Reading Assignment for
The Physics of Galaxy Clusters

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in preparation of lecture 5

Answers to be uploaded to moodle

Please read and work through the lecture notes, covering the following topics:

3.1.1 Adiabatic Processes and Entropy

3.1.2 Basic Conservation Equations

I prepared the following questions that should help you to understand the topics. Please read a topic first, think about it and then work through my set of questions on this topic. Some questions are going beyond what you have read in the lecture notes (indicated by *Bonus* questions). I do not expect you to answer these questions as well, but I would like you to start thinking about them and they will certainly be the starting point for our next lecture. Ideally you can come up with many more questions yourself!

- **Adiabatic Processes and Entropy**
  - How are the specific heat at constant pressure and at constant volume related?
  - Why does changing the temperature at constant pressure require more heat than at constant volume?
  - What is the adiabatic index for a diatomic gas that has five degrees of freedom (3 translational and 2 rotational)?
  - Derive the polytropic equation of state from the first law of thermodynamics and the equation of state for an ideal gas.
  - How are the constant for adiabatic changes, K, and the thermodynamic entropy, s, related? What is the advantage of using the quantity K?
- **Basic Conservation Equations**
  - **Fluid description.** When can a system be well described as a fluid? Is this justified in a low-mass and a high-mass cluster (with $k_\mathrm{B}T = 1$ keV and 10 keV) on scales of a few kpc? What does this imply for modelling clusters?
  - **Conservation laws.** Why does the collision term on the right-hand side vanish for the mass, momentum and energy equation?
  - Show explicitly, that the Navier-Stokes equation conserves momentum.
  - Why is the anisotropic viscous stress tensor, $\bar{\mathbf{\Pi}}$, traceless and symmetric? Explain, what viscosity does to shearing and interpenetration motion.
  - *Bonus:* derive the energy conservation equation.
  - Name and explain the condition for a non-vanishing conductive heat flux $\boldsymbol{Q}$.
  - Are the energy and entropy equations linearly independent?
  - Write down the full system of hydrodynamic equations. How many equations do you have for how many unknowns? Discuss whether you need additional information on the medium properties.

- **Gas in an NFW Halo** (not graded)

  In class, we will do this problem together. If you like, you can try solving it yourself.

  The NFW density profile diverges in the center, i.e., for $x \to 0$. Gas filled in the halo's gravitational potential Φ satisfies Euler's equation

  $$\frac{\nabla P_{\rm gas}}{\rho_{\rm gas}} = -\nabla\Phi(r), \tag{1}$$

  where $P_{\rm gas}$ is the gas pressure.

  1. Assuming an isothermal and ideal gas, show that the gas density profile is

     $$\rho_{\rm gas} = A \exp\left(-\frac{\bar{m}\Phi}{kT}\right), \tag{2}$$

     where T is the temperature, k is Boltzmann's constant, $\bar{m}$ is the mean particle mass, and A is a constant.

  2. On the execise sheet 2, you confirmed that the gravitational potential of an NFW halo is

     $$\Phi(r) = -\frac{GM_{\rm s}}{r_{\rm s}} \frac{\ln(1+x)}{x}. \tag{3}$$

     Using Equation (3), show that

     $$-\frac{\bar{m}\Phi}{kT} = 3\frac{\ln(1+x)}{x} \tag{4}$$

     if the gas is in equilibrium with the gravitational potential.

  3. Is the gas density finite in the halo's center? Compare the density profiles of gas and dark matter and explain the differences.

  4. What happens to the gas-to-dark matter mass density ratio $\rho_{\rm gas}/\rho$ at large radii? Is this a realistic behavior and if not, what would have to be changed in the model to make it more realistic?