Exercises for Cosmology (WS2014/15)

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Exercise sheet 7

Due: Dec. 5, 2014 11:00

To be handed in before the exercise class or emailed as a pdf or scanned hand-written document (via e-mail to walther@mpia.de). Remember to put your names on the document; you can work in groups of ≤ 2 but every student should hand in his/her own solution sheet and indicate clearly who contributed to it. All problems discuss relevant content for the final exam; however, you are only required to solve two out of the three problems and hence you can only earn a **maximum of 20 points per problem set** (which can be accumulated from any of the three problems).

1. **Power Spectrum and Correlation Function** (10 points)

   The power spectrum is conveniently expressed in dimensionless form as the variance per $\ln k$,

   $$\Delta^2(k) \equiv \frac{\mathrm{d}\langle\delta^2\rangle}{\mathrm{d}\ln k} = \frac{4\pi}{(2\pi)^3} k^3 P(k). \tag{1}$$

   Consider a dimensionless power spectrum with small-scale truncation:

   $$\Delta^2(k) = \left(\frac{k}{k_0}\right)^{n+3} \exp(-k/k_\mathrm{c}). \tag{2}$$

   Show that the corresponding correlation function is

   $$\xi(r) = \frac{(k_\mathrm{c}/k_0)^{n+3}}{y(1+y^2)^{1+n/2}} \Gamma(2+n) \sin[(2+n)\arctan y], \tag{3}$$

   where $y = k_\mathrm{c} r$. For this model, explain why ξ is negative at large r for $n > 0$. For what values of n does ξ stay positive at large r?

   Hint: to solve the final integral analytically, use the following identity

   $$\exp(-x)\sin(xy) \equiv \mathcal{I}m\{\exp[-(1+iy)x]\}, \tag{4}$$

   which suggests the change of variables $z = (1+iy)x$ in your integral.

2. **The Zel'dovich Approximation** (10 points)

   Consider a one-dimensional plane parallel density perturbation. Show that in this case, the Zel'dovich approximation provides an exact solution at all times before particle trajectories intersect.

   Hint: one way to go about this is to substitute the Zel'dovich approximation trajectories into the true equation of motion, and then to demonstrate that the gravitational potential $\vec{\nabla}\delta\Phi$ implied by the resulting equation is agrees with the solution for $\vec{\nabla}\delta\Phi$ that one gets from the Poisson equation.

3. **Imprints of Primordial Non-Gaussianity on the Distribution of Halos** (10 points)

Here, we derive an analytic expression for the peak height and clustering of dark matter halos in the presence of local non-Gaussianity in the primordial density field,

$$\Phi_{\rm NG}(\boldsymbol{x}) = \Phi(\boldsymbol{x}) - f_{\rm NL}\left(\Phi^2(\boldsymbol{x}) - \langle\Phi^2\rangle\right), \tag{5}$$

where Φ denotes the Newtonian potential, which is connected to the overdensity δ through Poisson's equation on subhorizon scales. With this choice of convention, positive $f_{\rm NL}$ corresponds to a positive skewness of the density probability distribution, and hence an increased number of massive objects.

(a) Since we are interested in the formation of halos, we focus on high peaks in the density field, where the derivative of Φ vanishes. Thus, we can neglect $|\nabla\Phi|^2$ in comparison to the curvature term $\Phi\nabla^2\Phi$ in the vicinity of rare, high peaks of the density distribution. Use Poisson's equation in combination with equation (5) to show that *in the vicinity of high peaks*, the density contrast in a model with non-Gaussianity ($\delta_{\rm NG}$) is connected to the matter density contrast of a purely Gaussian model (δ) via the equation

$$\delta_{\rm NG} \approx \delta\,(1 - 2f_{\rm NL}\Phi)\,. \tag{6}$$

This shows that non-Gaussianity *enhances* the peak height by a factor that is proportional to the primordial potential $|\Phi| = |\Phi_{\rm late}|a/D_+(a)$, where $\Phi_{\rm late}$ is the evolved potential at late times and $D_+(a)$ is the linear growth factor.

(b) The halo correlation function is parametrized in terms of the halo bias b, which is the rate of change of the halo abundance as the background density is varied. Writing the halo overdensity as $\delta_{\rm h}$ and the matter overdensity (in either model) as δ, we can define the halo bias as

$$\delta_{\rm h} = b\delta. \tag{7}$$

Consider a long-wavelength mode that provides a background density perturbation δ and corresponding potential fluctuation Φ. In the absence of non-Gaussianity, this perturbation raises (small-scale) subthreshold perturbations above the threshold, and thereby enhances the abundance of superthreshold peaks by $b_{\rm L}\delta$, where $b_{\rm L}$ is the (Gaussian) Lagrangian bias. As shown in part (a), for non-zero $f_{\rm NL}$, the long-wavelength mode also enhances the peak height by $2f_{\rm NL}|\Phi|\delta_{\rm pk}$, and we will focus on peaks near the threshold such that $\delta_{\rm pk} \simeq \delta_{\rm c}$. Show that in this case, the Lagrangian bias acquires the *scale-dependent* correction

$$\Delta b_{\rm L}(k) = 3b_{\rm L}f_{\rm NL}\frac{\delta_{\rm c}\Omega_{\rm m}H^2}{D_+(a)k^2}, \tag{8}$$

where the total Lagrangian bias is $b_{\rm L}(k) = b_{\rm L} + \Delta b_{\rm L}(k)$. If you want to maximize your constraining power on the local non-Gaussianity parameter $f_{\rm NL}$, which type of objects would you target for a survey?

(c) Calculate the corresponding Eulerian bias in the presence of local non-Gaussianity.