Reading Assignment for The Physics of Galaxy Clusters

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in preparation of lecture 9

Answers to be uploaded to moodle

Please read and work through the lecture notes, covering the following topics:

Start with *3.2 Radiative Physics* and continue with

3.2.1 Radiative Cooling

3.2.2 Cooling versus Heating

3.2.3 Feedback by Supernovae

3.2.4 Feedback by Active Galactic Nuclei

I prepared the following questions that should help you to understand the topics. Please read a topic first, think about it and then work through my set of questions on this topic. Some questions are going beyond what you have read in the lecture notes (indicated by *Bonus* questions). I do not expect you to answer these questions as well, but I would like you to start thinking about them and they will certainly be the starting point for our next lecture. Ideally you can come up with many more questions yourself. **If you have problems with a derivation or if something is unclear, please email me those points well before the lectures**!

- **Radiative cooling**
  - Explain the physics of bremsstrahlung emission and motivate the main dependencies in the bremsstrahlung emissivity, $j \propto n^2\sqrt{k_\mathrm{B}T}$ via physical arguments.
  - Calculate the radiative cooling time t_cool of the X-ray-emitting gas for the core region of a cool-core cluster ($n_\mathrm{e} = 3\times10^{-2}\,\mathrm{cm}^{-3}$ and $k_\mathrm{B}T = 1$ keV) and non-cool core cluster ($n_\mathrm{e} = 3\times10^{-3}\,\mathrm{cm}^{-3}$ and $k_\mathrm{B}T = 4$ keV). Compare your result to the age of the universe and discuss it.
  - Rewrite the cooling time 1. as a function of entropy and temperature and 2. as a function of entropy and cluster mass (using the ideal temperature-mass scaling relation).
- **Cooling versus heating**
  - The density distribution of a **cool core cluster** is characterized by a double beta profile,

$$n(r) = \sum_{i=1,2} n_i \left[1 + \left(\frac{r}{r_{\mathrm{c},i}}\right)^2\right]^{-3\beta_i/2}. \qquad (1)$$

    Plot the density profile of this cluster as a function of radius in a double-logarithmic representation for $\beta_{1,2} = 1$, $(n_1, n_2) = (10^{-1}, 10^{-2})\,\mathrm{cm}^{-3}$, and $r_{\mathrm{c},1}, r_{\mathrm{c},2} = (10, 100)$ kpc.
  - Plot the cooling time as a function of radius in a double-logarithmic representation and assume a constant temperature of 2 keV. Where in a cluster is cooling time minimized and where is it largest? Use your result to calculate the cooling radius for this cluster, which is the radius where the cooling time equals 1 Gyr.

  - The density distribution of a **non-cool core cluster** obeys a single beta profile. If you drop the first term and only account for the second term (with subscripts 2), calculate the cooling radius (if there is one). Discuss your results in comparison to cool-core clusters and argue whether you also have a severe overcooling problem in a non-cool core cluster.

- **Feedback by supernovae**
  - How much energy per nucleon can be delivered by core-collapse SNe and by thermonuclear SNe to the ICM over its entire life time? Show your calculation.
  - Compare this energy input to the gravitational binding energy of a nucleon in our Milky Way (with circular velocity $v_{\mathrm{gal}} = 220$ km s^{-1}) and in a massive galaxy clusters (with velocity dispersion $\sigma_{\mathrm{cluster}} = 1200$ km s^{-1}) and discuss the thermodynamic impact of SNe in either system.

- **Feedback by active galactic nuclei**
  - Estimate the mass of a supermassive black hole in a massive elliptical BCG with $M_{\star,\,\mathrm{BCG}} \sim 10^{12}\,\mathrm{M}_{\odot}$ using the stellar bulge-black hole mass relation. Why can we identify the bulge mass in this relation with the stellar mass of the elliptical galaxy?
  - Estimate the released gravitational energy per nucleon of an AGN that accretes its own mass and adopt a radiative efficiency of $\eta \sim 0.1$. Use the supermassive black hole mass corresponding to a BCG galaxy with $M_{\star,\,\mathrm{BCG}} \sim 10^{12}\,\mathrm{M}_{\odot}$.
  - Derive the sound crossing time, the buoyant rise time and the time required to refill the volume as the bubble rises. Express the latter two time scales in units of the sound crossing time. Calculate the sound crossing time for $R = 20$ kpc and a temperature of 6 keV.
  - Calculate the work done by two AGN bubbles in one outburst and adopt the following properties of the AGN bubbles: $r_{\mathrm{b}} = 20\,\mathrm{kpc}$, $n_{\mathrm{a}} = 10^{-2}\,\mathrm{cm}^{-3}$, $k_{\mathrm{B}}T = 3$ keV. Use this to calculate the AGN heating rate and compare this to the X-ray “cooling” luminosity in the lectures.
  - Calculate the Schwarzschild radius for the supermassive black hole in M87 ($M_{\mathrm{M87}} = 5\times10^{9}\,\mathrm{M}_{\odot}$) and compare this to the cooling radius of our cool core cluster characterized by the double beta profile in Eqn. (1). By how many orders of magnitude does this differ?