Final Reading Assignment for
The Physics of Galaxy Clusters

Lecturer: Christoph Pfrommer

in preparation of lecture 15

Answers to be uploaded to moodle

Please read and work through the lecture notes, covering the following topics:

4.3.5 Magnetic Draping and Cold Fronts

4.5 Radio Emission: Shocks and Plasma Physics

4.6 Cluster Cosmology

I prepared the following questions that should help you to understand the topics. Please read a topic first, think about it and then work through my set of questions on this topic. Some questions are going beyond what you have read in the lecture notes (indicated by *Bonus* questions). I do not expect you to answer these questions as well, but I would like you to start thinking about them and they will certainly be the starting point for our next zoom meeting. Ideally you can come up with many more questions yourself. **If you have problems with a derivation or if something is unclear, please email me those points well before the lectures**!

- **Magnetic draping and cold fronts**
  - Explain the physics underlying Eq. (4.162): what is the role of magnetic pressure and magnetic tension in establishing this equilibrium (Sect. 3.3.2.3) and what is the role of magnetic flux freezing in establishing magnetic draping (Sect. 3.3.2.5)?
  - Convince yourself that Eq. (4.168) is valid. Explain why this equation is a necessary consequence of magnetic flux freezing.
- **Cosmological shocks**
  - Compute the gravitational energy that is dissipated in a cluster merger. Compare this to a galaxy merger, a supernova explosion and other energetic astrophysical events. What do you conclude?
  - How do supernova shocks differ from cosmological shocks that form in the process of cluster formation? Discuss the corresponding Mach numbers and dissipated energy.
- **Radiative processes and cooling times**
  - Derive to order of magnitude the ion Coulomb cooling time, i.e., Eq. (4.248). To this end, you may like to revise the discussion on electron scattering in the Coulomb field of a proton in Sections 3.2.5.1 and 3.2.5.2.
  - Derive to order of magnitude the inverse Compton energy loss rate, i.e., Eq. (4.250). To this end, you may want to revise the discussion at the beginning of the *Relativistic SZ effect (Sect. 4.4.2)* and of course make use of the formula of particle interaction rates we encountered multiple times in the lectures.
  - Rewrite the cooling time as a function of B and $\nu_{\rm syn}$ by using the formula of the synchrotron frequency (Eq. 4.250). Show that the cooling time has a maximum and determine the magnetic field strength of this maximum. How do electrons primarily cool above and below this critical magnetic field strength?

- **Equilibrium electron distribution**
  - Derive the equilibrium electron distribution in Eq. (4.253) from Eq. (4.252).
  - Using this result, compare the synchrotron emissivity j_ν and its spectral index α_ν for two cases: (i) a freshly injected population of relativistic electrons and (ii) an equilibrium electron population where injection is balanced by synchrotron/inverse Compton cooling. What are the limiting cases for a strong and a weak shock.
- **Radio relics, halos and head-tail radio galaxies**
  - ***Relic AGN bubble.*** How do you form ghost bubbles and how do they differ from relic AGN bubbles?
  - ***Radio phoenix.*** How can you measure the compression factor that describes the transition of an aged AGN bubble into a radio phoenix? By working out the adiabatic compression factor of the magnetic field and the CR electron pressure that is associated with the radio phoenix formation, by how much is the radio emissivity $j_\nu \propto P_{\mathrm{CRe}} B^2$ (assuming a radio spectral index $\alpha_\nu = 1$) increased?
  - ***Radio shocks.*** Assume that the relativistic electrons are accelerated at a strong cluster merger shock and are advected with the post-shock gas. Assume that the incoming gas had a pre-shock velocity of $v_{\mathrm{pre}} = 1200$ km/s in the shock frame, calculate the maximum cooling length $L_{\mathrm{cool}} = v_{\mathrm{post}} t_{\mathrm{cool}}$ at 1.4 GHz for a shock Mach number of $\mathcal{M} = 3$. What is L_{cool} for 5 μG and how does this visually compare to the radial extends of radio shocks?
  - ***Radio halos.*** Compare this cooling length of electrons to the spatial extend of radio halos. What does this imply for the acceleration process of the radio-emitting electrons?
  - Compare the cooling times of protons and electrons above 10 GeV. What does this imply for the distributions of (primary) electrons and protons in clusters? How does the distribution of secondary electrons that are injected by hadronic reactions look like?
  - ***Radio galaxies.*** How do you form head-tail radio galaxies and which astrophysics can you learn by studying them? Derive the ratio of the bending-to-jet radius of head tail radio galaxies.
- ***Bonus:*** **Cluster cosmology**
  - Which cosmological parameters can you estimate with cluster surveys?
  - Explain how you use the cluster abundance to measure the fluctuation amplitude σ_8 and discuss the problems associated with this method.
  - Explain how you use the baryon fraction in clusters to estimate $\Omega_{\mathrm{b}}/\Omega_{\mathrm{m}}$.
  - How can you use the cluster core structure to scrutinize the nature of dark matter?
  - Explain how the different components (galaxies, X-rays and gravitational weak lensing) of the bullet cluster can be used to probe the strength of dark matter self-interactions.