Master Course Astrophysics - Galaxies & Cosmology
SS 2018

Lecture 12: Structure and Dynamics II

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Non-circular motions in axisymmetric disc galaxies as 1st order perturbations:

$\boxed{\rightarrow \text{Binney \& Tremaine p. 159 f}}$

Adopt cylinder coordinates

$\Rightarrow$ separate equations of motion into (r, φ, z):

$$\ddot{r} - \dot{\varphi}^2 r = -\frac{\partial \Phi}{r} \qquad \Phi = \text{potential}$$

$$r\ddot{\varphi} + 2\dot{r}\dot{\varphi} = 0$$

$$\ddot{z} = -\frac{\partial \Phi}{\partial z} \longrightarrow$$ completely separable ~~we~~ will consider this later.

~~for now~~

for now we set $z=0$!

Expand potential into Taylor series around $r \simeq r_0$, $z \simeq 0$; take derivative:

$$\frac{\partial \Phi}{\partial r} \approx \left(\frac{\partial \Phi}{\partial r}\right)_{r=r_0} + (r - r_0)\left(\frac{\partial^2 \Phi}{\partial r^2}\right)_{r=r_0} + \dots$$

and define $\omega_0^2 = \frac{1}{r_0^2} V_c^2(r_0) = \frac{1}{r_0}\left(\frac{\partial \Phi}{\partial r}\right)_{r=r_0}$

circular frequency (angular speed) at r_0

we also set $x := r(t) - r_0$, $x \ll r_0$

and $\theta := \varphi(t) - \omega_0 t$

inserted into equations of motion:

$$\ddot{x} - 2\omega_0 r_0 \dot{\theta} - \omega_0^2 x = -x \left(\frac{\partial^2 \Phi}{\partial r^2}\right)_{r=0} \qquad (1)$$

$$r_0 \ddot{\theta} + 2\dot{x}\,\omega_0 = 0 \qquad (2)$$

Integrate (2) over t, setting integration constant to 0:

$r_0 \dot{\theta} + 2x\,\omega_0 = 0$ – insert into (1):

$$\ddot{x} = -\underbrace{\left[\left(\frac{\partial^2 \Phi}{\partial r^2}\right)_{r=r_0} + 3\omega_0^2\right]}_{\varkappa^2} x = -\varkappa^2 \cdot x \qquad (3)$$

[harmonic oscillator]

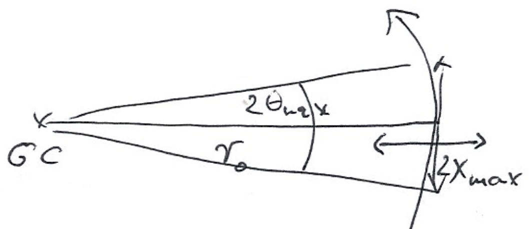
Solution: $x(t) = \underbrace{X_{max} \cos(\varkappa t + \alpha)}$ constants of integration.

(without proof:) $\theta(t) = -\frac{2\omega_0}{\varkappa}\frac{X_{max}}{r_0} \sin(\varkappa t + \alpha)$ θ_{max}

$\Rightarrow$ superposition of 2 motions:

1) circular orbit at $r = r_0$ with $v_c = \omega_0 r_0$

2) oscillation around point on circ. orbit

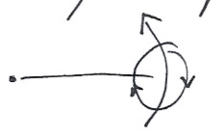


let $y = \theta \cdot r_0$:

$y(t) = -\frac{2\omega_0}{\varkappa} X_{max} \sin(\varkappa t + \alpha)$ y_{max}

$\Rightarrow$ Ellipse with axes X_{max} and y_{max}, where $y_{max}/X_{max} = \frac{2\omega_0}{\varkappa}$

⇒ Circular orbit of "guiding centre"
+ superimposed ellipse ⇒ "Epicycle"



(similar to ancient Greek / also Copernican concepts of planetary motions)

Digression: How does the orbit of the Moon around the Sun look like?

(∗ slide:) lunar orbit around the sun

Force on moon by Sun is 2x larger than by Earth:

$$\frac{F_{E-M}}{F_{S-M}} = \frac{M_E}{M_\odot}\left(\frac{a_M}{a_E}\right)^{-2} \approx 0.48 \rightarrow \text{lunar orbit always convex towards sun!}$$

How does a combined epicycle orbit in a Galactic potential look like?

- depends critically on value of $\varkappa$ (or ratio: on ratio $\varkappa/\omega_0$)

Useful to consider extreme cases first:

(a) Central point mass M → Keplerian problem

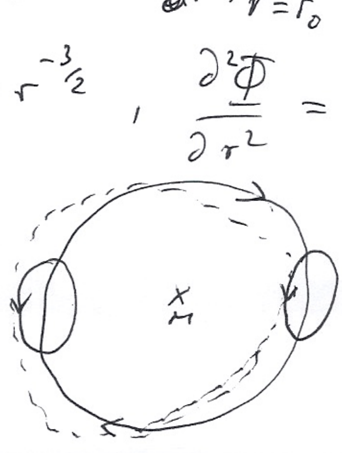
$$\Phi(r) \propto r^{-2} \rightarrow v_c^2(r_0) = r_0\left(\frac{\partial\Phi}{\partial r}\right)\bigg|_{r=r_0} = \frac{GM}{r_0}$$

$$\rightarrow \omega_0 = \frac{v_c}{r_0} = GM\, r^{-3/2}, \quad \frac{\partial^2\Phi}{\partial r^2} = -4\omega_0^2$$

$$\rightarrow \varkappa = \omega_0 \, !$$

orbit = excentric ellipse!

- recovery of Kepler I.



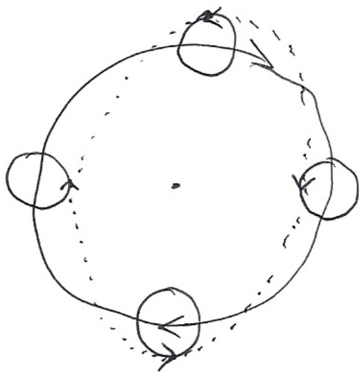
Note: $y_{max} = 2\, x_{max}$

(b) Constant mass density $\rho(r) = \text{const.}$; $M(r) = \frac{4}{3}\pi r^3 \rho$

$$v_c^2 = r_0\left(\frac{\partial \Phi}{\partial r}\right) = \frac{G\,M(r)}{r} \propto r^2 \quad , \; v_c \propto r$$

$$\omega_0 = \frac{v_c}{r_0} = \text{const.} \rightarrow \text{solid body rotation.}$$

$\rightarrow \varkappa = 2\,\omega_0$ for all r.



$y_{max} = x_{max}$: circular epicycles

orbits = ellipses again, but now <u>concentric</u>: "ovals"

Important for inner regions of galaxies. Possible orbital resonances can be cause of spiral structure ("density wave theory").

(*) Slide: how to make spiral arms by twisted ovals.

In most parts of a disc galaxy we know that $v_c(r) \simeq \text{const.} \rightarrow \omega_0 \propto r_0^{-1}$

then: $\varkappa = \sqrt{2}\,\omega_0$

Between the extremes (a) and (b): nearly always

$$\omega_0 \leq \varkappa \leq 2\,\omega_0$$

epicycle frequency of the same order of magnitude as ~~orbital~~ circular orbital frequency of guiding centre.

– Generally: non-closed orbits!

"rosetta pattern" after several revolutions.

* slide: rosetta pattern in non-closed orbit

Individual orbits are however unobservable!

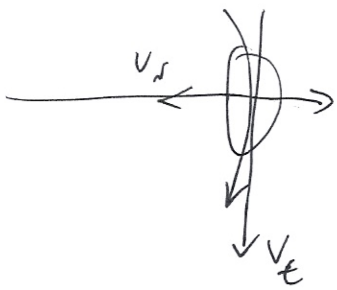
Observable: – mean rotation speed $\bar{v}$ at radius r

– line-of-sight velocity dispersion σ at r

more general: line-of-sight velocity distribution

LOSVD

we thus need velocities on the epicycles:



$$v_r = \dot{x} = \varkappa \cdot x_{max} \cos(\varkappa t+\alpha) \quad \left(\text{since } \frac{dv_c}{dr}=0\right)$$

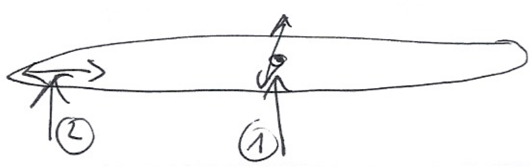
$$v_t = v_c + \dot{y} = v_c + \frac{2\omega_0^2}{\varkappa} x_{max} \sin(\varkappa t+\alpha)$$

Governing parameter: epicycle amplitude x_{max}

Averaging over all phases α:

① $\overline{v_r^2} = \frac{1}{2}\varkappa^2 x_{max}^2$ = vel. disp. edge-on central l.o.s.

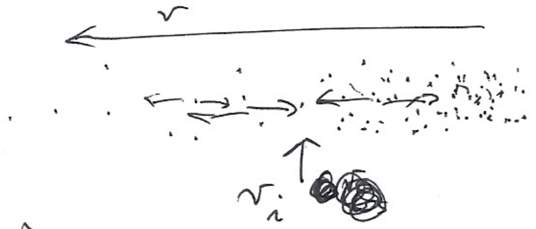
② $\overline{(v_t - v_c)^2} = \frac{\varkappa^4}{8\omega_0^2} x_{max}^2$ = vel. disp. edge-on tangential l.o.s.



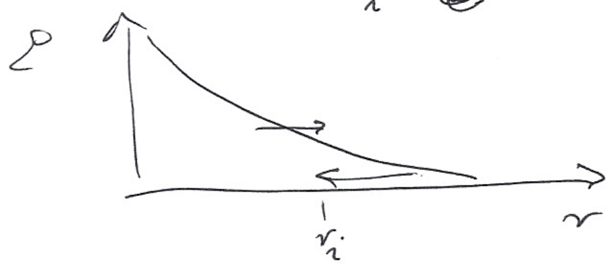
What about the mean l.o.s. velocity $\bar{v}$?

v_r symmetric about guiding centre, cancels out?

No: need to consider exponential density distribution of galactic discs!



at arbitrary point r_i more likely that star from $r_0 < r_i$ is on an outer excursion on epicycle than from $r_0 > r_i$ on an inner excursion!



since epicycles are always retrograde:
"outer" = too slow w.r.t. circular
"inner" = " fast " " " " "

$\Rightarrow$ mean ~~...~~ tangential velocity of stars at given r_i is always less than circular speed v_c at r_i

"asymmetric drift".

Difference depends on typical amplitude X_{max} ≈ amount of quasi-random motion.

cf. Galactic orbit of the Sun:
Δv_t relative to "Local Standard of Rest" (LSR)
= +5 km/s → guiding centre of the solar orbit lies at r_0 > current radius.

Now for the vertical axis (z):

$$\ddot{z} = -\left(\frac{\partial^2 \Phi}{\partial z^2}\right)_{\substack{r=r_0\\ z=0}} \cdot z$$

again a harmonic oscillator with

frequency $\nu(r_0) = \sqrt{\left(\frac{\partial^2 \Phi}{\partial z^2}\right)_{\substack{r=r_0\\ z=0}}}$

thus: $z(t) = z_{max} \cos(\nu t + \beta)$

→ vertical thickening of thin disc with typical width z_{max}

l.o.s. velocity dispersion in face-on disc

$$\overline{V_z^2} = \frac{1}{2} \nu^2 z_{max}$$

Qualitative description of ~~[illegible]~~ "heated disc", but not good enough to explain observations of ~exponential density distributions in z direction.

— limitation of epicycle approximation.