Reading Assignment for
The Physics of Galaxy Clusters

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in preparation of lecture 6

Answers to be uploaded to moodle

Please read and work through the lecture notes, covering the following topics:

3.1.3 Buoyancy Instabilities

3.1.4 Vorticity

3.1.5 Turbulence

I prepared the following questions that should help you to understand the topics. Please read a topic first, think about it and then work through my set of questions on this topic. Some questions are going beyond what you have read in the lecture notes (indicated by *Bonus* questions). I do not expect you to answer these questions as well, but I would like you to start thinking about them and they will certainly be the starting point for our next lecture. Ideally you can come up with many more questions yourself!

- **Buoyancy Instabilities – Perturbation analysis on a hydrostatic background**
  - Derive the most general background temperature profile of a hydrostatic background. Is this a strict constraint or can you violate it? If so, under which condition?
  - Derive the first-order conservation equations (3.63) – (3.65).
  - Why is the gas nearly incompressible here? What are the underlying approximations?
  - Project the momentum equation (3.68) into a purely vertical and perpendicular part.
  - Discuss the solution of the dispersion relation for gravity waves (3.75). Why do g-modes exhibit a maximum possible frequency?
  - *Bonus:* Derive the entropy condition for a stably stratified atmosphere (the "Schwarzschild condition") purely through thermodynamical considerations. How do these two derivations differ in their assumptions? Which result teaches us more about the stratified atmosphere and why? *Hint:* check out Appendix A2 in the lecture notes.
- **Vorticity**
  - Why can an incompressible vector field be described as a pure vortex field?
  - Derive Eq. (3.77).
  - Explain why vorticity (in the approximation $\mathcal{M} \ll 1$) is principally generated in the horizontal plane (perpendicular to gravity).
- **Turbulence**
  - Derive Eqn. (3.81) from the general Navier-Stokes equation (3.45).
  - Explain the physical meaning of the Reynolds number.
  - Explain the energy cascade and derive the steady-state scaling of velocity v_λ with scale λ.
  - Why does this imply strong intermittency on small scales? *Bonus:* Can you describe a turbulent velocity field by a purely Gaussian process?
  - Derive the Kolmogorov turbulent power spectrum of driven turbulence.