Reading Assignment for
The Physics of Galaxy Clusters

Lecturer: Christoph Pfrommer

in preparation of lecture 8

Answers to be uploaded to moodle

Please read and work through the lecture notes, covering the following topic:

3.1.7 Entropy Generation by Accretion

3.1.8 Cluster Scaling Relations

I prepared the following questions that should help you to understand the topics. Please read a topic first, think about it and then work through my set of questions on this topic. Some questions are going beyond what you have read in the lecture notes (indicated by *Bonus* questions). I do not expect you to answer these questions as well, but I would like you to start thinking about them and they will certainly be the starting point for our next lecture. Ideally you can come up with many more questions yourself!

- **Entropy Generation by Accretion**
  - Motivate the connection between the phase space density and entropy. When is a gas degenerate and why is the ICM very far from this?
  - To describe accretion onto a galaxy cluster, we put ourselves into the post-shock rest system – why?
  - Derive each of the governing equations (3.140) to (3.143). You can start with the Rankine-Hugoniot jump conditions that we derived in Sect. 3.1.6.4.
  - Derive the entropy generated at the cluster accretion shock in dimensional and dimensionless form, Eqns. (3.147) and (3.152).
  - What happens to the cluster entropy profile for lumpy accretion?
- **Ideal Cluster Scaling Relations**
  - Derive the ideal cluster scaling relations $k_\mathrm{B}T$-M_Δ, M_gas-M_Δ, $M_\star$-M_Δ, Y-M_Δ, and L_X-M_Δ, where Δ denotes the overdensity with respect to the critical density of the universe (typically taken to be $\Delta = 200$).
  - Using our order of magnitude numbers for a $10^{15}\,\mathrm{M}_\odot$ cluster of Section 1.2, derive $k_\mathrm{B}T$, M_gas, $M_\star$, and Y for a $10^{14}\,\mathrm{M}_\odot$ cluster and a group of $10^{13}\,\mathrm{M}_\odot$, each at a redshift of $z = 0$ and $z = 1$. To this end assume $\Omega_\mathrm{m0} = 0.3$, $\Omega_\Lambda = 0.7$, which implies that the universe has zero curvature $\Omega_\mathrm{K} = 0$ and we can neglect the radiation term at late times.
  - Argue for each wave band, why the halo mass scale of $\sim 10^{14}\,\mathrm{M}_\odot$ is a good choice for calling an object a galaxy cluster.
  - Sketch observed and theoretically expected scaling relations of clusters. Note that they agree at the high-mass end of around $10^{15}\,\mathrm{M}_\odot$.

- **Real Cluster Scaling Relations**
  - Assume an isothermal gas in a cluster with a beta profile of the gas density:

$$\rho = \rho_0 \left[\frac{1}{1 + (r/r_{\mathrm{c}})^2)} \right]^{3/2\beta},$$

    where ρ_0 is the central density, r_{c} is the core radius, and β is the scaling parameter, calculate the X-ray luminosity for $\beta = 2/3$ and 1 (typical values in the X-ray literature). Compare your result to our approximation of Eqn. (3.172) and discuss it.
  - Explain how pre-heating the gas before it gets accreted onto the cluster can solve the problem of the X-ray luminosity scaling relation.
  - Explain how AGN feedback and radiative cooling can theoretically also solve the problem of the X-ray luminosity scaling relation.