Exercises for
The Physics of Galaxy Clusters

Lecturer: Christoph Pfrommer

Exercise sheet 1

To be uploaded to Moodle. Remember to put your name on the document. You may work in groups of up to 2 but every student should hand in his/her own solution sheet and indicate clearly who contributed to it. The exercises are based on the lecture notes. Thus, **studying the lecture notes carefully** will help you immensely in solving the exercises!

1. **Sunyaev-Zel'dovich Effect** (10 points)

   (a) Why does the intracluster medium (ICM) inside a galaxy cluster dominate the line-of-sight integral of the Compton-y parameter? To answer this, let's do an *order of magnitude problem:* compute the Compton-y parameter ...

   - ... of a a galaxy cluster of mass $10^{15}\,\mathrm{M}_\odot$ and radius 3 Mpc,
   - ... of the ionized intergalactic medium from us to the epoch of reionization at $z_{\rm reion} = 9$ ($n_{\rm e} \sim 2\times10^{-7}\,\mathrm{cm}^{-3}$ and $T \sim 10^4$ K) and neglect the intervening cluster (why can we neglect the integral of the redshift range from $z \sim 9$ to 1100, where CMB photons are released?),
   - ... of our own galactic halo ($n_{\rm e} \sim 2\times10^{-4}\,\mathrm{cm}^{-3}$ and $T \sim 10^6$ K).

   Which contribution dominates and by how much?

2. **The Growth of Perturbations – 1** (10 points)

   For this problem, assume that recombination happens instantaneously at $z = 1100$ and assume a matter density parameter today $\Omega_{\rm m,0} = 0.3$.

   (a) The sound speed in matter *before* recombination is $c_{\rm s}^2 \approx c^2/3$.

      i. Compute the sound speed, $c_{\rm s}^2 = \gamma P/\rho$, shortly *after* recombination assuming that matter is all monoatomic hydrogen.

      ii. What is providing the restoring force for sound waves in both epochs, respectively?

   (b) Calculate the Jeans mass shortly before and after recombination, where the Jeans mass is defined by

   $$M_{\rm J}(a) \equiv \frac{4\pi}{3}\bar\rho(a)\left(\frac{2\pi a}{k_{\rm J}(a)}\right)^3, \quad \text{where} \quad k_{\rm J}(a) = \sqrt{\frac{4\pi G\bar\rho(a)a^2}{c_{\rm s}^2}}$$

   is the comoving Jeans wave number and $\bar\rho(a) = \Omega_{\rm m0}\rho_{\rm cr0}a^{-3}$. Discuss which implications this has for structure formation, e.g., which structure formed first; star clusters, galaxies, galaxy clusters, super clusters.

3. **Spherical Collapse** (10 points)

   Starting with the equations of motion for a collapsing sphere (2.44), please derive the parametric solution $R(\theta)$ (2.48 and 2.49) and discuss it.

4. ***Bonus problem:*** **The Growth of Perturbations – 2** (10 points)

   (a) If you have time and are motivated, you may want to derive equation (2.13) in the notes. To make things a little easier, you may want to follow my set of cosmology notes. Equations (2.1) to (2.19) in these notes guide your through this (lengthy) derivation.

   (b) After this, you deserve a break. You may watch the following movies of the Millennium simulation, which visualize cosmic scales and and show a fly-through the highly structure large-scale structure of the universe.