Reading Assignment for The Physics of Galaxy Clusters

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in preparation of lecture 7

Answers to be uploaded to moodle

Please read and work through the script, covering the following topic:

3.1.6 Shocks

I prepared the following questions that should help you to understand the topics. Please read a topic first, think about it and then work through my set of questions on this topic. Some questions are going beyond what you have read in the lecture notes (indicated by *Bonus* questions). I do not expect you to answer these questions as well, but I would like you to start thinking about them and they will certainly be the starting point for our next lecture. Ideally you can come up with many more questions yourself!

- **Shocks.** These are the questions for your reading assigment:
  - Explain how you can generate a shock.
  - If a shock is a true discontinuity in hydrodynamic quantities, are our partial derivatives in our evolution equations for mass, momentum and energy well defined at the shock? How does one deal with this issue in practice? What is really happening at a shock?
  - Derive an evolution equation for the kinetic energy density $\rho v^2/2$.
  - Start with the general form of the conservation equations (3.93) to (3.95) supplemented by Eq. (3.97) and simplify to a steady state and a plane-parallel geometry, i.e., confirm Eqs. (3.100) to (3.103).
  - Use the equation for mass conservation (3.100), derive the first Rankine-Hugoniot jump condition, i.e., Eq. (3.104).
  - How does a tangential discontinuity differ from a shock? Name at least two of the three differences.
  - What is the physical interpretation of the Mach number?
  - Using Eqs. (3.118) to (3.120), derive the Rankine-Hugoniot jump conditions for strong shocks of Eqns. (3.121) to (3.123). Why does the density only jump by a factor of 4?
  - Why is energy conservation apparently violated across the shock in the shock rest frame (see Eqn. 3.126)? How is this conundrum resolved?
  - Verify the statement "A shock converts supersonic gas into denser, slower moving, higher pressure, subsonic gas" through equations (for simplicity, you can use the strong-shock limit $\mathcal{M}_1 \gg 1$).
  - Explain the difference between an adiabatic curve and a shock adiabat in the $P-V$ diagram.
  - Explain what changes qualitatively at an oblique shock in comparison to a plane-parallel shock.