Lecture 4: Galaxy distribution functions and clustering

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Introductory slides:

- ⊛ Galaxies in the Hubble Ultra Deep Field
- ⊛ The large-scale distribution of galaxies in projection

→ Wide range of galaxy brightnesses, sizes, ..
but to decode this information we need distances, i.e. redshifts of the galaxies!

→ Clearly non-homogeneously distributed in the sky but again redshifts required for de-projection.

Today: Statistical approach of "cosmography"
= mapping the density distribution in the universe, assuming galaxies as "points" in space!
Focus on: a) Distribution of luminosities
b) Statistical description of clustering

a) The luminosity (distribution) function of galaxies

Let n_{gal} := number density of galaxies per unit volume, e.g. per Mpc^3

Definition luminosity function (LF):

$$\phi_{gal}(L) = \frac{d n_{gal}}{dL}$$

L = Luminosity (bolometric or per spectral band)

Frequently the LF is taken per log luminosity interval.

$$\phi_{gal}(\log L) = \frac{dn_{gal}}{d(\log L)}$$

or equivalent in absolute magnitudes, with $M_{abs} = \text{const.} - 2.5 \log L$

As measurement: samples have finite sizes

→ derivatives replaced by binning and ratios:

$$\phi(\log L) \simeq \frac{1}{\Delta \log L} \underbrace{\frac{1}{V}}_{\text{survey volume}} \overbrace{N_{gal}}^{\text{number of galaxies per bin}}\left(\log L - \frac{\Delta(\log L)}{2}, \log L + \frac{\Delta(\log L)}{2}\right)$$

– basically a histogram.

But this doesn't work in practice, because real samples are never volume-complete, but flux-limited (or similar).

? How to get a volume-related quantity such as the LF from a flux-limited sample?

(*) Slide: Example for a flux-limited galaxy sample

Sample contains all galaxies within a certain solid angle Ω (footprint of the survey) and with magnitudes $m_i < m_{lim}$.

The issue is: m_{lim} corresponds to different absolute magnitudes, depending on the redshift.

Let's convert $z \to d_L$ (luminosity distance) and $m \to M_{abs}$

(*) Slide: Same as before, but now converted to d_L, M_{abs}.

Consider the different available survey volumes:

for $M_{abs} = -18$: m_{lim} is at $d_L \simeq 120$ Mpc

-20: m_{lim} is at $d_L \simeq 360$ Mpc

$\Rightarrow$ survey volume is $3^3 = 27 \times$ larger for $M = -20$

Def: $d_{max,i}$:= distance out to which an object i with ~~[illegible]~~ absolute mag. $M_{abs,i}$ could have been found, given m_{lim}.

then: $V_{max,i} := \frac{4}{3}\pi \, d_{max,i}^3 \times \Omega/4\pi$ "survey volume"

~~[illegible]~~

How to use this as LF estimator? Very simple: replace the summation over unit contributions by summation over $\frac{1}{V_{max,i}}$

i.e. instead of

$$\phi = \frac{1}{\Delta \log L} \sum \frac{1}{V} = \frac{1}{\Delta \log L} \frac{N}{V} \longrightarrow \phi = \frac{1}{\Delta \log L} \sum \frac{1}{V_{max,i}}$$

for each luminosity bin separately.

since low-L galaxies have smaller V_{max}, $\sum \frac{1}{V_{max}}$ becomes larger $\rightarrow$ higher number densities despite smaller number of objects.

Results from large redshift surveys:

(*) Slide: Galaxy LF from 2dF survey

Common approximation to ~~Φ~~LF ~~·~~ with analytical expression

$$\phi(L) = \phi^* \left(\frac{L}{L^*}\right)^\alpha e^{-L/L^*}$$

introduced by Schechter (1976), ApJ 203, 297
motivated by the previous theoretical model of ~~·~~ self-similar gravitational collapse (Press & Schechter ~~·~~1976, ApJ 187, 425)
— but its application to the galaxy LF is only heuristic, providing basically a fitting function!

Free parameters of the Schechter function:
L^* : characteristic luminosity, "knee" of the LF
α : power-law slope of the faint end
ϕ^* : overall normalisation;

Note: When transforming ~~from~~ $L \to \log L$ or M_{abs}, the exponent effectively changes from $\alpha \to \alpha - 1$. [Proof as exercise!]

<u>Luminosity density</u>:

= Volume-integrated emissivity of all galaxies as function of ~~·~~ lower luminosity limit, $L > L_{min}$:

$$\rho_L(L_{min}) = \int_{L_{min}}^{\infty} L' \, \phi(L') \, dL'$$

units: $[\mathrm{erg\ s^{-1}\ Mpc^{-3}}]$

What happens for $L_{min} \to 0$ for different values of α?
→ Check this yourself!

Distribution function of stellar masses of galaxies

M_s = Total mass in stars of a galaxy
- not a direct measurable quantity, but it can be estimated from the spectral energy distribution + stellar population models.

Construction of $\phi(M_s)$: similar to $\phi(L)$, but need much more data per galaxy in the sample (at least photometry in several bands, and of course redshifts)

(*) Slide: stellar mass function of galaxies

Shape ~similar to LF at first sight, but somewhat more complex.

While single Schechter function is a poor fit, a "double Schechter" seems to work well:

$$\phi(M_s) = \left[\phi_1^* \left(\frac{M_s}{M_s^*}\right)^{\alpha_1} + \phi_2^* \left(\frac{M_s}{M_s^*}\right)^{\alpha_2}\right] e^{-\frac{M_s}{M_s^*}}$$

with 5 free parameters: $\alpha_1, \alpha_2, \phi_1^*, \phi_2^*$, and M_s^* (same for both Schechter components).

Initially just a fitting trick, but probably astrophysically meaningful (→ later)

Mass density in stars: analogous to luminosity density:

$$\rho_s = \int_{M_{min}}^{\infty} M_s \, \phi(M_s) \, dM_s \qquad \text{units: } [M_\odot \, Mpc^{-3}]$$

b) Galaxy clustering

As seen, the distribution of galaxies in space is non-uniform on scales of $\approx 1 \ldots 100$ Mpc. This becomes much more evident when using redshift surveys to de-project.

* Slide: Galaxy clustering in the CfA redshift survey.

How to measure the non-uniformity quantitatively?
- Approach: Consider galaxies as point realisations of an underlying probability distribution.

Let's take N galaxies in volume V.

i) distributed in a statistically uniform way (i.e. no clustering).
The probability to find 1 galaxy in $\begin{pmatrix} x, & x+dx \\ y, & y+dy \\ z, & z+dz \end{pmatrix}$

is $P_1 = N \cdot \frac{dV}{V} = n\,dV$ with $dV = dx \cdot dy \cdot dz$ and $n = \frac{N}{V}$

In this model, the probability to find 2 galaxies in the same volume element is

$$P_2 = P_1^2$$

ii) The distribution is uniform on large scales (cosmological principle), but with an excess probability to find close pairs.

$$\Rightarrow \quad P_2 = P_1^2 \left(1 + \xi(r)\right)$$

Def: $\xi(r)$ is the excess probability for finding pair at separation r, where $r = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2+(z_2-z_1)^2}$

$\xi(r)$ is called the "two-point correlation function". It is an observable!

→ Take a galaxy catalogue, count the number of pairs as a function of separation (every galaxy against every other galaxy) and compare with the expected number for a pure random distribution.

(*) Slide: The observed two-point correlation function of galaxies.

In good approximation, the 2pcf is a simple power law:

$$\xi(r) \approx \left(\frac{r}{r_0}\right)^{-\gamma}$$

with r_0: "correlation length"

and γ: slope of the power law.

→ Galaxies are strongly clustered, even on scales much larger than individual groups or clusters. For a typical $r_0 \simeq 6-7$ Mpc and $\gamma \approx 1.7$, the excess probability is significantly > 0 already for $r \lesssim$ some 10 Mpc.

Generalisation to density fields:

Consider $\rho(\vec{x}) = \bar{\rho}(1+\delta(\vec{x}))$ with $\delta(\vec{x})$: Density contrast relative to mean density $\bar{\rho}$

then the cross-correlation is defined

as $\underbrace{\langle \rho(\vec{x}) \rho(\vec{y}) \rangle}_{\text{expectation value}}$

$= \bar{\rho}^2 \langle (1+\delta(\vec{x}))(1+\delta(\vec{y})) \rangle$

$= \bar{\rho}^2 \, (1 + \underbrace{\langle (\delta(\vec{x})\, \delta(\vec{y})) \rangle}_{\text{depends only on } |\vec{x}-\vec{y}| = r})$ since $\langle \delta(\vec{x}) \rangle = 0$ by definition

$= \bar{\rho}^2 (1 + \xi(r))$

Alternatively, the cosmic density field can be characterised by its power spectrum. It can be shown (not here!) that there is a 1:1 relation between power spectrum and two-point correlation function:

$$P(k) = 2\pi \int_0^\infty dr \; r^2 \frac{\sin(kr)}{kr} \xi(r)$$

→ Connection between cosmological theory (density field) and astronomical observable! (clustering)

This is the reason why galaxy redshift surveys have become one of the most important tools of observational cosmology!

Peculiar velocities and redshift-space distortions

The measured radial velocity of a galaxy is

$$v_r = cz = H_0\, d_{true} + v_{pec} \qquad (z \ll 1, \text{ local universe})$$

(not independently measurable)

but from redshift surveys we infer $d_z = \frac{cz}{H_0} = d_{true} + \frac{v_{pec}}{H_0}$

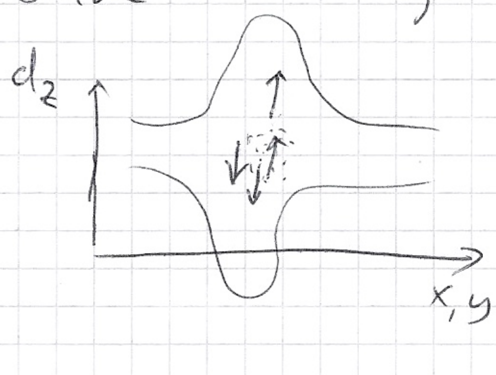
(for $z \not\ll 1$: use cosmological relations)

Def. "Redshift space":
use transverse coordinates x, y (or α, δ) unchanged, but take d_z as radial distance instead of unknown d_{true}.

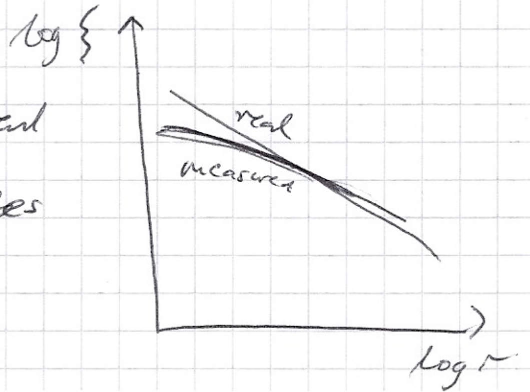
This implies some characteristic patterns, known as "Redshift space distortions":

i) Enhanced v_{pec} due to the higher velocity dispersions in galaxy groups and (especially) clusters.

(X) Slide: "Finger of god" in SDSS data.

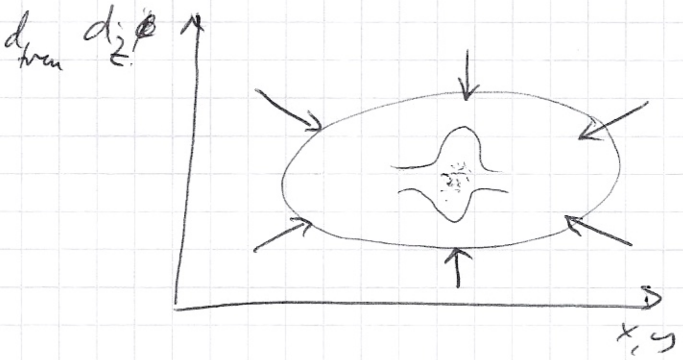


extends structure in radial direction.



Net effect: reduction of apparent clustering strength at small scales

ii) Large-scale infall into overdense cosmic structures:

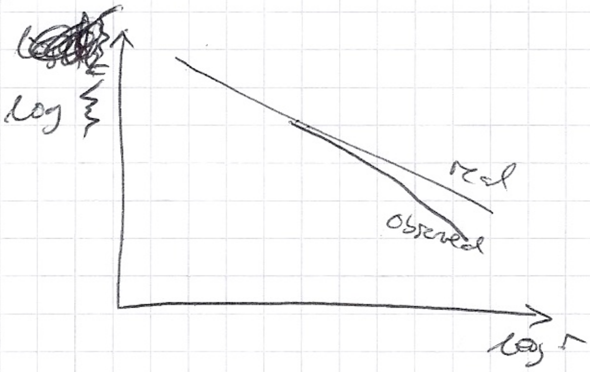


more subtle effect than "fingers of god", but measured.

"Kaiser effect" after N. Kaiser (1987)

(*) Slide: Kaiser effect

Net effect on 2pcf:



Technical trick to get rid of redshift space distortions in clustering measurements: "Projected correlation function":

Def: $$w_p(r_p) = \int_{-\infty}^{\infty} \xi(r_p, \pi)\, d\pi$$

where $\pi = \frac{c\Delta z}{H_0}$ = apparent line-of-sight separation in redshift space

Equivalently: $$w_p(r_p) = \int_{-\infty}^{\infty} \xi\left(\sqrt{r_p^2 + r_3^2}\right) dr_3$$

where r_3 = true line-of-sight separation

One can show that if $\xi(r)$ is a power law with (r_0, γ), then $w_p(r_p)$ is also a power law with $(r_0, \gamma - 1)$.

→ possibility to measure r_0, γ despite of redshift space distortions.

How does the 2pcf behave on <u>very</u> large scales?

Naive expectation: should go to zero (cosmological principle)

Observations: yes, but not monotonically.

(X) Slide: Baryonic acoustic oscillations

slight increase around 100/h Mpc

- generally interpreted as structure remnant of acoustic waves in early universe before cosmic recombination - also seen in CMB.

Can be used as "cosmic ruler" to measure the expansion of the universe since $z = 1000$.

- Big topic for future redshift surveys.