Master Course Astrophysics - Galaxies & Cosmology
SS 2018

Lecture 11: Structure and Dynamics I

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Introduction: Some general properties of self-gravitating systems (consisting of point masses = "stars")

* $F_G \propto r^{-2}$, no self-shielding → large distances

* many distant objects exert same gravitational pull as a single nearby one

e.g. for $\rho = \text{const}$: $M(r, r+dr) = 4\pi r^2 dr \rho$

$$\Rightarrow F(r) \propto r^2 \cdot \frac{1}{r^2} \propto \text{const.}$$

* No natural & stable equilibrium state:

Virial theorem: $K = -\frac{V}{2}$ for full system

→ total energy $E = K + V = \frac{V}{2} < 0$ always!

in case of energy loss (e.g. from collisions)

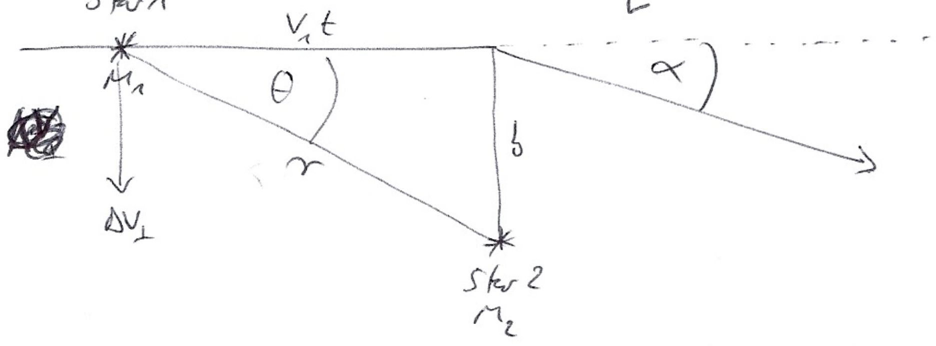
→ E even more negative

→ K increases

→ probability for further energy losses increases.

* Direct interactions between 2 stars are rare; but they get important over long time spans:

Consider a single "distant encounter"
[≈ Coulomb scattering → standard literature]



In small deflection approximation (α very small):

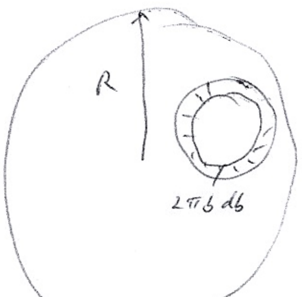
$$\Delta v_\perp = \frac{2GM_1}{b v_1}$$ change of velocity of star 1

$$\alpha = \frac{2GM_1}{b v_1^2}$$ deflection angle

Number of interactions for a star crossing a stellar system

R = radius of system

N = number of stars



Probability for passing 1 star in $[b, b+db]$: $P_1 = \frac{2\pi b\,db}{\pi R^2}$

passing N stars: $dn_b = \frac{2N b\,db}{R^2}$

average change of velocity: $\langle dv_\perp \rangle = 0$

but mean square deflection is $\langle dv_\perp^2 \rangle = \left(\frac{2GM_1}{b v_1}\right)^2 dn_b > 0$!

→ slow increase of quasi-random motions: "heating up"

* relaxation time:

after many interactions: $\langle \Delta v_\perp^2 \rangle \simeq \langle v \rangle^2$

this is reached after

$$t_{relax} \simeq \frac{N}{8 \ln N} \frac{R}{\langle v \rangle}$$

crossing time for 1 star

$\boxed{\text{derivations} \rightarrow \text{literature \& exercises!}}$

=> in many cases much larger than age of the universe.

=> Initial conditions are not erased for many typical stellar systems (exceptions: very dense systems with strongly enhanced interaction rates)

Once more: stellar systems can be long-lived (large relaxation times) but they are not stable configurations.

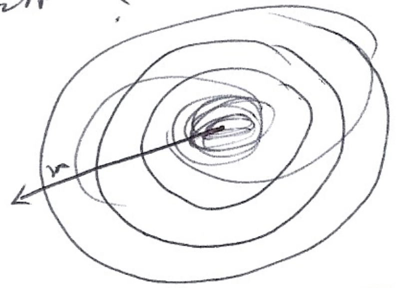
Disc galaxies

* Evidence: Spiral galaxies often thin axisymmetric discs with vertical extent $H \ll$ radial extent R

$\boxed{\text{Slide:}}$ Hubble sequence face-on / edge-on

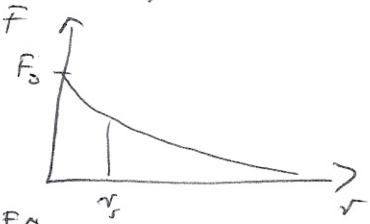
- How to define "size" or "extent"?

  surface brightness profiles

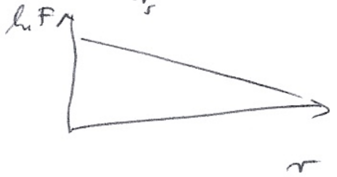


for disc galaxies mostly well approximated by single exponential $\boxed{\text{slide}}$

$$F(r) = F_0 \, e^{-r/r_s}$$

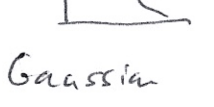


$$\ln F(r) = \ln F_0 - \frac{r}{r_s}$$

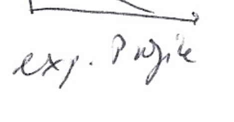


special case of "sersic profile":

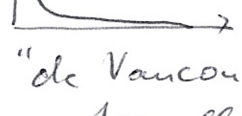
$$F(r) = F_0 \, e^{-(r/r_s)^{1/n}} \quad \text{where } n > 0$$



Gaussian



exp. Profile



"de Vaucouleurs" profile for elliptical galaxies

Total brightness of galaxy:

$$S_{total} = \int_0^\infty 2\pi r \, F(r) \, dr \; ; \quad \text{half-light radius: } \int_0^{r_{50}} 2\pi r F(r) \, dr = \frac{S_{total}}{2}$$

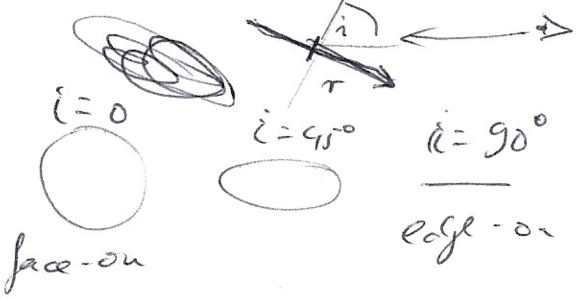
Vertical surface brightness distribution
for edge-on galaxies $\boxed{\text{slide}}$

exponential + flattening near midplane

→ "Extent" of galaxies:

- scale length (for exponentials)
- half-light radius
- isophotal at given surface brightness

How do we know that spirals = thin discs?

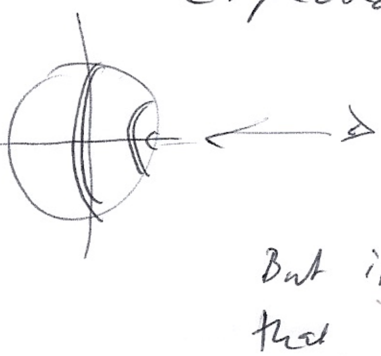


projected shape = ellipse
major axis $a = r$
minor axis $b = a \cdot \cos i$

$\rightsquigarrow \cos i \approx \frac{b}{a}$

no way to test this for individual galaxies!
but for large statistical ensemble: inclinations to line of sight should be random.

Expected distribution from solid angle covered



$d\Omega(i) = \sin i \, di$ ⇝ much larger for ° → non-uniform in i (many more at $i \approx 90°$ than at $i \approx 0°$)

But it can be shown (exercises / seminar) that $\frac{b}{a}$ is ~ uniformly distributed in $[0, 1]$ for thin circular discs.

☆ Slides: $\frac{b}{a}$ for spiral galaxies / for ellipticals

Simplest model for disc galaxies:
- circular coplanar concentric orbits
- orbital speed to match centrifugal forces from grav. potential

Rotation curve $V(r)$: certainly non-Keplerian unless far away from the galaxy

* For self-gravitating exponential disc:

slide lengthy calculation [→ Binney & Tremaine, BT]

but little practical relevance: we know that

$V(r) \sim$ const.

and potential is dominated by Dark Matter, which is ~ spherically symmetric.

* Spherical potential for $\rho(r) \propto r^{-2}$

→ $V(r) =$ const.

→ angular speed $\omega(r) = \frac{V}{r}$

→ differential rotation $\frac{dv}{dr} \propto \frac{1}{r}$

Since 2-body interactions depend on relative velocities:

→ interaction rate initially very low