Reading Assignment for The Physics of Galaxy Clusters

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in preparation of lecture 14

Answers to be uploaded to moodle

In the next lecture, we will discuss a selection of the following topics: *4.3 X-ray Cluster Astrophysics* and *4.4 Sunyaev-Zel'dovich Effect*. Please prepare the following sections:

4.3.1 Hydrostatic Equilibrium Masses and Biases

4.3.2 Cluster Population and Evolution

4.3.3 Intracluster Medium Turbulence

4.3.4 Merger Shocks and Electron Equilibration

4.4.1 Thermal and Kinetic SZ Effect

4.4.3 Self-similar SZ Scaling Relation

4.4.4 SZ Power Spectrum

Thinking about the following questions should help you to understand the topics. Please read a topic first, think about it and then work through the set of questions on this topic. Some questions are going beyond what you have read in the lecture notes, but I would like you to start thinking about them and they will certainly be the starting point for our next discussion meeting. Ideally you can come up with many more questions yourself. **If you have problems with a derivation or if something is unclear, please email me those points well before the lectures!**

- Please read Sections 1.2.2, 1.2.4 and 1.2.5 again to recapitulate some of the general properties of X-ray and SZ emission and observations.
- **Hydrostatic Equilibrium Masses and Biases**
  - Derive a typical value for the speed of sound in a galaxy cluster. How does this compare to typical galaxy velocities? What is the reason for this?
  - Equation (4.120) can either be obtained from the Euler equation (Eq. 2.2; show it!) or is directly obtained from considering the balance of hydrodynamical and gravitational forces on a gas parcel in the intracluster medium. In the latter case, show why the hydrodynamic force per unit volume can be written as $\boldsymbol{\nabla} P$.
  - Derive the X-ray surface brightness profile from a spherically symmmetric emission profile, i.e., derive Eq. (4.137). How is the line-of-sight integral along z converted to an integral over r?
- **Cluster Population and Evolution; ICM Turbulence**
  - How do X-ray images of cool core and non-cool core clusters differ? How is this related to the differences of the central entropy values of cool core and non-cool core clusters?
  - What is a typical ratio of the mean free path to the Larmor radius? How does the mean free path compare to the distance from the cluster center in different parts of the cluster (centers of cool core and non-cool core clusters, cluster outskirts)?

- **Merger Shocks and Electron Equilibration**
  - In the absence of energy exchange between ions and electrons, to which temperatures would a shock heat the ions and the electrons for a velocity change Δv of 1000 km/s across the shock?
  - Why is the X-ray emission not sensitive to the ion temperature T_i?
- **Thermal and Kinetic SZ Effect**
  - Show that Eq. (4.182) is a solution of Eq. (4.179).
  - Show that Eq. (4.184) reduces to Eq. (4.186) for $x \ll 1$.
  - Explain how the thermal SZ effect differs from the kinetic SZ effect microphysically.
- **Self-similar SZ Scaling Relation**
  - Derive the Y–M scaling relation (Eq. 4.220). What processes could cause deviations from the derived $Y - M$ relation?
  - How could the Y–M scaling relation be used to learn about cosmology?
- **SZ Power Spectrum**
  - Derive the Fourier transform of the Compton-y parameter (Eq. 4.238), starting from Eq. (4.231). Why can we use a Fourier analysis instead of spherical harmonics, which would be the appropriate basis functions on the sphere?
  - How could the SZ power spectrum be used to learn about cosmology?