{\partial \omega} + n + n^2 \right) \right]$$

The Kompaneets equation

$$\frac{\partial n}{\boxed{\partial t}} = \frac{\boxed{\sigma_{\mathrm{T}} n_{\mathrm{e}}} \hbar}{\boxed{m_{\mathrm{e}} c}} \frac{1}{\omega^2} \frac{\partial}{\partial \omega} \left[\omega^4 \left(\frac{\boxed{k_{\mathrm{B}} T_{\mathrm{e}}}}{\hbar} \frac{\partial n}{\partial \omega} + n + n^2 \right) \right]$$

related to Bose-Einstein statistics

advection term

diffusion term (depending on energy transfer)

$$\boxed{\frac{d\tau}{dt} = \sigma_{\mathrm{T}} n_e c}$$

optical depth

$$\frac{\langle \Delta E_\gamma \rangle}{E_\gamma} = \boxed{\frac{k_{\mathrm{B}} T_{\mathrm{e}}}{m_{\mathrm{e}} c^2}} \quad \text{for} \quad p \ll m_{\mathrm{e}} c$$

energy transfer per scattering

The Kompaneets equation

$$\frac{\partial n}{\partial t} = \frac{\sigma_{\mathrm{T}} n_{\mathrm{e}} \hbar}{m_{\mathrm{e}} c} \frac{1}{\omega^2} \frac{\partial}{\partial \omega} \left[\omega^4 \left(\frac{k_{\mathrm{B}} T_{\mathrm{e}}}{\hbar} \frac{\partial n}{\partial \omega} + n + n^2 \right) \right]$$

convenient to use a variable y (called Compton-y parameter) and photon energy in units of the thermal energy

$$\mathrm{d}y = \frac{k_{\mathrm{B}} T_{\mathrm{e}}}{m_{\mathrm{e}} c^2} n_{\mathrm{e}} \sigma_{\mathrm{T}} c \mathrm{d}t \quad \text{and} \quad x_{\mathrm{e}} = \frac{\hbar \omega}{k_{\mathrm{B}} T_{\mathrm{e}}}$$

-> $$\frac{\partial n}{\partial y} = \frac{1}{x_{\mathrm{e}}^2} \frac{\partial}{\partial x_{\mathrm{e}}} \left[x_{\mathrm{e}}^4 \left(\frac{\partial n}{\partial x_{\mathrm{e}}} + n + n^2 \right) \right]$$

The Kompaneets equation

$$\frac{\partial n}{\partial t} = \frac{\sigma_{\mathrm{T}} n_{\mathrm{e}} \hbar}{m_{\mathrm{e}} c} \frac{1}{\omega^2} \frac{\partial}{\partial \omega} \left[\omega^4 \left(\frac{k_{\mathrm{B}} T_{\mathrm{e}}}{\hbar} \frac{\partial n}{\partial \omega} + n + n^2 \right) \right]$$

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example - equilibrium:

$$\frac{\partial n}{\partial x_{\mathrm{e}}} + n + n^2 = 0$$

$$n(x_{\mathrm{e}}) = \frac{1}{\mathrm{e}^{x_{\mathrm{e}} + \mu_{\mathrm{c}}} - 1}$$

The thermal SZ effect

$$\frac{\partial n}{\partial y} = \frac{1}{x_e^2}\frac{\partial}{\partial x_e}\left[x_e^4\left(\frac{\partial n}{\partial x_e} + n + n^2\right)\right]$$

Kompaneets eq.

for $x_e \ll 1$

(as CMB temperature ~ 10^7 times smaller than electron temperature)

$$\frac{\partial n}{\partial y} = \frac{1}{x_e^2}\frac{\partial}{\partial x_e}\left(x_e^4\frac{\partial n}{\partial x_e}\right)$$

$$= \frac{1}{x^2}\frac{\partial}{\partial x}\left(x^4\frac{\partial n}{\partial x}\right) = 4x\frac{\partial n}{\partial x} + x^2\frac{\partial^2 n}{\partial x^2}$$

$$n(x) = \frac{1}{e^x - 1}$$

Planckian distribution of CMB

with $x = \dfrac{\hbar\omega}{k_B T_{cmb}}$

-> $$\frac{\partial n}{\partial y} = \frac{x e^x}{(e^x - 1)^2}\left(x\frac{e^x + 1}{e^x - 1} - 4\right)$$

The thermal SZ effect

using $$\frac{\partial n}{\partial y} = \frac{x\mathrm{e}^x}{(\mathrm{e}^x - 1)^2}\left(x\,\frac{\mathrm{e}^x + 1}{\mathrm{e}^x - 1} - 4\right)$$ and $$\frac{\Delta I}{I} = \frac{\Delta n}{n}$$

->

$$\Delta I_{\mathrm{tSZ}}(x, \boldsymbol{\theta}) = i_0 y(\boldsymbol{\theta}) g(x), \quad \text{where}$$

$$g(x) = \frac{x^4\mathrm{e}^x}{(\mathrm{e}^x - 1)^2}\left(x\,\frac{\mathrm{e}^x + 1}{\mathrm{e}^x - 1} - 4\right), \quad \text{and}$$

$$y(\boldsymbol{\theta}) = \frac{\sigma_{\mathrm{T}}}{m_{\mathrm{e}}c^2}\int n_{\mathrm{e}}(\boldsymbol{r}) k_{\mathrm{B}} T_{\mathrm{e}}(\boldsymbol{r}) c\,\mathrm{d}t$$

or in terms of the brightness temperature of the modified CMB

$$\frac{\Delta T_{\mathrm{tSZ}}}{T}(\boldsymbol{\theta}) = y(\boldsymbol{\theta})\left(x\,\frac{\mathrm{e}^x + 1}{\mathrm{e}^x - 1} - 4\right) \equiv y(\boldsymbol{\theta}) f(x)$$

The thermal SZ effect

using $$\frac{\partial n}{\partial y} = \frac{x e^x}{(e^x - 1)^2}\left(x\frac{e^x + 1}{e^x - 1} - 4\right)$$ and $$\frac{\Delta I}{I} = \frac{\Delta n}{n}$$

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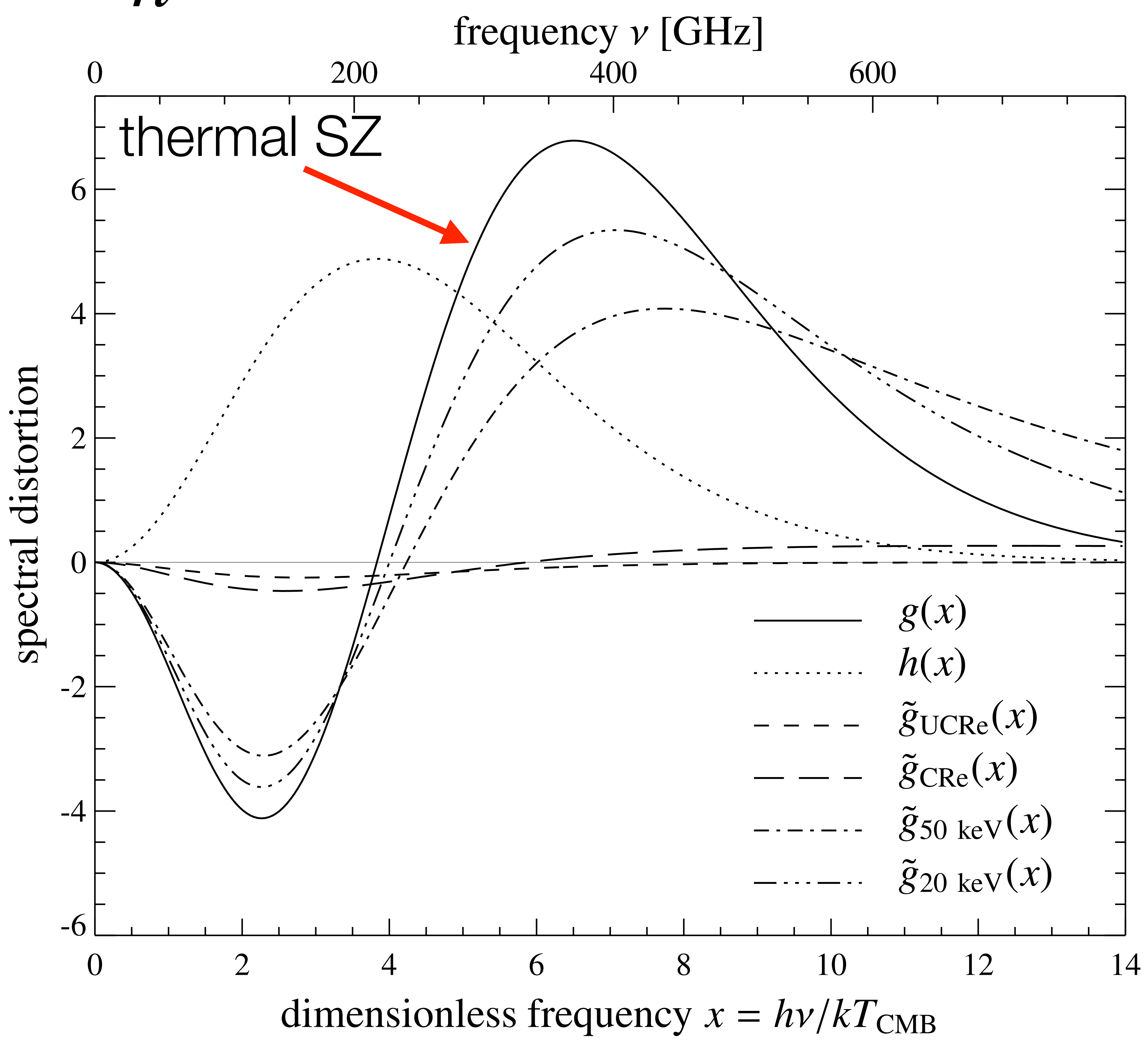

Pfrommer et al. 2005

The thermal SZ effect

using $$\frac{\partial n}{\partial y} = \frac{x e^x}{(e^x - 1)^2}\left(x\frac{e^x + 1}{e^x - 1} - 4\right)$$ and $$\frac{\Delta I}{I} = \frac{\Delta n}{n}$$

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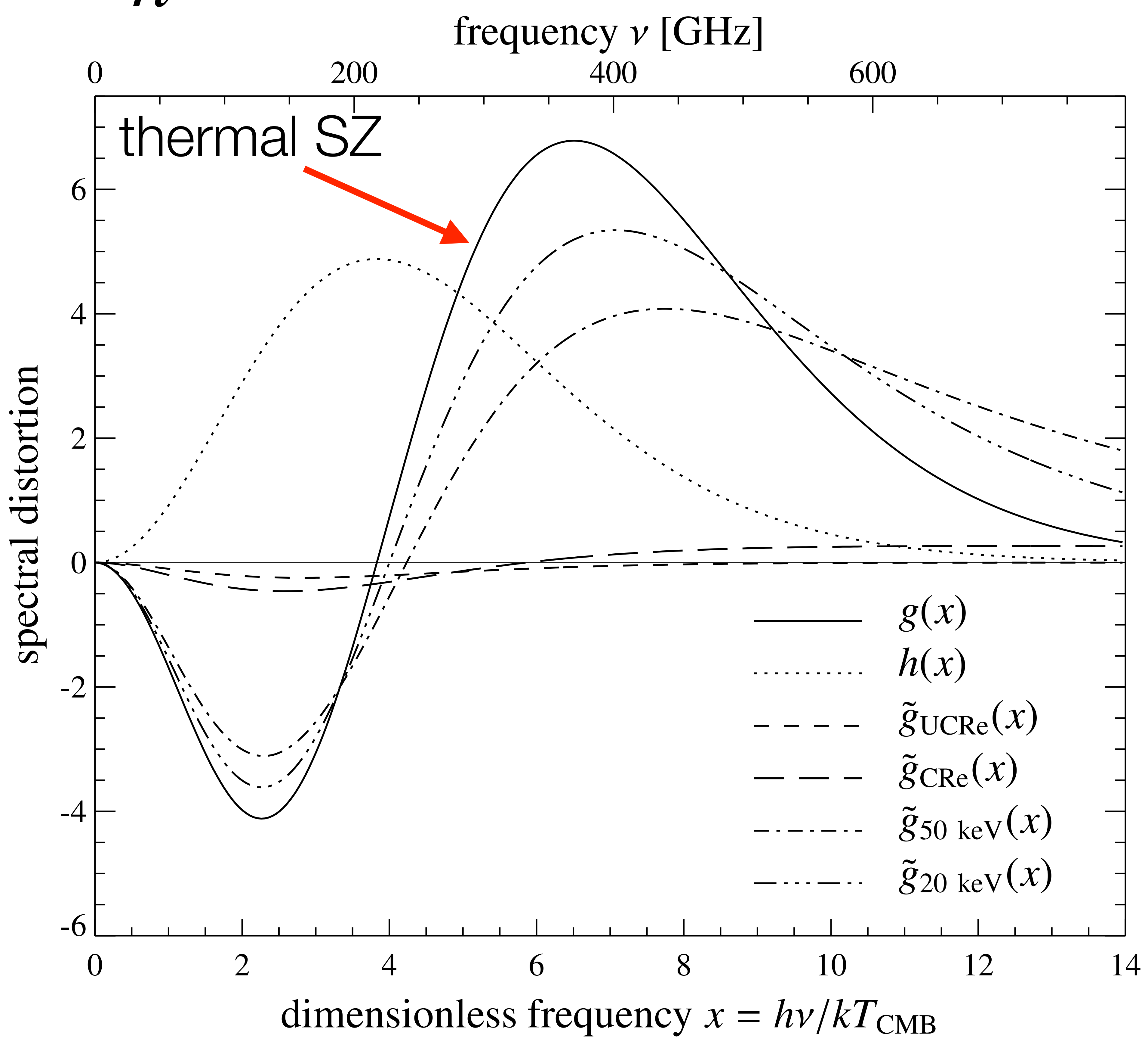
linear in density

or in terms of the brightness t... f the n

$$\frac{\Delta T_{\mathrm{tSZ}}}{T}(\boldsymbol{\theta}) = y(\boldsymbol{\theta})\left(x\frac{e^x + 1}{e^x - 1} - 4\right) \equiv y(\boldsymbol{\theta}) f(x)$$



Pfrommer et al. 2005

The thermal SZ effect

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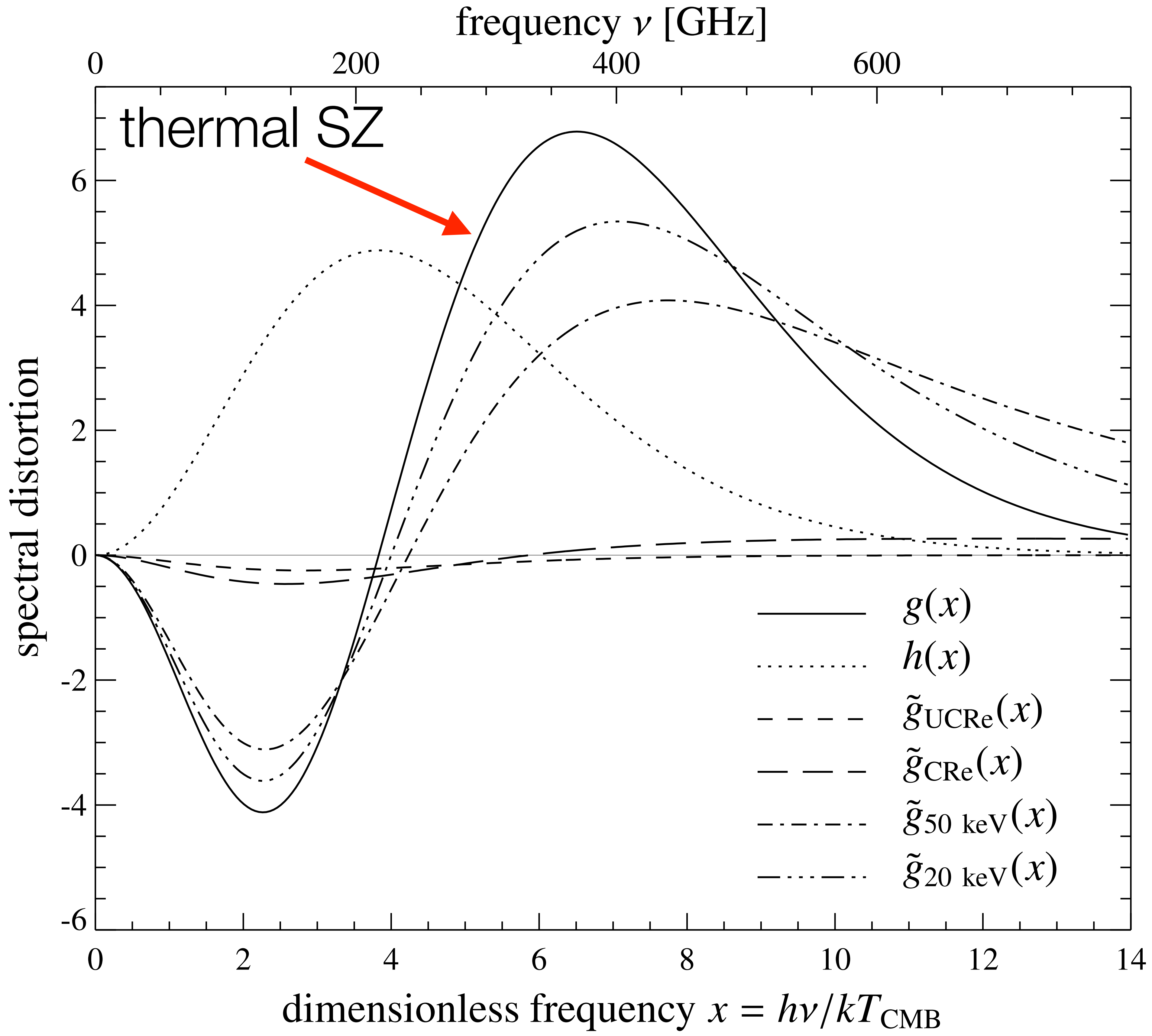
$$y(\boldsymbol{\theta}) = \frac{\sigma_{\mathrm{T}}}{m_e c^2}\int n_e(\boldsymbol{r}) k_B T_e(\boldsymbol{r}) c\,\mathrm{d}t$$

linear in density

or in terms of the brightness t f the n

$$\frac{\Delta T_{\mathrm{tSZ}}}{T}(\boldsymbol{\theta}) = y(\boldsymbol{\theta})\left(x\frac{e^x + 1}{e^x - 1} - 4\right) \equiv y(\boldsymbol{\theta}) f(x)$$

relative effect -> independent of redshift

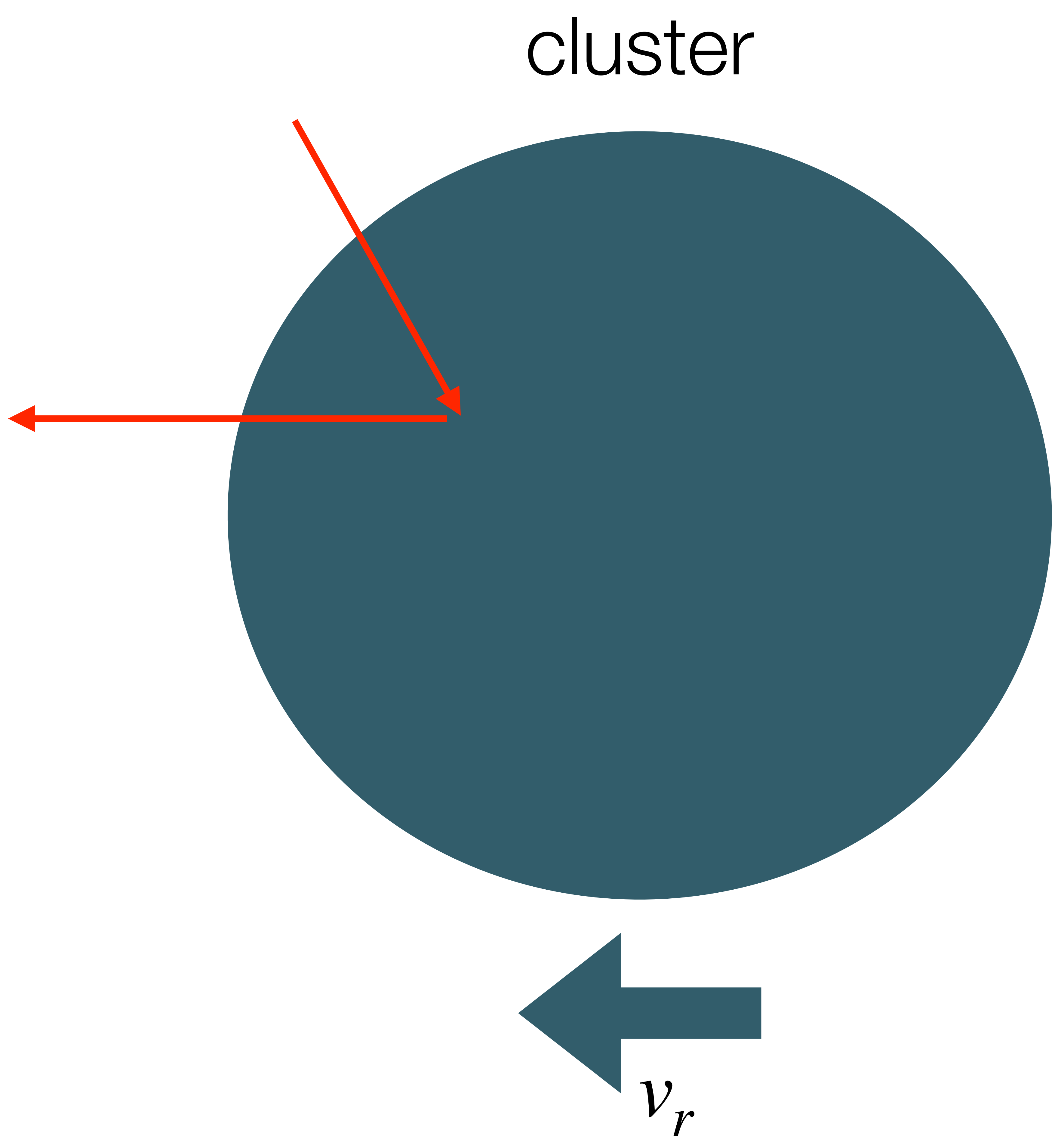


Pfrommer et al. 2005

The kinetic SZ effect

Doppler shift due to radial velocity of cluster:

$$\frac{\Delta T_{\mathrm{kSZ}}}{T}(\boldsymbol{\theta}) = -w(\boldsymbol{\theta}),$$
$$w(\boldsymbol{\theta}) \equiv \sigma_{\mathrm{T}} \int \mathrm{d}l\, n_{\mathrm{e}}(\boldsymbol{r})\, \frac{v_r}{c}$$



The kinetic SZ effect

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$$\frac{\Delta T_{\mathrm{kSZ}}}{T}(\boldsymbol{\theta}) = -w(\boldsymbol{\theta}),$$

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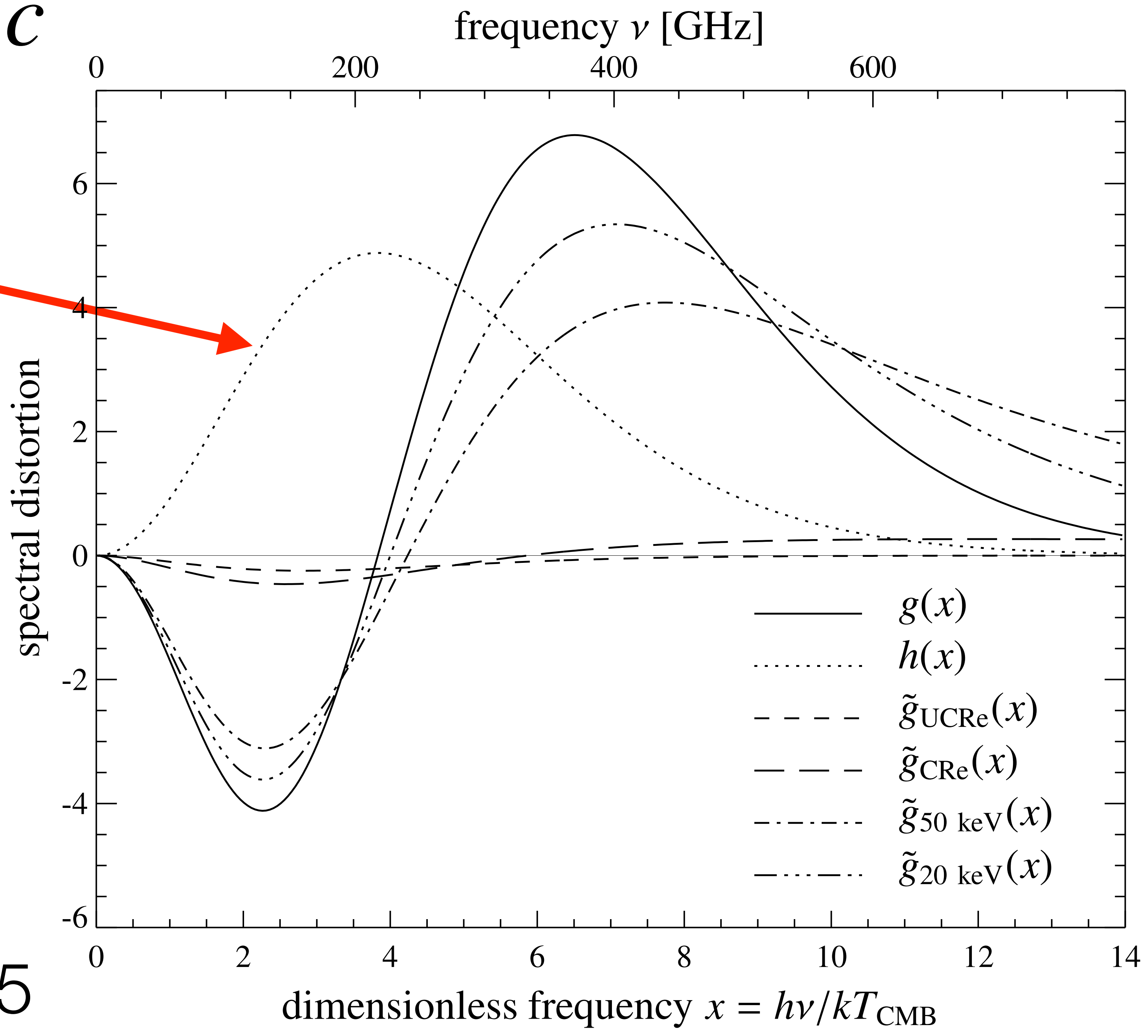
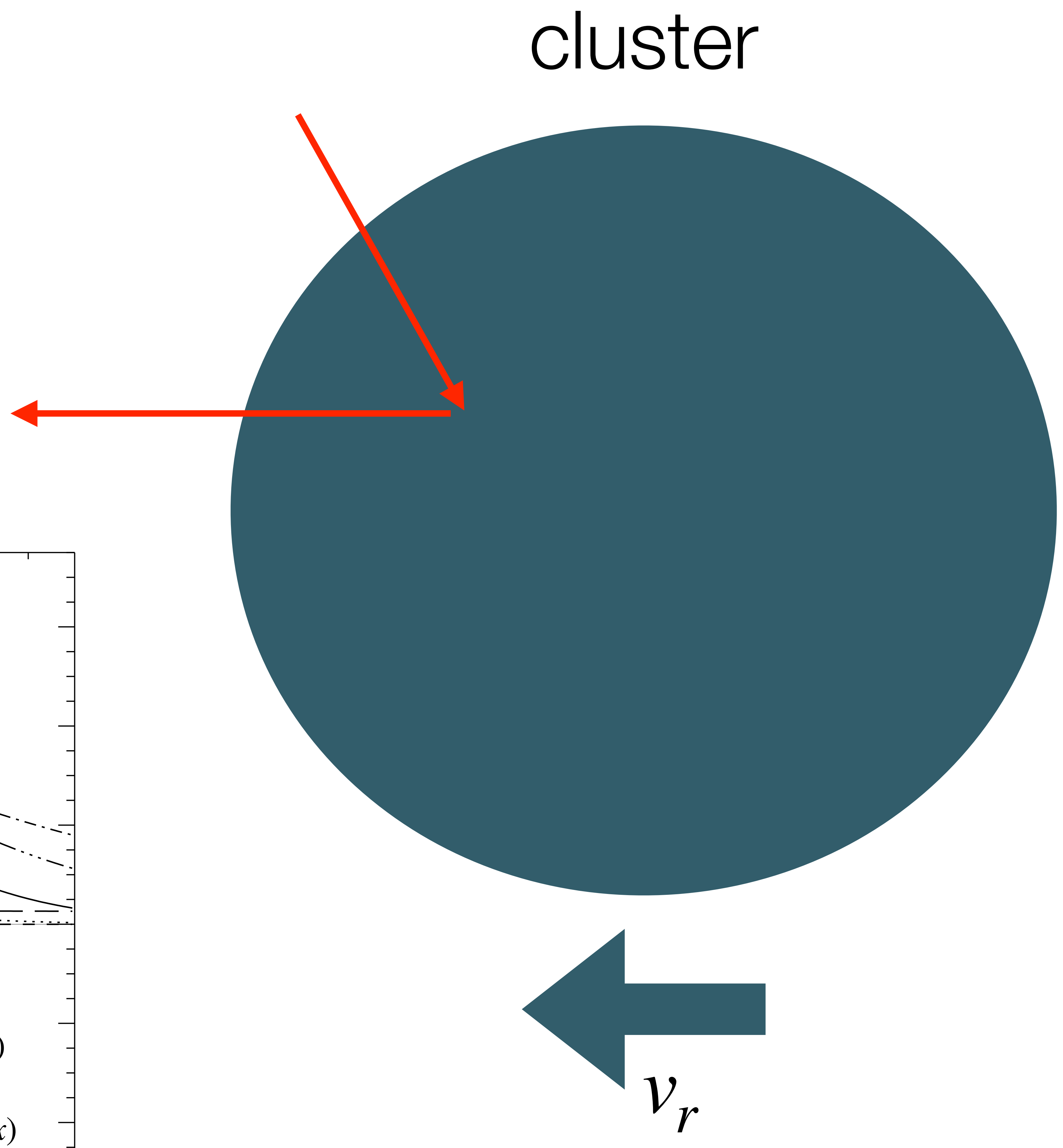
kinetic SZ



Pfrommer et al. 2005



The relativistic SZ effect

energy transfer in scattering:

$$\frac{\langle \Delta E_\gamma \rangle}{E_\gamma} = \frac{k_B T_e}{m_e c^2} \quad \text{for} \quad p \ll m_e c$$ non-relativistic electrons

$$\frac{\langle \Delta E_\gamma \rangle}{E_\gamma} = \frac{4}{3}\gamma_e^2 \quad \text{for} \quad p \gtrsim m_e c$$ relativistic electrons

full SZ effect:

$$\begin{aligned} \delta i(x) &= g(x)\, y\, [1 + \delta(x, T_e)] - h(x)\, w \\ &\quad + [\underbrace{j(x)}_{\text{scattering in}} - \underbrace{i(x)}_{\text{scattering away}}]\, \tau_{\rm rel}\,, \end{aligned}$$

$$\tau_{\rm rel} = \sigma_T \int \mathrm{d}l\, n_{\rm e,rel}$$

The relativistic SZ effect

energy transfer in scattering:

$$\frac{\langle \Delta E_\gamma \rangle}{E_\gamma} = \frac{k_B T_e}{m_e c^2} \quad \text{for} \quad p \ll m_e c$$

non-relativistic electrons

$$\frac{\langle \Delta E_\gamma \rangle}{E_\gamma} = \frac{4}{3} \gamma_e^2 \quad \text{for} \quad p \gtrsim m_e c$$

relativistic electrons

full SZ effect:

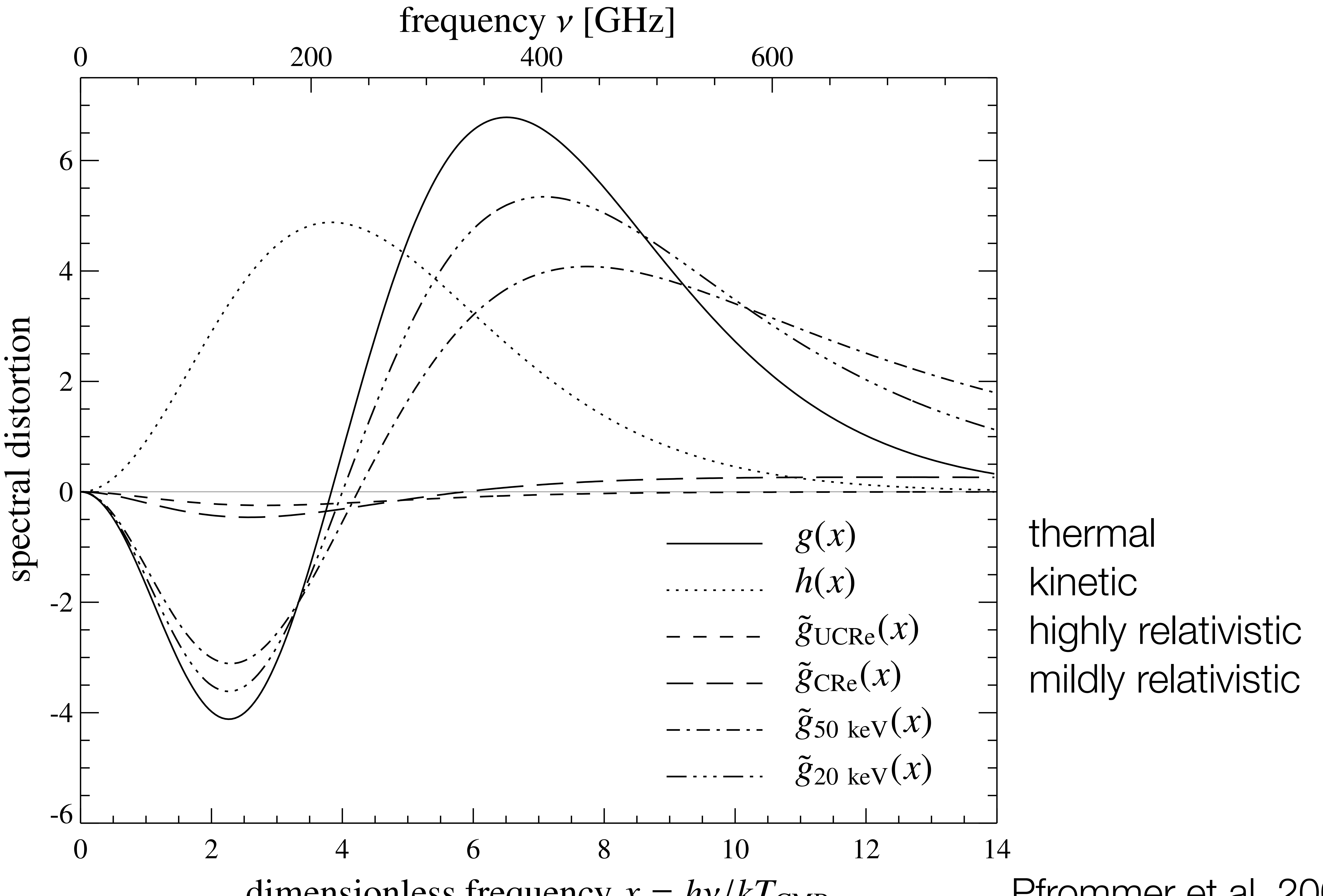
$$\begin{aligned} \delta i(x) &= g(x)\, y \, [1 + \delta(x, T_e)] - h(x)\, w \\ &\quad + [\underbrace{j(x)}_{\text{scattering in}} - \underbrace{i(x)}_{\text{scattering away}}]\, \tau_{rel} \,, \end{aligned}$$

$$\tau_{rel} = \sigma_T \int dl\, n_{e,rel}$$

-> not local anymore:

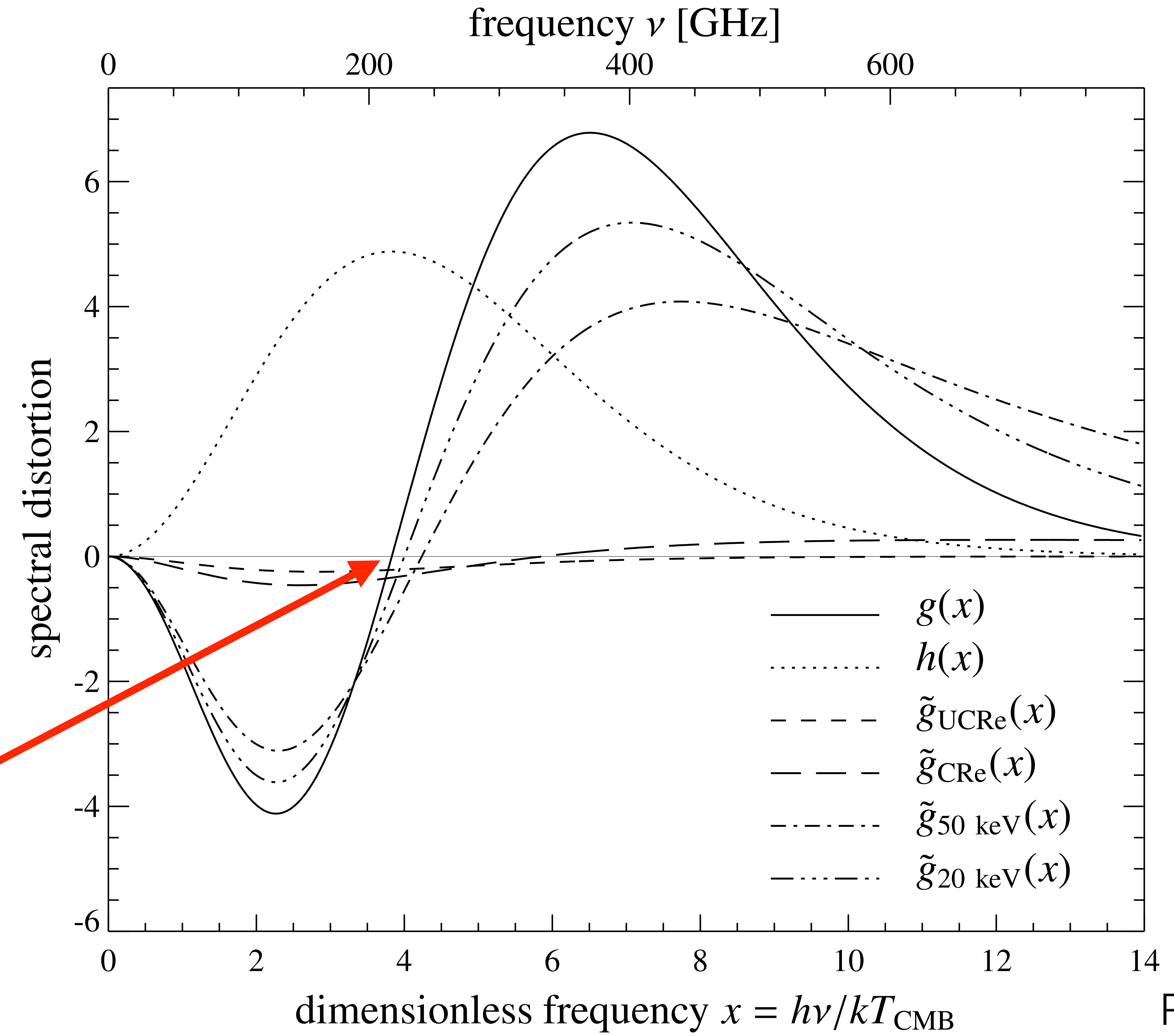
$$j(x) = \int_0^\infty ds \int_0^\infty dp\, f_e(p)\, P(s,p)\, i(x/s)$$

The complete SZ effect



Pfrommer et al. 2005

The complete SZ effect



thermal
kinetic
highly relativistic
mildly relativistic

Note that all additions (kinetic, relativistic) modify the zero point!

Pfrommer et al. 2005

Integrated parameters

$$Y = \int_\Omega \mathrm{d}\Omega\, y = \frac{1}{D_\mathrm{ang}^2} \int_A \mathrm{d}^2 r\, y = \frac{1}{D_\mathrm{ang}^2} \frac{\sigma_\mathrm{T}}{m_\mathrm{e} c^2} \int_A \mathrm{d}^3 r\, P_\mathrm{e}, \quad \propto E_\mathrm{thermal}$$

overall "amplitude" of thermal SZ effect of cluster

$$W = \int_\Omega \mathrm{d}\Omega\, w = \frac{1}{D_\mathrm{ang}^2} \int_A \mathrm{d}^2 r\, w, = \frac{1}{D_\mathrm{ang}^2} \sigma_\mathrm{T} \int_A \mathrm{d}^3 r\, n_\mathrm{e} \frac{v_r}{c}, \quad \propto \text{momentum}$$

overall "amplitude" of kinetic SZ effect of cluster

$$Y_\mathrm{X} = \frac{1}{D_\mathrm{ang}^2} \frac{\sigma_\mathrm{T}}{m_\mathrm{e} c^2} \int_A \mathrm{d}^3 r\, n_\mathrm{X} k_\mathrm{B} T_\mathrm{X}$$

for comparison/matching similar quantity defined for X-ray measurements

$$Y_\mathrm{sph} = \frac{\sigma_\mathrm{T}}{m_\mathrm{e} c^2} \int_0^{R_{200}} \mathrm{d}V P_\mathrm{e} = \frac{(\gamma - 1)\, \sigma_\mathrm{T}}{m_\mathrm{e} c^2} \tilde{x}_\mathrm{e}\, X \mu\, E_\mathrm{gas}$$

integrated *Y* can also be restricted, e.g., to within virial radius

Integrated parameters

$$Y = \int_\Omega \mathrm{d}\Omega\, y = \frac{1}{D_\mathrm{ang}^2} \int_A \mathrm{d}^2 r\, y = \frac{1}{D_\mathrm{ang}^2} \frac{\sigma_\mathrm{T}}{m_\mathrm{e} c^2} \int_A \mathrm{d}^3 r\, P_\mathrm{e}, \quad \propto E_\mathrm{thermal}$$

overall "amplitude" of thermal SZ effect of cluster

$$W = \int_\Omega \mathrm{d}\Omega\, w = \frac{1}{D_\mathrm{ang}^2} \int_A \mathrm{d}^2 r\, w, = \frac{1}{D_\mathrm{ang}^2} \sigma_\mathrm{T} \int_A \mathrm{d}^3 r\, n_\mathrm{e} \frac{v_r}{c}, \quad \propto \mathrm{momentum}$$

overall "amplitude" of kinetic SZ effect of cluster

$$Y_\mathrm{X} = \frac{1}{D_\mathrm{ang}^2} \frac{\sigma_\mathrm{T}}{m_\mathrm{e} c^2} \int_A \mathrm{d}^3 r\, n_\mathrm{X} k_\mathrm{B} T_\mathrm{X}$$

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$$= \frac{n_\mathrm{e}}{n_\mathrm{H} + n_\mathrm{He} + n_\mathrm{e}}$$

integrated *Y* can also be restricted, e.g., to within virial radius

Self-similar SZ scaling relations

want to relate this to cluster mass:

$$Y_{\mathrm{sph}} = \frac{\sigma_{\mathrm{T}}}{m_{\mathrm{e}}c^2} \int_0^{R_{200}} \mathrm{d}V P_{\mathrm{e}} = \frac{(\gamma - 1)\,\sigma_{\mathrm{T}}}{m_{\mathrm{e}}c^2}\, \tilde{x}_{\mathrm{e}}\, X\, \mu\, E_{\mathrm{gas}}$$

characteristic temperature:

$$kT_{200} = \frac{G M_{200}\, \mu\, m_{\mathrm{p}}}{3 R_{200}} = \frac{\mu\, m_{\mathrm{p}}}{3} \left[10\, G\, M_{200}\, H_0\, E(z)\right]^{2/3}$$

$$M_{200} = \frac{4}{3}\pi r_{200}^3 \times 200\, \rho_{\mathrm{crit}} \qquad \rho_{crit} = \frac{3H^2}{8\pi G} = \frac{3H_0^2 E^2(z)}{8\pi G}$$

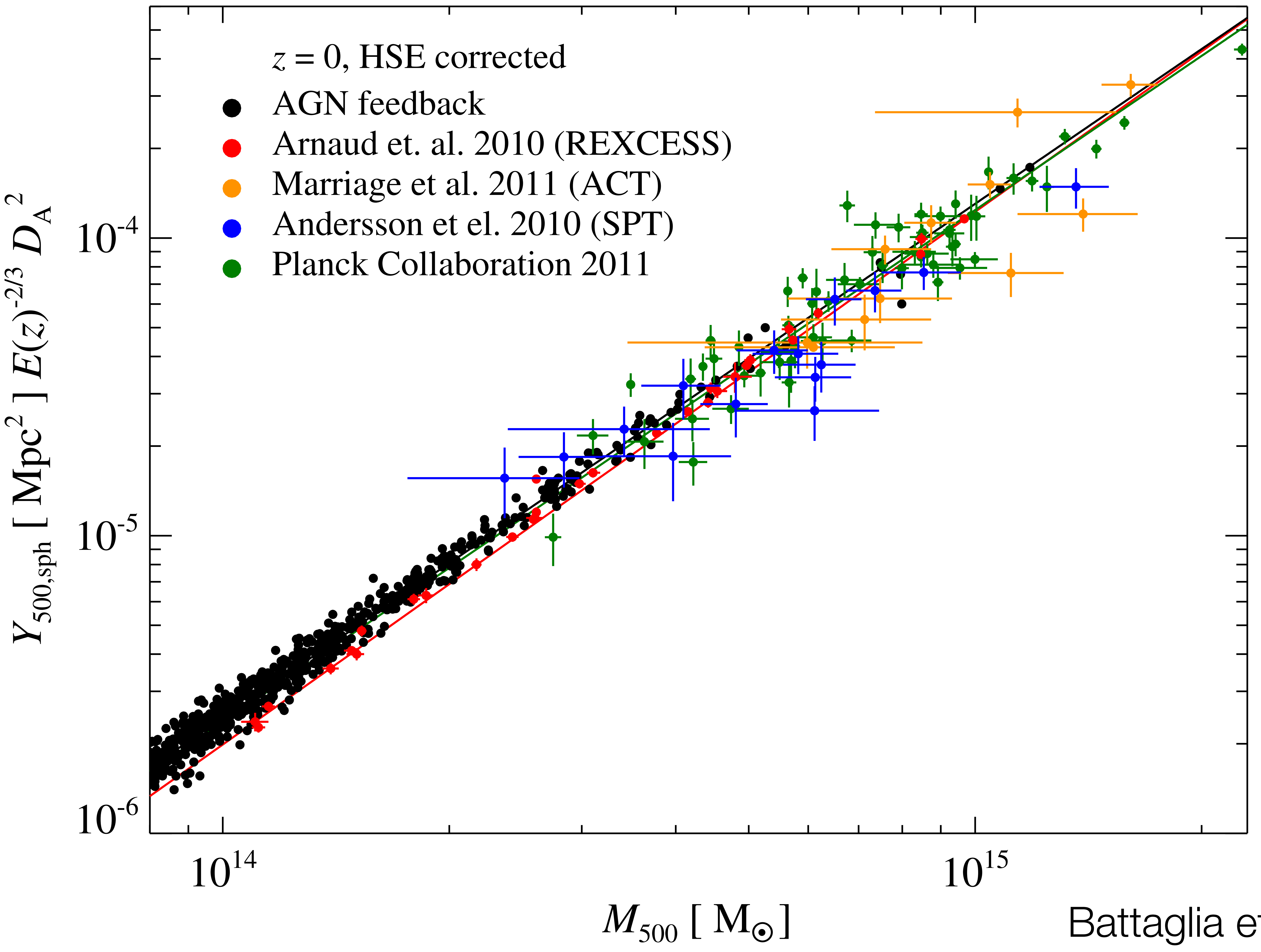
$$E_{\mathrm{gas}} = \frac{3}{2} N_{\mathrm{gas}}\, k_{\mathrm{B}} T_{200}$$

thermal energy

the Y — M relation:

$$Y_{\mathrm{sph}} = \frac{(\gamma - 1)\sigma_{\mathrm{T}}}{m_{\mathrm{e}}c^2} \tilde{x}_{\mathrm{e}} X \mu (1 - f_*) f_{\mathrm{b}} f_{\mathrm{c}}\, G \left[\frac{\pi}{3} 100 \rho_{\mathrm{cr}}(z)\right]^{1/3} M_{200}^{5/3}$$
$$= 97.6\, h_{70}^{-1}\, \mathrm{kpc}^2 E(z)^{2/3} \left(\frac{M_{200}}{10^{15}\, h_{70}^{-1} \mathrm{M}_\odot}\right)^{5/3} \frac{\Omega_{\mathrm{b}}}{0.043} \frac{0.25}{\Omega_{\mathrm{m}}}$$

The $Y - M$ relation



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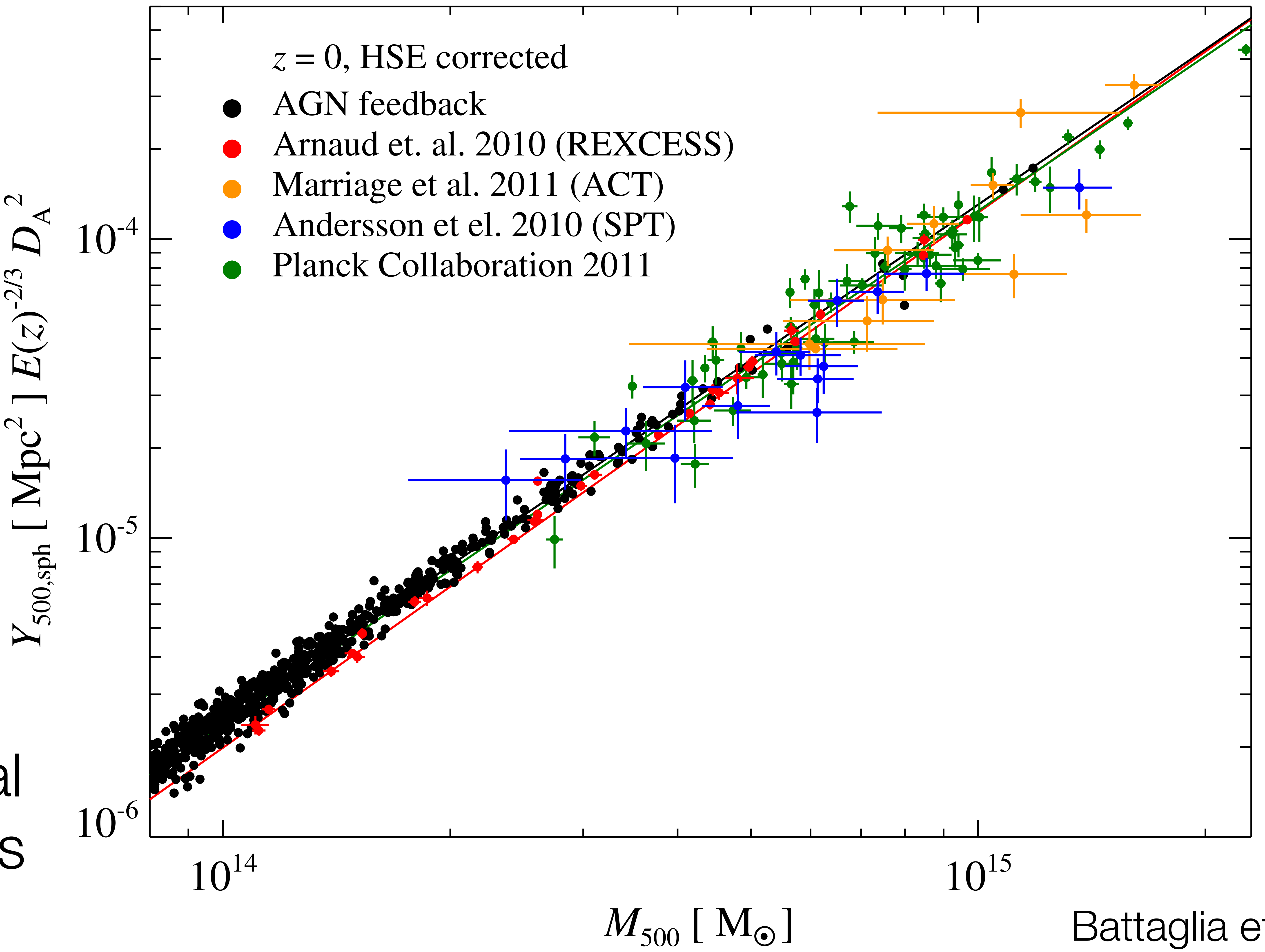
The $Y-M$ relation



-> can be used to compare theoretical halo mass functions to cluster counts

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The SZ power spectrum

$\Theta_{\mathrm{tSZ}}(\boldsymbol{\theta}) \equiv \frac{\Delta T_{\mathrm{tSZ}}}{T}(\boldsymbol{\theta}) = \sum_{\ell=0}^{\infty} \sum_{m=-\ell}^{\ell} a_{\ell m} Y_\ell^m(\boldsymbol{\theta})$ can expand an SZ map in spherical harmonics

$\left\langle a_{\ell_1 m_1} a^*_{\ell_2 m_2} \right\rangle \equiv \delta_{\ell_1,\ell_2} \delta_{m_1,m_2} C_{\ell,\mathrm{tSZ}}$ <- defines power spectrum

for small angles one can use the flat sky approximation
-> 2D Fourier transform instead of spherical harmonics

SZ power spectrum of single cluster:

$C_{(\ell),\mathrm{tSZ}} = f(x)^2 |\hat{y}_{(\ell)}(M,z)|^2$ with *f* defined by $\frac{\Delta T_{\mathrm{tSZ}}}{T}(\boldsymbol{\theta}) = y(\boldsymbol{\theta}) \left(x \frac{\mathrm{e}^x + 1}{\mathrm{e}^x - 1} - 4 \right) \equiv y(\boldsymbol{\theta}) f(x)$

2D Fourier transform of Compton-*y*

The SZ power spectrum

summing this over all clusters in considered volume:

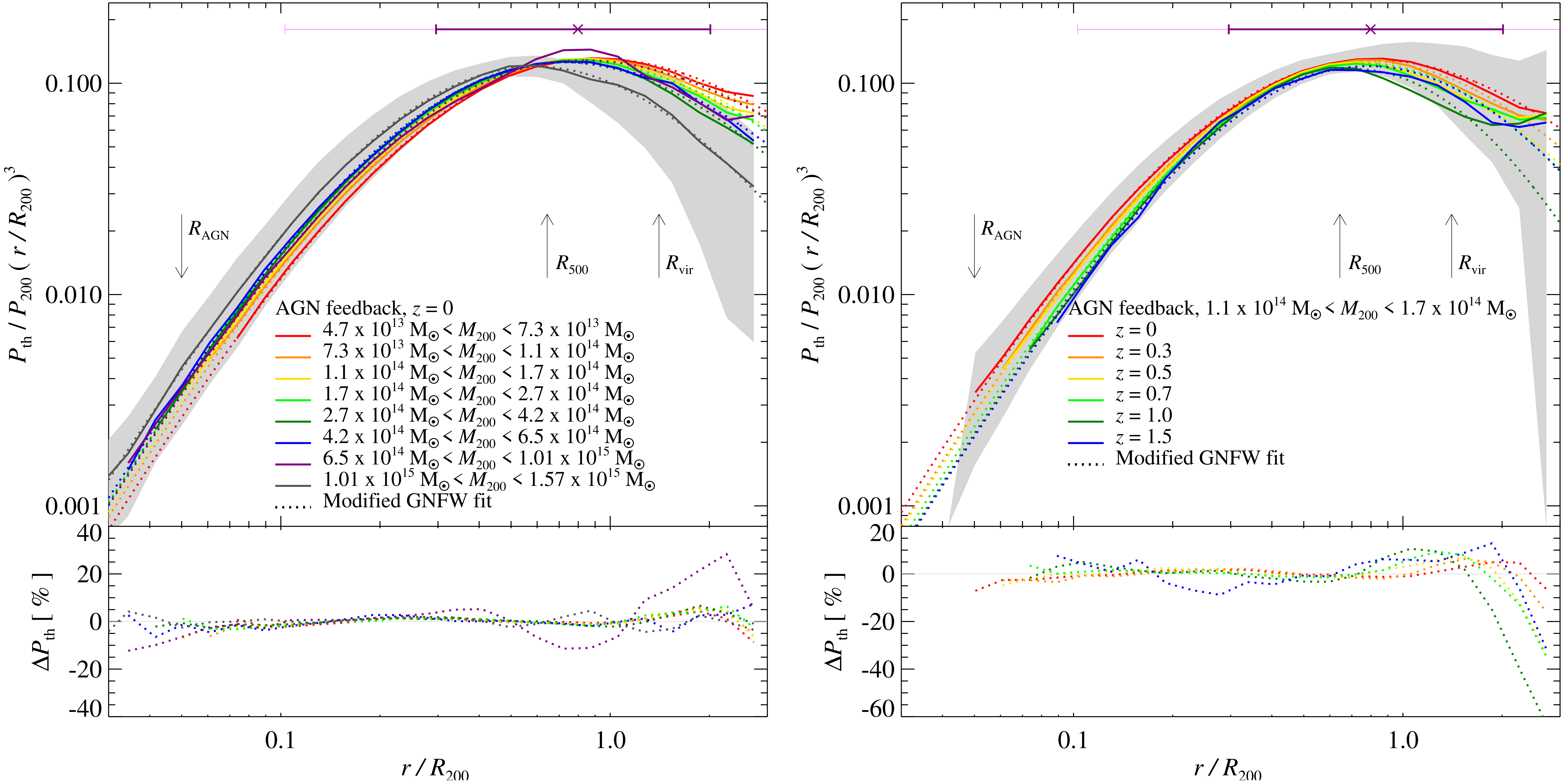
$$C_{(\ell),\mathrm{tSZ}} = f(x)^2 \int_0^{z_{\max}} \frac{\mathrm{d}V}{\mathrm{d}z}\mathrm{d}z \int_{M_{\min}}^{M_{\max}} \mathrm{d}M \frac{\mathrm{d}n(M,z)}{\mathrm{d}M} |\hat{y}_{(\ell)}(M,z)|^2$$

$$\hat{y}_{(\ell)} = \frac{1}{D_{\mathrm{ang}}^2} \int \mathrm{d}^3 r \frac{\sigma_{\mathrm{T}}}{m_{\mathrm{e}} c^2} P_{\mathrm{e}}(r) \mathrm{e}^{\mathrm{i}\boldsymbol{k}\cdot\boldsymbol{r}}$$

can, e.g., use fits to pressure profiles of simulated clusters to evaluate this:

$$\bar{P}_{\mathrm{fit}} = P_0\,(x_r/x_{\mathrm{c}})^{\delta}\left[1+(x_r/x_{\mathrm{c}})^{\alpha}\right]^{-\beta},\; x_r \equiv r/R_{200}$$ in units of $$P_{200} \equiv \frac{GM_{200} 200\,\rho_{\mathrm{cr}}(z) f_{\mathrm{b}}}{2R_{200}}$$

Simulated pressure profiles



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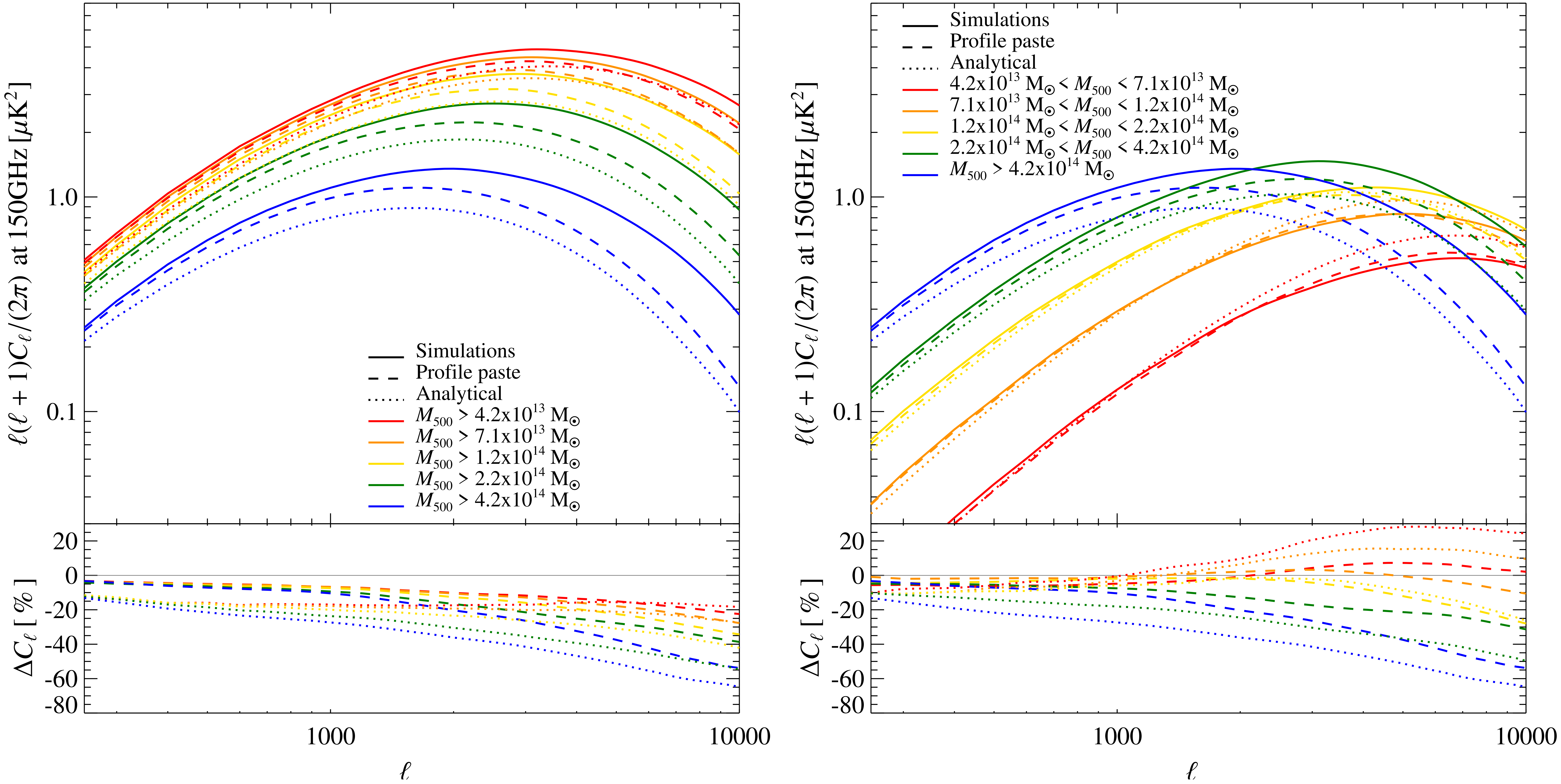
Contributions of different halo masses to SZ powerspectrum



-> sensitive to massive halos and hence σ_8

Battaglia et al. 2012

Recap - X-ray & SZ effect

- **intracluster medium can be observed in X-ray emission:**
  - allows inference of ICM state (temperature, density) including turbulence
  - allows hydrostatic mass estimates
  - can distinguish cool core and non-cool core clusters (different central entropy, density and temperature); cool cores seem not to evolve much over time and not to be disrupted by AGN outbursts
- **intracluster medium can be observed via the spectral distortions it imprints on the CMB (SZ effect):**
  - contributions from thermal, kinetic and relativistic SZ effects
  - integrated Compton-y correlates tightly with cluster mass
  - SZ power spectrum useful for comparing theoretical models (e.g., for different cosmologies) to observations