Exercises for
Cosmology (WS2013/14)

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Exercise sheet 11

Due: Jan. 14, 2014 16:00

To be handed in before the exercise class or emailed as a pdf or scanned hand-written document (via e-mail to walther@mpia.de). Remember to put your names on the document; groups of ≤ 3 are allowed.

1. **The Monopole Problem** (10 points)

   Many Grand Unified Theories (GUTs) – quantum field theories that unify the electromagnetic, weak nuclear and strong nuclear forces – predict that numerous magnetic monopoles should be produced during the GUT era, when the cosmic temperature $T_{\mathrm{GUT}} \simeq 3 \times 10^{28}\,\mathrm{K}$. Once created, these monopoles behave as non-relativistic matter.

   (a) Suppose that at the point when they are created, the total energy density in magnetic monopoles is $\Omega_{\mathrm{mon}}(T_{\mathrm{GUT}}) = 10^{-10}$. Using this value, calculate the value of $\Omega_{\mathrm{mon}}/\Omega_{\mathrm{r}}$ at the present day. Is your answer compatible with observations?

   (b) Suppose now that we have a period of inflation, starting during the GUT era (after the creation of the monopoles). The radiation, matter and monopole densities will all fall off exponentially quickly during this period of inflation, but the total energy density remains constant. At the end of inflation, the energy of the field driving inflation is converted into matter and radiation, in a process known as **reheating**. Assume that essentially all of the energy is converted into radiation, and use this fact to help you determine how much inflation is necessary in order for the present day energy density of monopoles to equal that of radiation.

   (c) Compare your answer to part (b) with the amount of inflation necessary to solve the flatness and horizon problems. Is inflation a viable solution for the monopole problem?

2. **The "Big Rip"** (10 points)

   Suppose that we have a model for dark energy that has an equation of state

   $$p = w\rho c^2, \tag{1}$$

   where $w < -1$ and $w = \mathrm{const}$.

   (a) Show that in such a model, the cosmic scale factor tends to infinity after a finite time, a phenomenon that has been dubbed the "Big Rip".

   (b) Estimate the time remaining before the Big Rip for a flat Universe with $\Omega_{\mathrm{m}} = 0.3$, $H_0 = 70\,\mathrm{km\,s^{-1}\,Mpc^{-1}}$, and $w = -3/2$.

3. **Chaplygin Gas** (10 points)

Another possible model for dark energy is the Chaplygin gas model. A Chaplygin gas is a perfect fluid with the following exotic equation of state:

$$p = -\frac{A}{(\rho c^2)^\alpha}, \tag{2}$$

where A is a positive constant.

(a) Show that the energy density of a Chaplygin gas evolves with time as

$$\rho(t) = \frac{1}{c^2}\left[A + \frac{B}{a^{3(1+\alpha)}}\right]^{1/(1+\alpha)}, \tag{3}$$

where B is a constant of integration.

(b) Discuss the behaviour of this solution in the limits $a \ll 1$ and $a \gg 1$. What feature does this model have that makes it appealing as a model of dark energy?