Reading Assignment for The Physics of Galaxy Clusters

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in preparation of lecture 4

Answers to be uploaded to moodle

Please read and work through the lecture notes, covering the following topics:

2.4.1 The Press-Schechter Mass Function

2.4.2 Halo Formation as a Random Walk

2.5 Halo Density Profiles

Note that we do not cover section *2.4.3 Extended Press-Schechter Theory* in the lectures and you can skip this section.

I prepared the following questions that should help you to understand the topics. Please read a topic first, think about it and then work through my set of questions on this topic. Some questions are going beyond what you have read in the lecture notes. I do not expect you to answer these questions as well, but I would like you to start thinking about them and they will certainly be the starting point for our next lecture. Ideally you can come up with many more questions yourself!

- **The Derivation of the Halo Mass Function**
  - How do the characteristic length scale $R(M)$ and its associated mass M differ from the characteristic length scale $R(M_*) \equiv R_*$ with its associated "non-linear mass" M_*?
  - *Bonus:* What is the physical basis for assuming that the density field obeys a Gaussian random process?
  - Justify the hypothesis of Press & Schechter that the probability of finding the filtered density contrast at or above the linear density contrast for spherical collapse, $\bar{\delta} > \delta_c$, is equal to the fraction of the cosmic volume filled with haloes of mass M.
  - Which substitution do you need to adopt in order to perform the integral in Eq. (2.78)?
  - Sketch and explain the main steps necessary to obtain the correct normalisation of the Press-Schechter mass function by considering halo formation as a random walk.
  - Explain the physical reason for the missing factor of two and why this has been missed in the first derivation.
- **Halo Density Profiles**
  - **General remarks.** Scrutinize the statement that *a self-gravitating system of particles does not have an equilibrium state*. Assume that you have a globular cluster of 10^6 stars and size 10 pc. Explain what happens to the system when you eject one star after each other. What is the theoretical end state?
  - **Isothermal sphere.** Show explicitly that a power-law ansatz in $\rho(r)$ yields Eq. (2.101).
  - What is the problem of the singular isothermal sphere at small and larger radii?
  - **Navarro-Frenk-White profile.** What is the logarithmic slope, $\mathrm{d}\log\rho/\mathrm{d}\log r$, of the NFW profile at the center, the scale radius r_s and at large radii?

– Compare the different definitions for halo mass, M_{200}, $M_{200\mathrm{m}}$, and M_{500} and order them by increasing size.

– Sketch qualitatively the scaled NFW density profiles $\log(\rho/\rho_{200})$ for two different halos which only differ by their concentration parameters. What do you observe? Which halo is on average more massive?