Reading Assignment for
The Physics of Galaxy Clusters

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in preparation of lecture 3

Answers to be uploaded to moodle

Please read and work through the lecture notes, covering the following topics:

2.2.2 Hierarchical Structure Formation

2.2.3 Non-linear Evolution

2.3 Spherical Collapse

I prepared the following questions that should help you to understand the topics. Please read a topic first, think about it and then work through my set of questions on this topic. Some questions are going beyond what you have read in the lecture notes. I do not expect you to answer these questions as well, but I would like you to start thinking about them and they will certainly be the starting point for our next lecture. Ideally you can come up with many more questions yourself!

- **Hierarchical Structure Formation**
  - Initially, $\sigma^2(k) < \sigma_*^2 = 1$ on all scales and structure grows linearly. At which scales do the fluctuations become first non-linear ($\sigma^2(k) > 1$) and why? Plot $\sigma^2(k)$ as a function of wave number for two consecutive times and determine the critical wave numbers at which the fluctuation strength becomes non-linear, respectively? Does this scale become smaller or larger with time?
  - What does this imply for the collapse of the first objects? Does structure formation in CDM cosmologies proceed top-down (first on large scales, then on progressively smaller ones) or bottom-up with smaller structures to form first?
  - How would you have to change the initial linear power spectrum in order for structure formation to proceed differently? *Bonus:* What physics would you have to change in such a universe?
  - By looking at the potential fluctuations, argue why there is a preferred spectrum of fluctuations. How would you have to modify the primordial power spectrum if you adopted a different spectral index for the Harrison-Zeldovich-Peebles spectrum to make it physical?
- **Non-linear Evolution**
  - Why do you need numerical simulations to study the non-linear phase of structure formation?
  - Plot the probability distribution function of the density contrast at early and at late times after non-linear structure formation has already begun. What is the physical reason that the distribution becomes skewed at late times?
  - *Bonus:* What does a "particle" in these numerical N-body simulations represent? What are the considerations when choosing the number of particles and what are the limitations in terms of a maximum and minimum number of particles?

To appreciate the insights into structure formation we can gain by means of cosmological simulations, you may want to watch a few movies of structure formation simulations, that you can find e.g., on the TNG web page. Which properties that we talked about so far can you recognize in these simulations? Which things do you not understand or would like to know more about?

- **Spherical Collapse**
  - Summarize the assumptions of the spherical collapse model. What is the benefit of doing this calculation if you have to assume so many simplifications?
  - The cycloidal solution of spherical collapse is periodic beyond $\theta = 2\pi$. Is this completely unphysical? Why is the solution for $\theta > 2\pi$ not realized in nature?
  - Assume that the sphere has a uniform initial overdensity (δ_i) at some early time (t_i), why does the sphere remain uniform as it collapses?
  - Why do perturbations collapse earlier if they are initially more over overdense?
  - We find values for the density contrast at collapse ($t = t_\mathrm{c} = 2t_\mathrm{max}$) of

$$\delta_\mathrm{c} \equiv \delta_\mathrm{lin}(t_\mathrm{c}) = \frac{3}{20}(12\pi)^{2/3} \approx 1.686, \quad (1)$$

$$\delta_\mathrm{v} \equiv \delta_\mathrm{coll} = 18\pi^2 - 1 = 177\,. \quad (2)$$

    Explain the difference of these results that describe the same quantity at the same time.
    We will later on use both results. Under which circumstances would you use the first and under which the second result?
  - Last week, I asked the question which process would determine the relation $\bar{\rho}_\mathrm{cl} \sim 10^3\bar{\rho}_\mathrm{m,0}$ (in the context of “Relation to the Average Universe”). By which factor does the average universe collapse in the spherical collapse model? What stops the collapse in virialized case?