

Galaxies are made of stars

- appearance of galaxies is due to their composition
  - type of stars
  - amount of gas
  - dark matter, bh....
- due to their kinematics & dynamics

⇒ SHOW IMAGES OF GALAXIES

- spectra: source of information

⇒ SHOW SPECTRA OF

- ELLIPTICAL
- SPIRAL
- STARBURST

→ why are these spectra different?

① amount of gas: emission lines

② very different shape

- E: no light blueward of  $4000 \text{ \AA}$
- SB: lots of light in the blue

- in order to understand galaxies one has to:
  - know Newtonian dynamics (last lecture)
  - understand stars and stellar populations

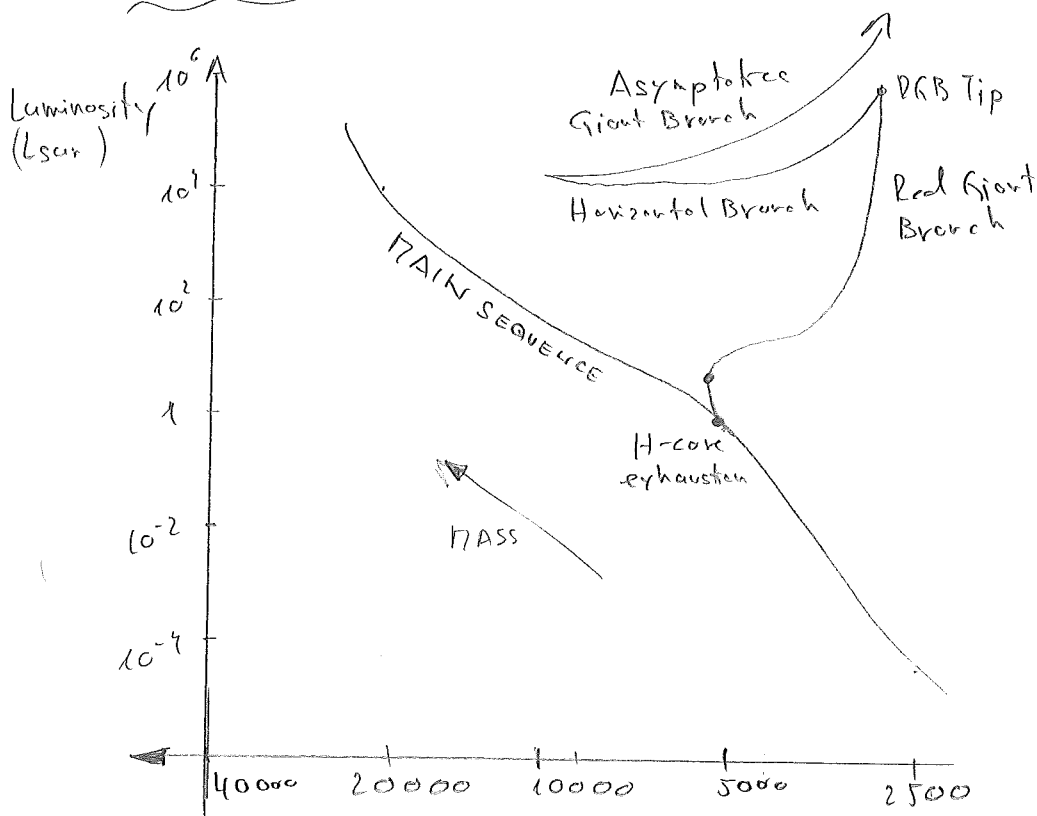
- stellar populations

- a link between stellar astrophysics and properties of galaxies

theory that explains the structure and working of stars

observed characteristics of galaxies

Phases of stellar evolution: a reminder



① Main sequence: (MS)

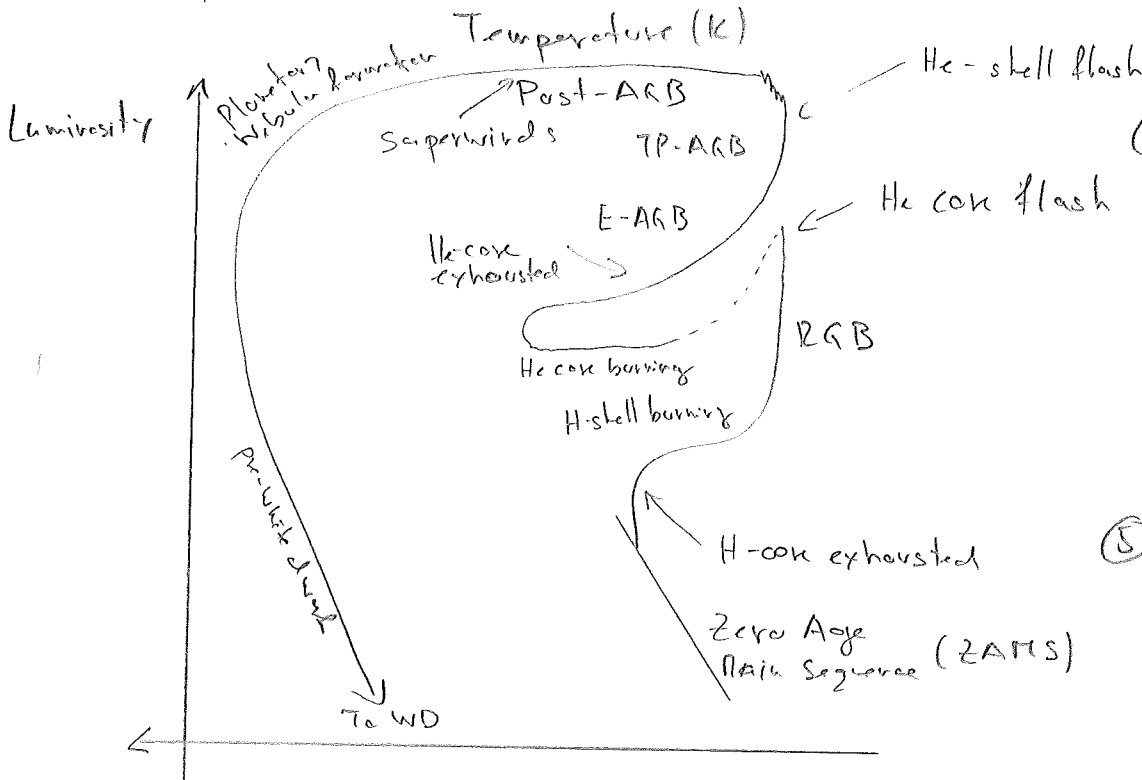
- core H burning
- longest phase of evolution

② Turn-off (TO)

- H exhausted in core
- > beginning of interesting evolution

③ Red Giant Branch (RGB)

- H burning in shell around inert He core
- growth of He core



④ RGB Tip

- end of RGB phase
- He core massive and hot enough to ignite He-burning (He flash)

⑤ Horizontal Branch (HB)

- core He burning
- details depend on metallicity & mass loss during RGB phase

⑥ Asymptotic Giant Branch (AGB)

- He burning in shell around inert C/O core
- very complex (not fully understood) evolution dependent on mass

⑦ White dwarf sequence (WD)

- low mass stars eject envelope and leave inert C/O core which cools as a WD

⑧ Supernovae [more on this later]

- massive stars explode
- neutron stars, black holes

Stellar lifetimes depend on their masses

- each evolutionary phase has its duration, which is sensitive to the initial mass -

- MS:  $t \sim 10^{10} \left( \frac{M}{M_{sun}} \right)^{-2.5}$

for  $M = 1 M_{\odot} \Rightarrow t \sim 10 \text{ Gyr}$

for  $M = 10 M_{\odot} \Rightarrow t \sim 30 \text{ Myr}$

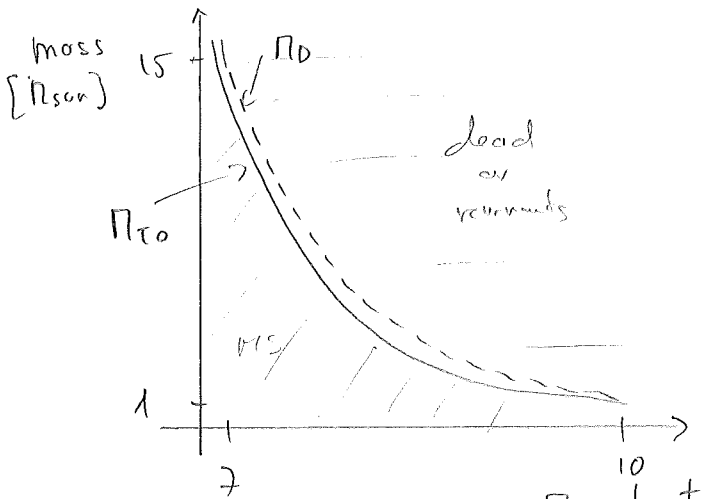
- subsequent phases start lived

- time on MS

$\log M_{TO} = 0.0439 \log t^2 - 1.146 \log t + 7.119$  turn off mass

- total life time of a star

$\log M_D = 0.0379 \log t^2 - 1.048 \log t + 6.119$  Mass of final star dying mass



$\Rightarrow M < M_{TO}$   
 $\rightarrow$  all stars are still on their sequence burning H

$\Rightarrow M > M_D$   
 $\rightarrow$  dead stars, remnants  
 WD, neutron stars, BH

[plot it, find what is the mass of star that has life time > turnoff]

$\Rightarrow M_{TO} < M < M_D$   
 $\rightarrow$  stars going through post-MS evolutionary stages

$\Rightarrow M \approx M_{TO} \approx M_D$  - initial mass for all post-MS phases is the same, and is equal to the turn-off mass  
 - it could be used as a time indicator

# Stellar evolutionary tracks and isochrones

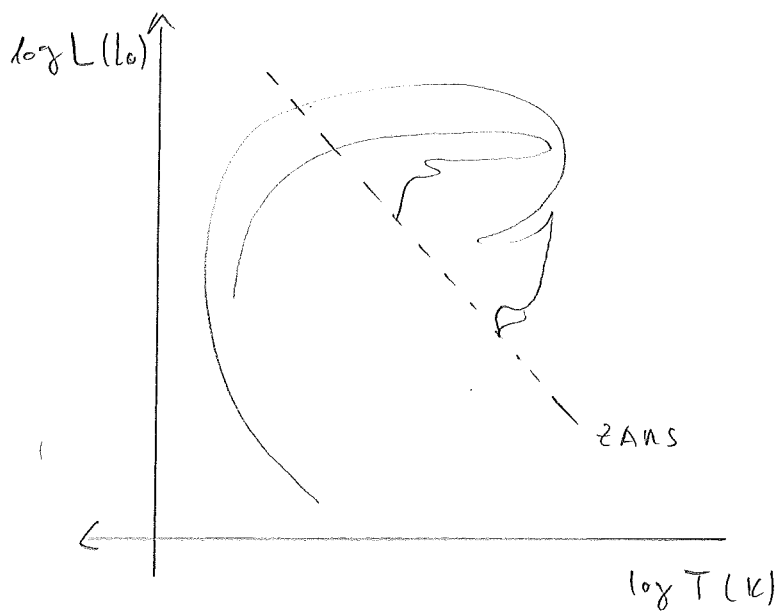
Tracks: describe evolution of a single star with time

- output of a stellar modelling code
- table with stellar parameters:
  - luminosity
  - temperature
  - surface gravity (dwarf / giant)
  - core temperature
  - core consumption
  - current mass
  - ....

track: locus of stars of given initial mass  $M_i$  as a function of age  $t$   
 isochrone: locus of stars of given age  $t$  as a function of initial mass  $M_i$

$$a(t) \Big|_{M_i = \text{cte}} \rightarrow a(M_i) \Big|_{t = \text{cte}}$$

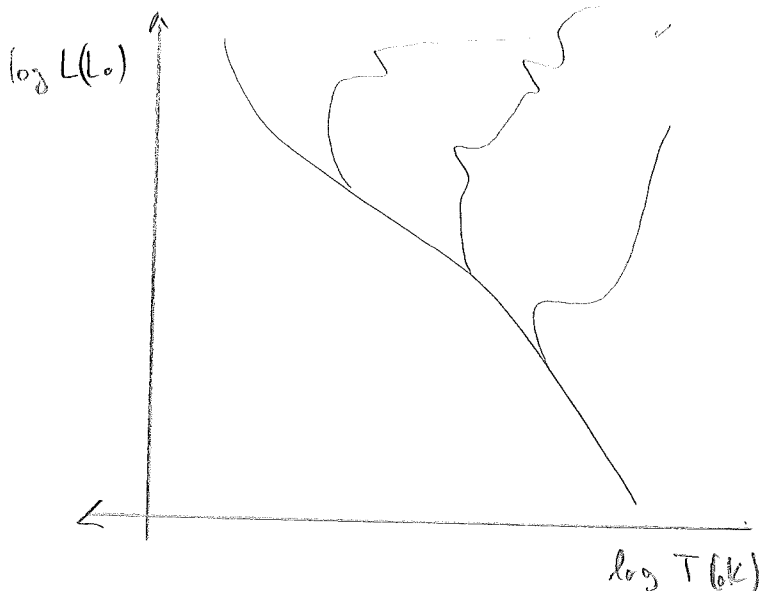
- grid of tracks has to be very dense  $\Delta M_i \rightarrow 0$
- interpolation between equivalent evolutionary status in adjacent tracks.



- tracks depend on mass
  - low mass stars  $M < 0.5 M_\odot$  stay on ZAMS
  - high mass stars move through different evolutionary phases
- $\Rightarrow$  SHOW example isochrones  
 $\Rightarrow$  SHOW example tracks

## ISCHRONES

- lines that join parameters of stars of different masses at the same age
- tables: same content as for evolutionary tracks but reorganized
- made from dense grids of evolutionary tracks  $M_i(t)$  via interpolation, for every evolutionary property ( $L, T_{\text{eff}}, M_{\text{core}}, \text{chemistry}$ )



# Single Stellar population = SSP

SSP - stellar population described by a single age, single metallicity  
or Initial Mass Function, and a total mass (initial or present)

imagine: convert a homogeneous package of gas instantaneously  
 into stars



SSP = essentially the isochrone  
 ↳ all stars of various mass that have the same age & metallicity

↳ this is given by IMF  
 (won later)

⇒ for example: Globular clusters  
 ↳ are typically SSP

## Composite stellar population

CSP - a sum of SSPs, each age and metallicity weighted by  
 the star formation history (SFH)

⇒ now we have to explain these terms:

1) metallicity

2) IMF

3) SFH & element abundances

What is metallicity?

"Metal" content of the gas the stars formed from

- core of stars changes during life time (as stars burn fuel)
- surface layers reflect the original composition (at least until late evolutionary phase)

"Metal" - for astronomers

- H
- He
- metals (oxygen is a metal...)

chemical mixtures of stars expressed as mass fractions

$X$  or  $H$  = mass fraction of H

$Y$  or  $He$  = mass fraction of He

$Z$  = mass fraction of everything else = metals

empirically: measurements of metallicity in stellar atmospheres usually expressed in terms of number density (NOT mass fraction)

$$[Z/H] = \log_{10} \frac{N_Z}{N_H} - \log_{10} \left( \frac{N_Z}{N_H} \right)_{\text{sun}}$$

$Z/H = 0 \equiv$  solar metallicity

$Z/H = +0.3 \equiv 2 \times$  solar

$Z/H = -1 = \frac{1}{10}$  solar

- what is  $Z$ ?  $\rightarrow$  which element should be used

$\Rightarrow$  many possibilities

$\Rightarrow O \rightarrow$  very important for stellar evolution, but hard to measure

Fe  $\rightarrow$  easier to measure in spectra

$[Fe/H]$  is preferred to  $[O/H]$  or  $[Z/Fe]$  if all elements are varied proportionally

# Where do metals come from?

## ① Big Bang nucleosynthesis

$\Rightarrow$   $4\text{He}$  + some traces of  $\text{D}$ ,  $3\text{He}$ ,  $3\text{Be}$ ,  $7\text{Li}$ ,  $6\text{Li}$

$\Rightarrow$  everything else comes from stars

$\Rightarrow$  Show slide with PN, SN II, SN Ia out

## ② Low-mass stars: AGB/planetary nebula

$1 < M < 8 M_{\text{sun}}$

- outer layers enriched in  $\text{C}$ ,  $\text{N}$ ,  $\text{O}$  (AGB phase of He-shell burning)  
 $\rightarrow$  ejected into ISM

- core:  $\text{C-O}$   $\rightarrow$  post AGB (luminous) cooling to WD

$\rightarrow$  outer layers ionised by p-AGB  $\rightarrow$  shine as PN

-  $t \sim 10^3 \text{ yr}$

## ③ High-mass stars: Type II supernova

$M > 8 M_{\text{sun}}$

- collapse of Fe core  $\rightarrow$  neutron star / BH  $\rightarrow$  shockwave ejects outer layers (SN explosion)

- major producer of  $\text{O}$ ,  $\text{Mg}$  ( $\alpha$ -elements)

- elements heavier than Fe synthesised in the SN explosion

-  $t \sim 10^7 \text{ yr}$  [massive stars]

## ④ Binary stars: Type Ia supernova

- accretion of a companion star to a WD pushes WD over Chandrasekhar limit [could be also a merger of WD]

- WD made of  $\text{C+O}$  undergoes nuclear fusion all the way to Fe

$\Rightarrow$  all ejected into ISM

-  $t \sim 10^3 \text{ yr}$

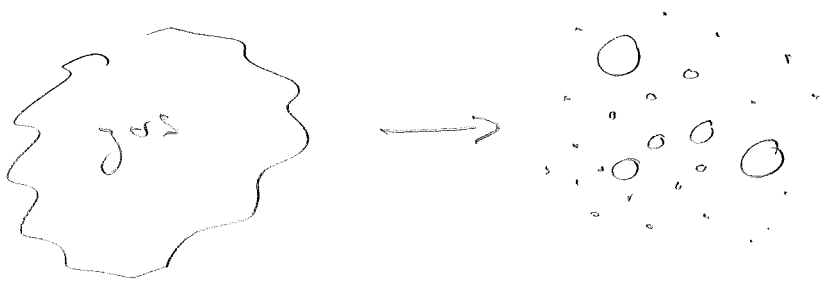
- producer of Fe

$\Rightarrow$  different types of SN give different types of metals!

SN II  $\rightarrow$   $\text{O}$ ,  $\text{Ne}$ ,  $\text{Mg}$   $\rightarrow$   $\alpha$ -elements [nuclei multiples of 2 protons, He nuc.]

SN Ia  $\rightarrow$  Fe [come back to this in SF]

# Initial mass function - IMF



- how does gas get distributed among stars
- what are the sizes of new born stars

- not yet predicted from first principles
- need to understand: fragmentation of gas cloud, accretion to pre-stellar cores, influence of turbulence, self gravity, magnetic fields, feedback from radiation...
- but we see the outcome
  - stars in open clusters have various sizes and masses

Initial mass function - initial mass distribution at the time of the birth of stars

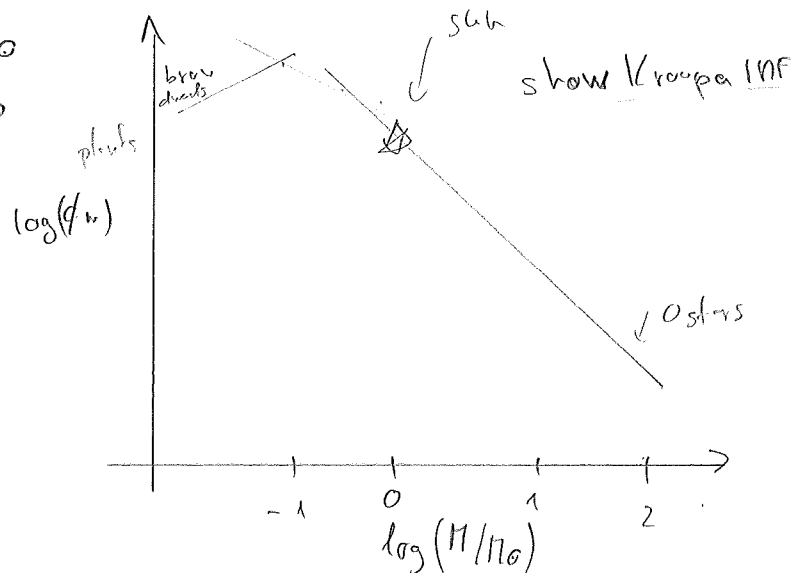
$\phi(m) dm$  is the number of stars born with masses between  $m$  and  $m+dm$

$$\phi(m) = \frac{dN}{dm} \sim m^{-(1+\alpha)}$$

$$\int_{m_L}^{m_U} dm m \phi(m) = 1 M_{\odot} \quad \text{normalization of IMF (arbitrary)}$$

$m_L$  } Lower & upper mass cutoffs are not well defined.

Typically  $m_L = 0.1 M_{\odot}$   
 $m_U = 100 M_{\odot}$



- constrained from detailed observations of star-forming regions in MW



IMF continuous

$$\phi(m) \equiv \frac{dN}{dm} = A m^{-(1+\alpha)}$$

is a good representation of the observed in the solar neighbourhood

there are several variants on the IMF

a) Salpeter - single power-law

$\alpha = 1.35$  over the entire mass range

b) Salpeter-dict (bi-modal distribution)

$\alpha = 1.35$   $m > 0.5 M_{\odot}$

$\alpha = 0.3$   $m < 0.5 M_{\odot}$

c) Kroupa

$\alpha = -0.7$   $m < 0.08$

$\alpha = 0.3$   $0.08 < m < 0.5$

$\alpha = 1.3$   $m > 0.5$

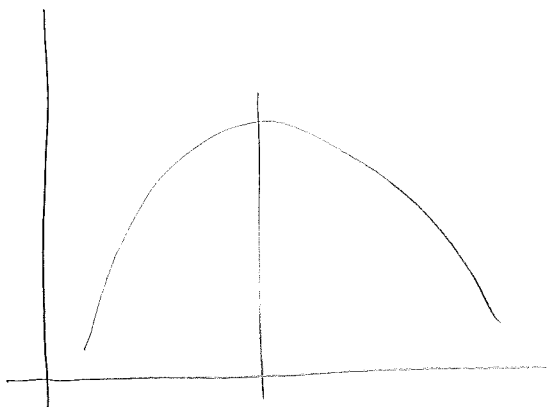
d) Chabrier

$$d(\log m) \sim e^{-\frac{[\log m - \log 3.5]^2}{2 \cdot 0.2^2}}$$

$m < 4.0 M_{\odot}$

$$d(\log m) \sim m^{-1.7}$$

$m \geq 4.0 M_{\odot}$



$$\chi(m) = \frac{dN}{dm} \sim m^{-\alpha} \left[ 1 - e^{-\left(\frac{m}{m_p}\right)^{\beta}} \right]$$

number of stars with mass  $m$ ,  $m \in [m, m+dm]$

$$dN = N_0 \phi(m) dm$$

-  $\phi$  normalized in sense of the total mass, not total number

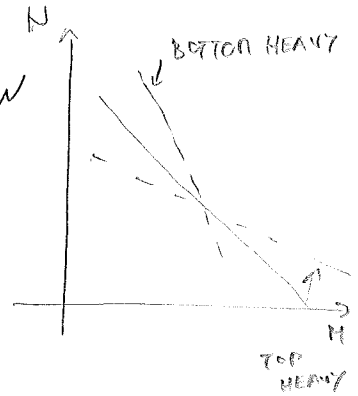
$$\int dm m \phi(m) = 1 M_{\odot}$$

-  $N_0$  is the number of solar masses contained in the SB

IMF continued

- is it universal  $\rightarrow$  the same <sup>SLOPE!</sup> in all star-forming regions in all galaxies!?
- $\rightarrow$  it could be
- $\rightarrow$  there is no reason to be universal
  - $\Rightarrow$  at high redshift SF conditions could be different (amount of gas, metallicity, turbulence)

- it is very challenging to observe IMF beyond MW
  - $\rightarrow$  new facilities E-ELT very important
  - $\rightarrow$  are there other ways to explore IMF

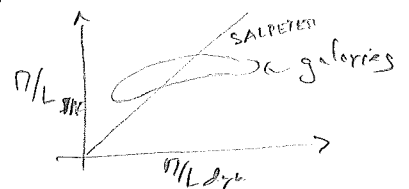


① Spectral features

- we will discuss spectra later, now just to say:
  - slope of IMF depends on how many massive (giants) and how many low-mass (dwarfs) stars there are
  - dwarfs & giants differ in SURFACE GRAVITY
    - $\Rightarrow$  they will have different spectra
    - $\Rightarrow$  look for spectral features that are sensitive to surface gravity (to coming from giants / dwarfs)
    - $\Rightarrow$  TiO, CaH, (FeH 936)
    - $\Rightarrow$  (NaI 8130), CaI, CaII
- prominent in dwarfs  
[SHOW SLIDE AFTER PAGE 16]

② determine total & stellar mass

- total mass: from kinematics + dynamics (stars, DM, gas...)
- stellar mass: from stellar populations (more about this in lecture 2)
- $\Rightarrow$  look what is the  $M/L$  predicted from dynamical models
- compare it to  $M/L$  from stellar populations given a certain IMF



## Star formation rate & Star formation history

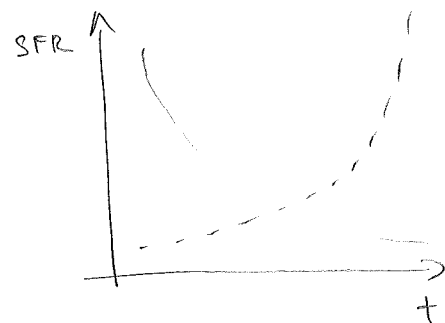
SFR  $\equiv$  gas mass converted to stars per unit time

$$\psi(t) = - \frac{dM_g}{dt}$$

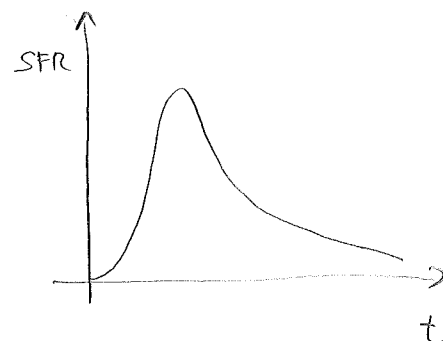
- SFH:
- how is gas converted to stars in a system as a function of time (a history of star formation)
  - describes the evolution of SF in a system (GC, galaxy)
  - traces variation of metals vs. time and interactions, accretions and other evolutionary processes.
  - number of stars (mass of stars) formed as a function of time

many possible SFH

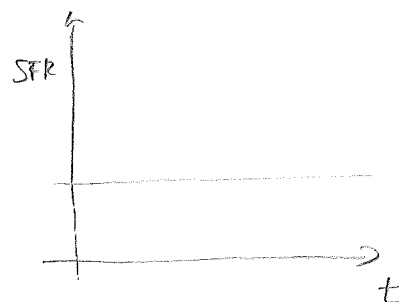
①  $SFR \sim e^{-\frac{t}{\tau}}$  decrease with time  
 $\sim e^{+\frac{t}{\tau}}$  increase with time



②  $SFR \sim \frac{t}{\tau^2} e^{-\frac{t^2}{2\tau^2}}$  galaxy that formed its stars long time ago

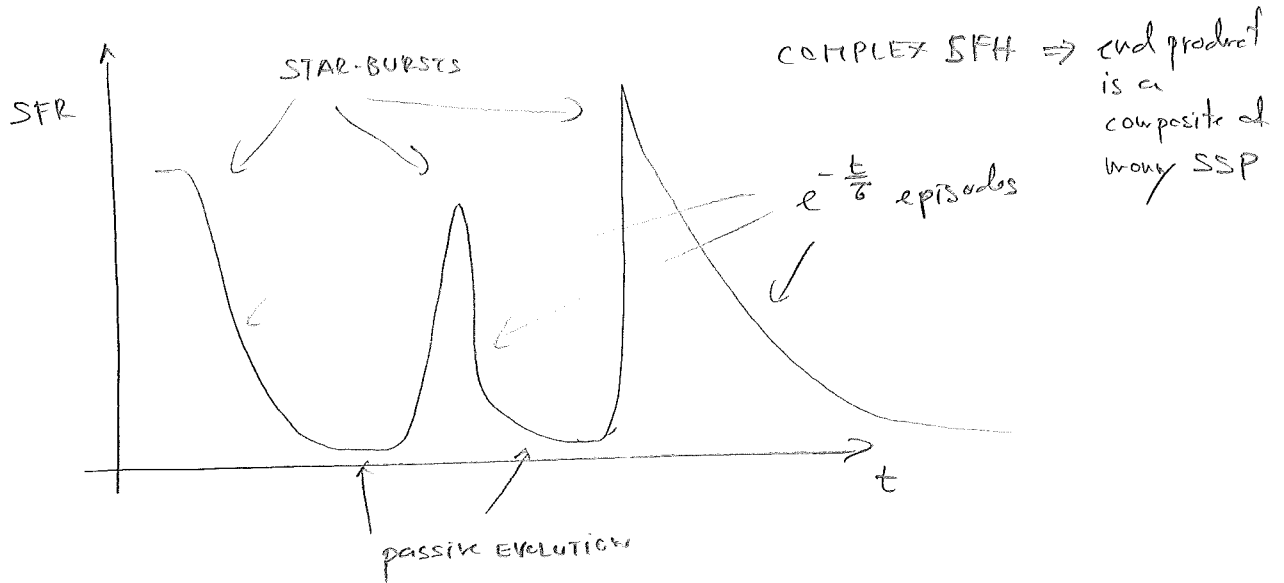


③  $SFR \sim \text{constant}$  galaxy that is forming stars now



SFR continued

- it is very unlikely that these simple mathematical functions are true representatives of SFH in galaxies } but are decent approximations
- SFH is likely characterised by EPISODES of SF which could be described as burst followed by exp. decay and passive evolution



Star formation and abundance of elements

1) progenitors of  $SN Ia$  &  $SN II$  have different life-times.  
 ( $10^9$  yr)      ( $10^7$  yr)

2)  $SN Ia$  &  $SN II$  synthesise different elements

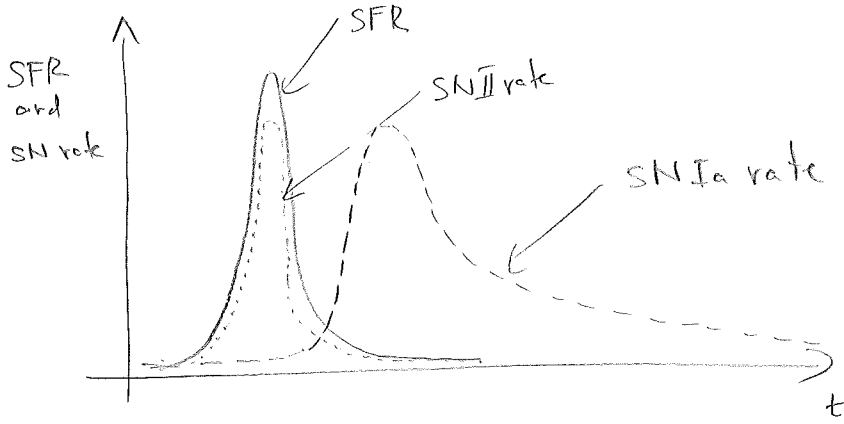
$\downarrow$  Fe       $\downarrow$  O, Mg  $\Rightarrow$   $\alpha$  elements (multiple of He nuclei =  $\alpha$  particles)  
 $\downarrow$  50% of all elements produced  
 $\hookrightarrow$   $\sim 60\%$  O,  $20\%$  Ne,  $10\%$  Mg  $\Rightarrow$  details depend on progenitor (mass)

$\Rightarrow$  timing of various SN will change the chemical composition of the system [chemical evolution]

SF & abundances continued I

Two types of SFHs:

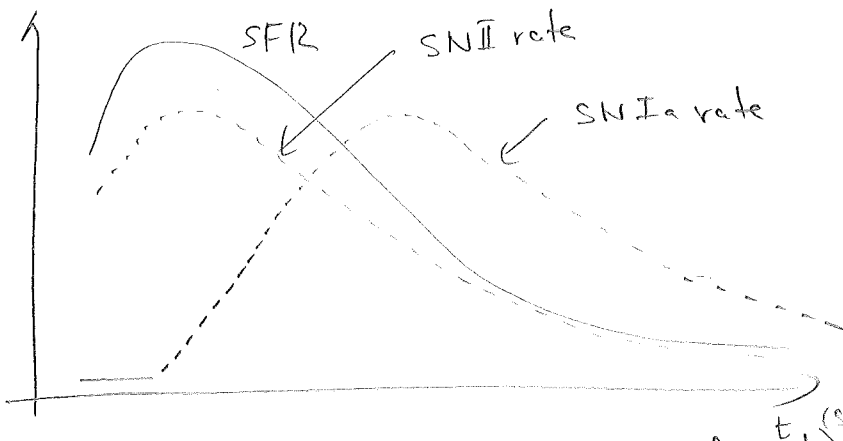
(I) short burst



- SNII rate follows SFR (as the progenitors live short)
- SNIa follow after a delay of  $\sim 10^{8-9}$  yr (broad distribution of precursor lifetimes)

- SNII make  $\alpha$ -elements
- as the burst is short, and SNIa products (Fe) is released after the stars are made, only  $\alpha$ -elements are incorporated into stars

(II) long burst



- SNII rate follows SFR
- SNIa again offset

- as the SB is longer, both  $\alpha$ -elements and Fe (from SNIa) can get absorbed in new stars

$\Rightarrow [\alpha/Fe]$  or  $[Mg/Fe] \rightarrow$  element abundances  
orig of element  
 $\rightarrow$  trace duration & type of SF (SFH)

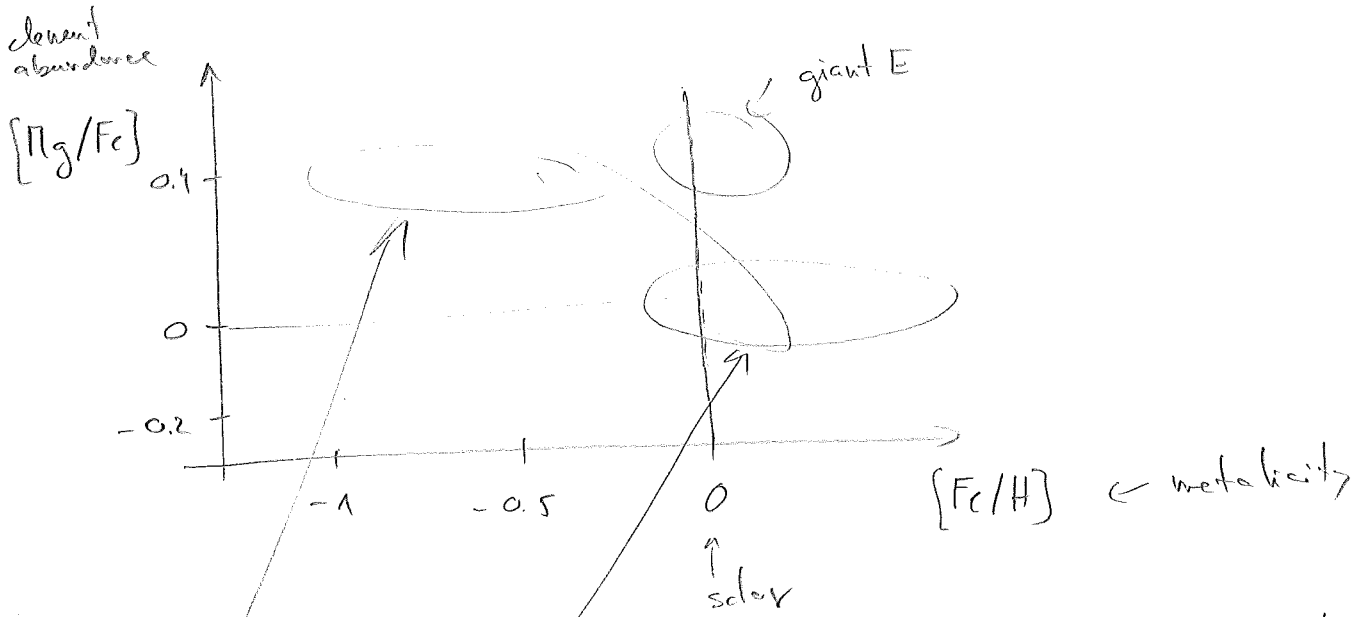
case I: high  $[Mg/Fe]$   
 case II: low  $[Mg/Fe]$

$\Rightarrow$  show MW

[skip if no time]

# SF & abundances continued II

(show slide)

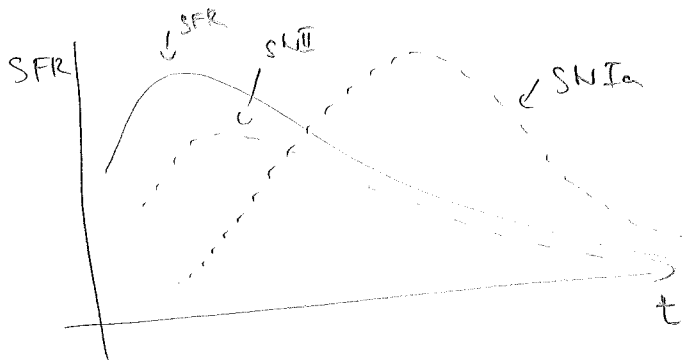


- earlier stars (low metallicity, high  $\alpha$  fraction): form from SNII material

- later stars "polluted" by SNIa Fe ejecta (higher metallicity, lower  $\alpha$  fraction)

$\Rightarrow$  MW SFH is in the "long burst" mode

- metallicity is rising



## Metal poor stars: Population III (a digression)

- first stars that were born: no metals  $\rightarrow$  Pop III

(very unusual properties  
- very massive (100 $M_{\odot}$ )  
- possible progenitors of SAGB)

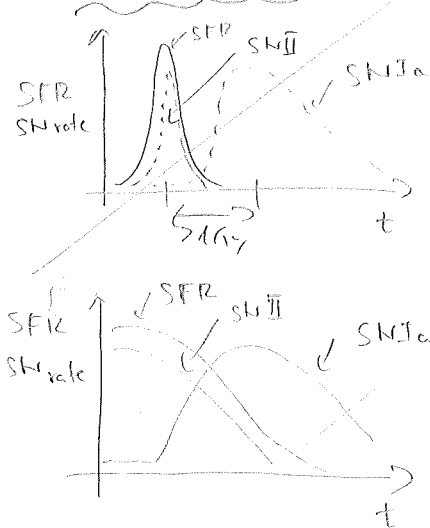
- most metal poor star:  $[Fe/H] \approx -5$   
 $\hookrightarrow$  already polluted

(show slide on stellar metallicities)

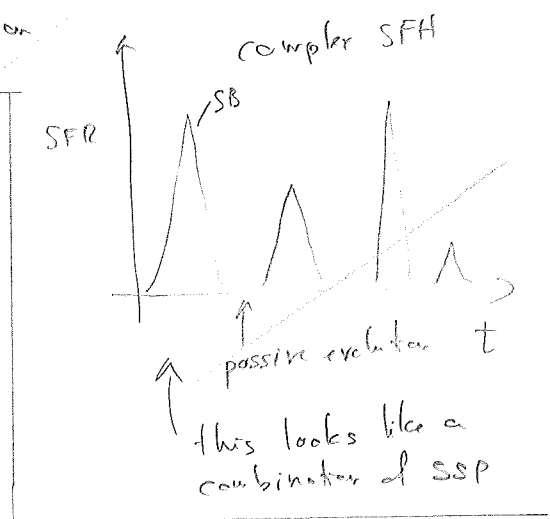
SFR continued

- it is very unlikely that these simple mathematical functions are true representation of SFH in galaxies
- but they seem to be decent approximation

SF & abundance of  $\alpha$ -elements



- short burst of SF
  - SNII make  $\alpha$  elements that get incorporated in stars  $\Rightarrow$  high  $[\alpha/Fe]$  ratios
- Longer burst
  - $\rightarrow$  SNIa happen while there is still star formation going on  $\Rightarrow$  Fe (from SNIa) can get incorporated in stars  $\Rightarrow$  lower  $[\alpha/Fe]$  ratios



Spectral energy distribution - spectra of stars

Spectrum - distribution of electro-magnetic radiation emitted or absorbed by a particular object

$S_\lambda$  - spectrum of a star

$S_\lambda = S_\lambda(X_i, g, T)$  ; a function of

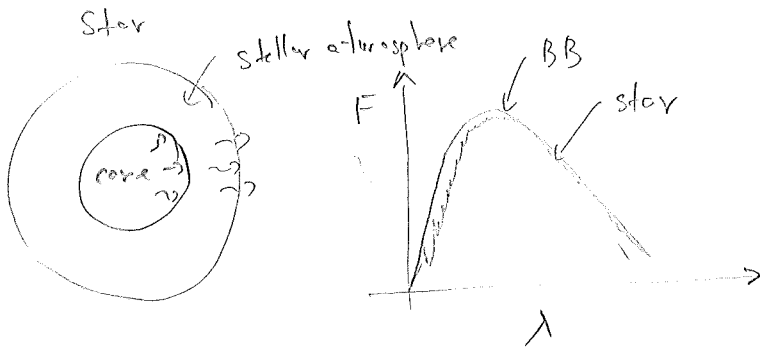
- chemical composition
- surface gravity
- effective temperature

often written as  $Z$  if given by one number

$\Rightarrow$  show spectra of stars (Mikis library) ~~Sánchez-Blázquez + 2006~~ ~~Falcon-Boraso + 2011~~ ~~Yau + 2003~~

SED continued

- ① effective temperature  $T_{eff}$  - dominant influence on stellar spectra
- spectral classes: O B A F G K M
  - continuum shape  $\rightarrow$  black body
  - modified by atomic/molecular processes
- LL) show star-BB comparison (Kortat 1996)



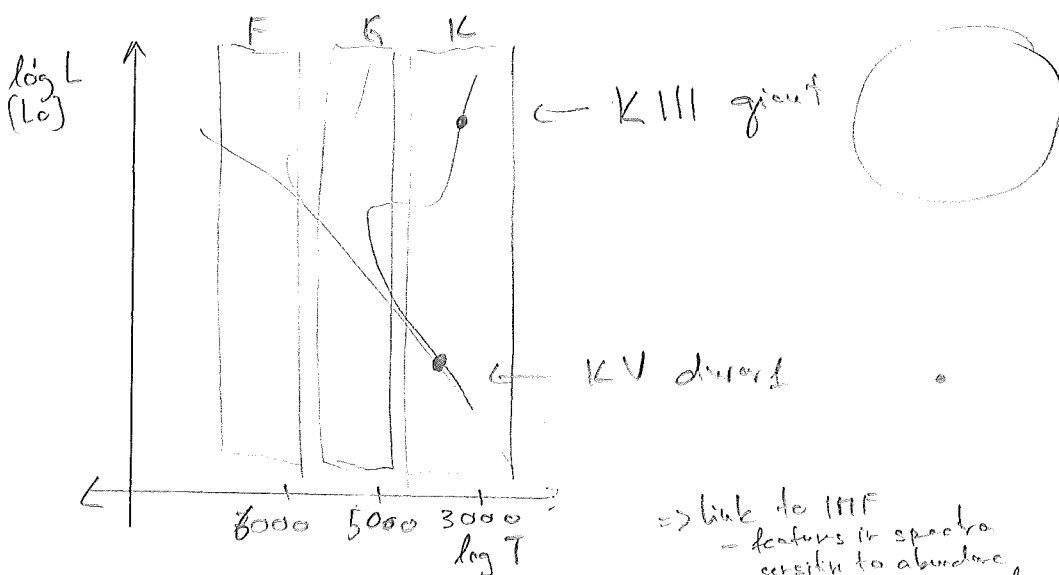
② metallicity effects

- increasing metallicity - changes opacities in stellar interiors
- increases absorption in atomic/molecular transitions in stellar atmosphere

$\Rightarrow$  show example of F star Young (2003)

③ surface gravity

- a measure of stellar size: dwarfs vs giants



$\hookrightarrow$  luminosity classes I - V  
 giants      dwarfs

$\rightarrow$  difference in size = difference in surface gravity

show slide Courvoisier & Doherty (also Royner + (1976 spectra))



SED continued

Stellar libraries

need: good signal-to-noise, flux calibrated, covering the full atmospheric window, covering the full range of  $(X_i, g, T)$

1) empirical libraries

- based on spectroscopic observations of nearby stars
  - > solar neighbourhood
- spread of ages,  $[Fe/H]$ ,  $[alpha/Fe]$ 
  - > typically in limited spectral range
  - 3500 - 7000 Å
- problems
  - limited spectral range: e.g. 3500 - 7500
    - > no continuous UV-IR spectrum
    - > also important for high z galaxies with redshifted spectrum
  - insufficient coverage at high-metallicity (e.g. only young stars, no old metal rich stars typical for Es)
  - only solar  $[alpha/Fe]$

2) synthetic (model) libraries

- only  $(X_i, g, T)$  parameter can be produced
- not all stages of stellar evolution are well understood
  - > chemical composition of cool stars ... ->
- still limited to produce spectra of all stars

tens of millions of abs. lines; many not measured in lab, but only predicted

what is better

- a matter of taste
- NOTE: - empirical libraries have to be fitted with synthetic spectra to derive  $(X_i, g, T_{obs})$ !
- synthetic libraries don't have contributions of all  $X_i$  well taken into account

Stellar population synthesisCASE FOR CONSTANT METALlicity  $\rightarrow$  SSP II

$F_\lambda(t)$  - spectral energy distribution, age  $t$ , of a stellar population with a SFR, a chemical enrichment rate [a spectrum representative for some stellar population]

$$F_\lambda(t) = \int_0^t \psi(t') \cdot S_\lambda(x_i, t-t') dt'$$

- ①  $S_\lambda(x_i, t)$  - spectrum of a SSP, of chemical composition  $x_i$ , born at  $t$
- ②  $\psi(t)$  - SFR, describing the SFH which defines the final stellar population

① Synthesis of SSP spectra

Ingredients:

- a) isochrones (stellar evolution theory)
- b) IMF (star formation observation - i.e. theory does not exist)
- c) spectral library (empirical or theoretical from stellar evolution theory)

How is it done:

- i) assign a spectrum  $S_\lambda$  to each point on the isochrone
- isochrone gives dependence on  $T_{\text{eff}}$ ,  $g$ , and metallicity as a function of stellar mass
  - $S_\lambda(T_{\text{eff}}(\pi), g(\pi) | t, Z)$
  - $\hookrightarrow$  this is what I called before  $x_i$

SSP synthesis eq.

ii) distribution of mass is given by assumed IMF  
 $\Rightarrow$  sum spectra of stars weighted by IMF,  $\phi(M)$ ,  
 along the isochrone

spectrum of a SSP

$$f_{SSP} = \int_{m_{LO}}^{m_{up}(t)} S_{\lambda} (T_{eff}(M), \log g(M) | t, z) \phi(M) dM$$

$M$  - initial mass on the MS

$\phi$  - IMF

$S_{\lambda}$  - spectrum which depends on age and metallicity

$m_{LO} \sim 0.1$  (depends on IMF)

$m_{up} \sim$  depends on stellar evolution

- relation between  $T_{eff}$ ,  $\log g$ ,  $M$ , for a given age & metallicity is defined by an isochrone  $\rightarrow$  integrate along a isochrone

from SSP to Composite Stellar Pop (CSP)

CSP has (and SSP doesn't):

- stars of a range of ages given by their SFH
- stars of a range of metallicities given by chemical enrichment function
- dust

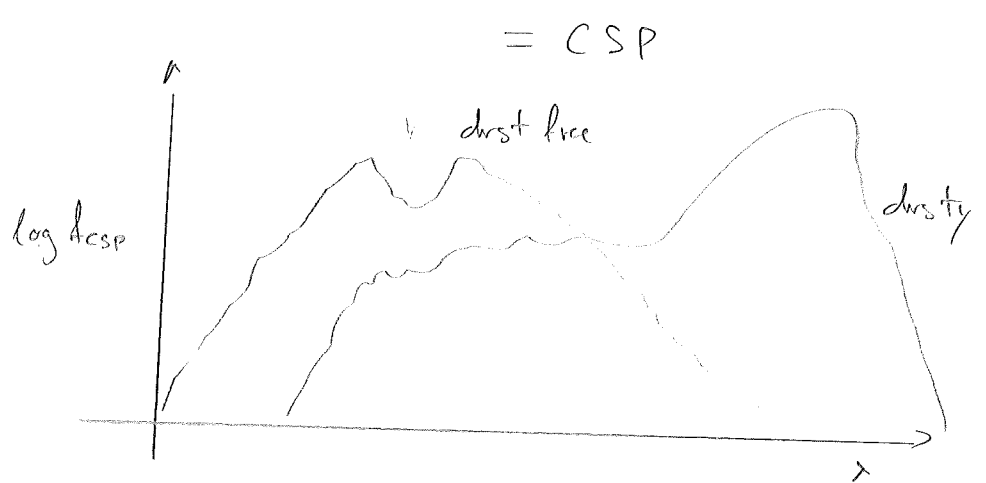
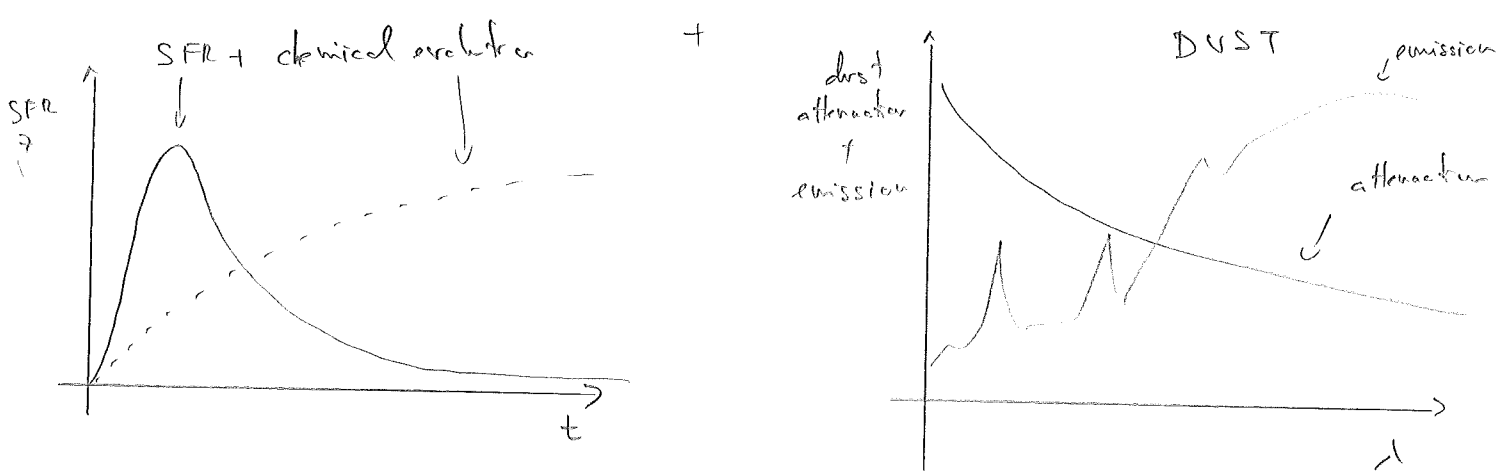
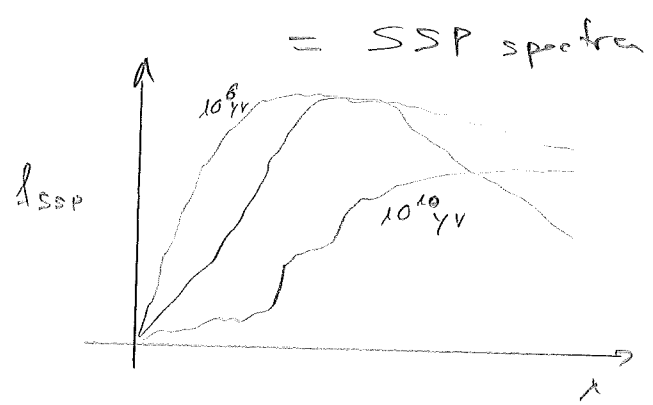
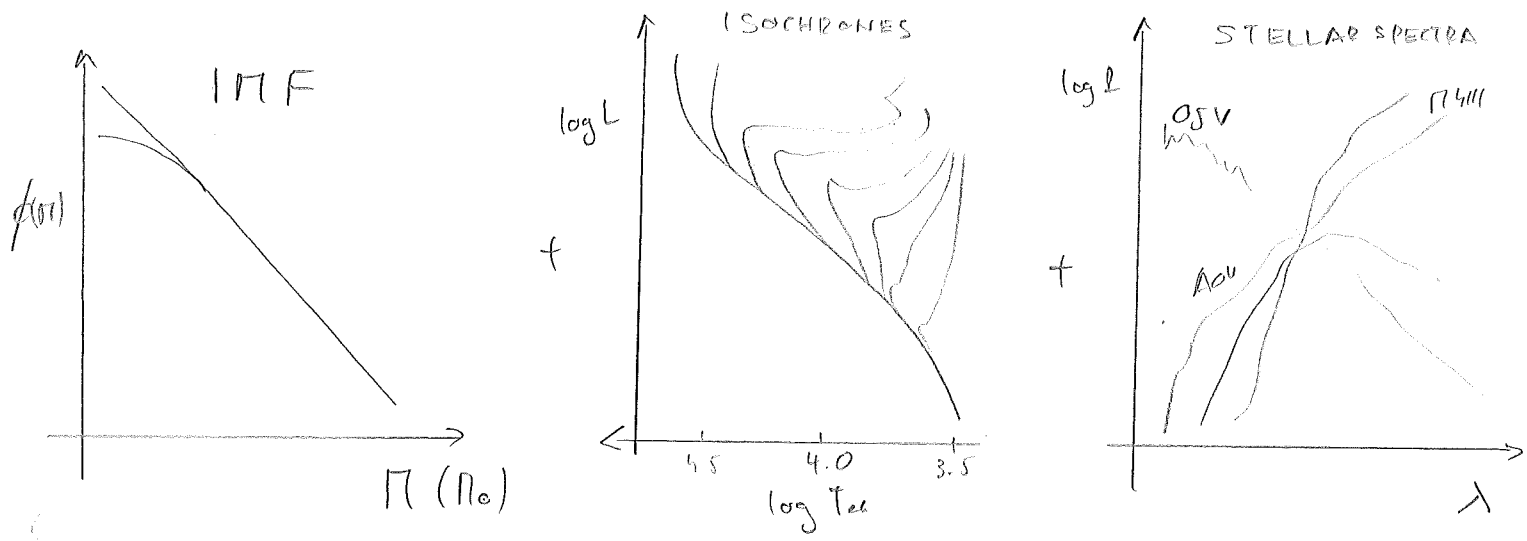
spectrum of a CSP

$$f_{CSP} = \int_{t'=0}^{t=t} \int_{z=0}^{z_{max}} \left[ \Psi(t-t') \cdot \mathcal{Y}(z, t-t') \cdot f_{SSP}(t', z) e^{-\tau_{opt}(t')} + A_{dust}(t', z) \right] dt' dz$$

$\Psi(t-t')$  (SFH)  
 $\mathcal{Y}(z, t-t')$  (chemical enrichment)  
 $f_{SSP}(t', z)$  (SSP spectrum)  
 $e^{-\tau_{opt}(t')}$  (absorption/attenuation by dust)  
 $A_{dust}(t', z)$  (dust emission)  
 $dt' dz$

Stellar pop. synthesis

What is all needed

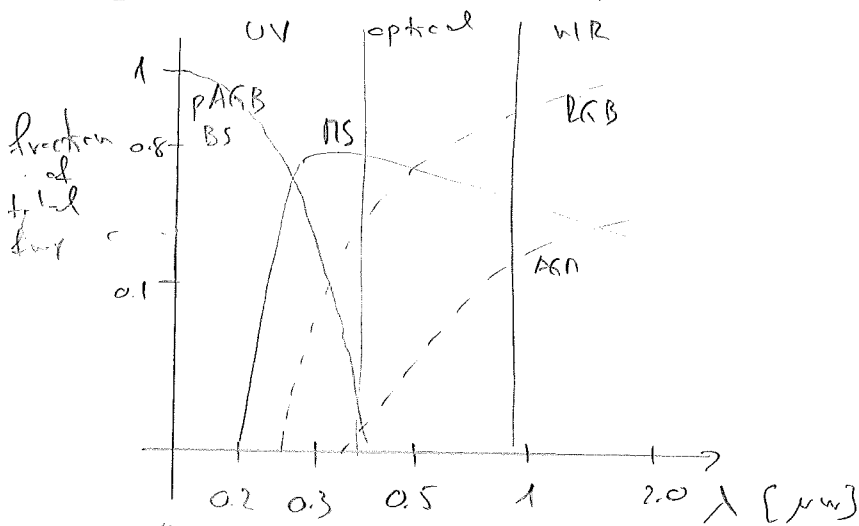


To worry about

- 1) are isochrones correct for stellar phases that contribute a lot
- 2) do we have spectra for all relevant phases & parameters ( $T_{\text{eff}}$ ,  $\log g$ ,  $M$ ,  $Z$ , etc) + that will be in galaxies we wish to investigate
- 3) can we attach the right spectra to right point on the isochrones (do we know the stellar parameters)
- 4) can we "interpolate" reasonably, or are there significant technical problems when doing the isochrone integration

Which stars contribute most to spectra

- stellar luminosity depends on age (time after star burst)
- contributions also depend on the wavelength

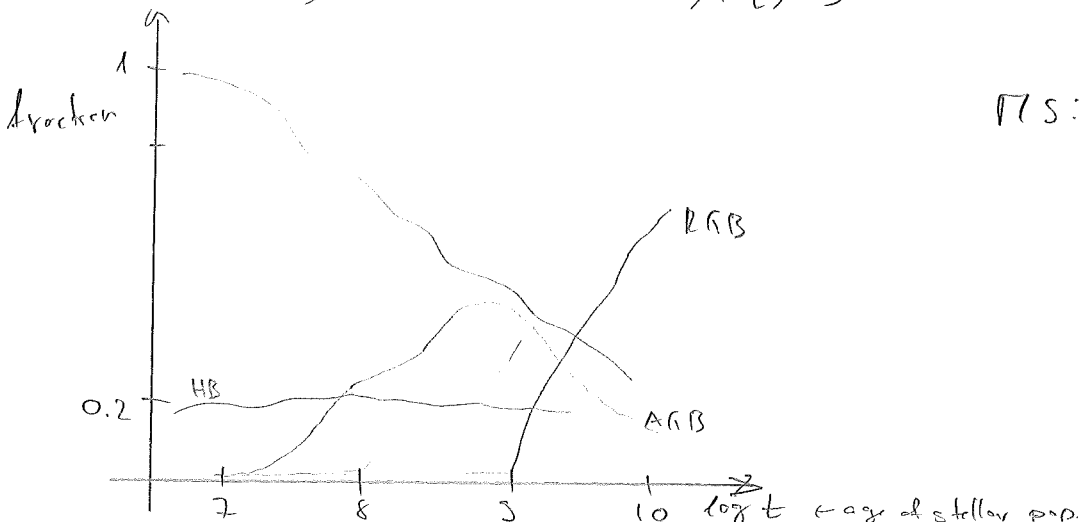


optical: - MS & RGB  
 ↓ star                      ↓ red star  
 - AGB → almost present

IR: RGB & AGB & MS

UV: post AGB  
 ↓ Horizontal Branch

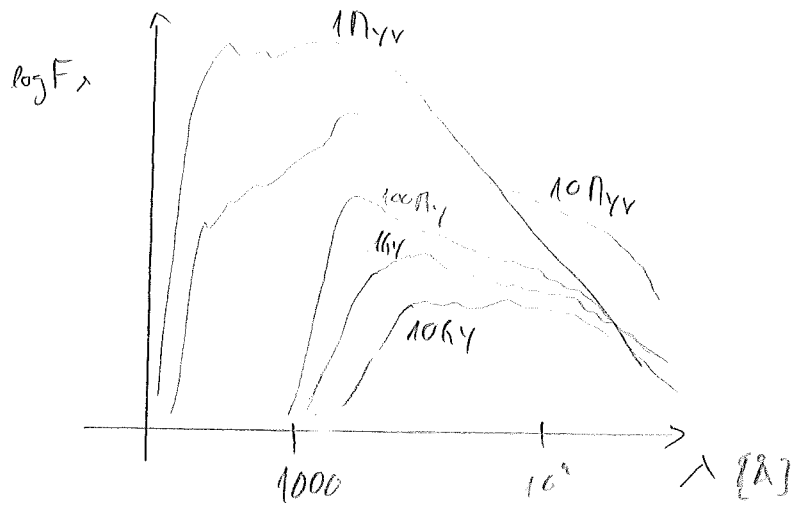
MS: important for young populations



(show slides)

## Spectral evolution of a SSP

- instantaneous burst stellar pop. at solar metallicity



- aging of an SSP

=> show fig 5.1 from Greggio & Rizzi 2011

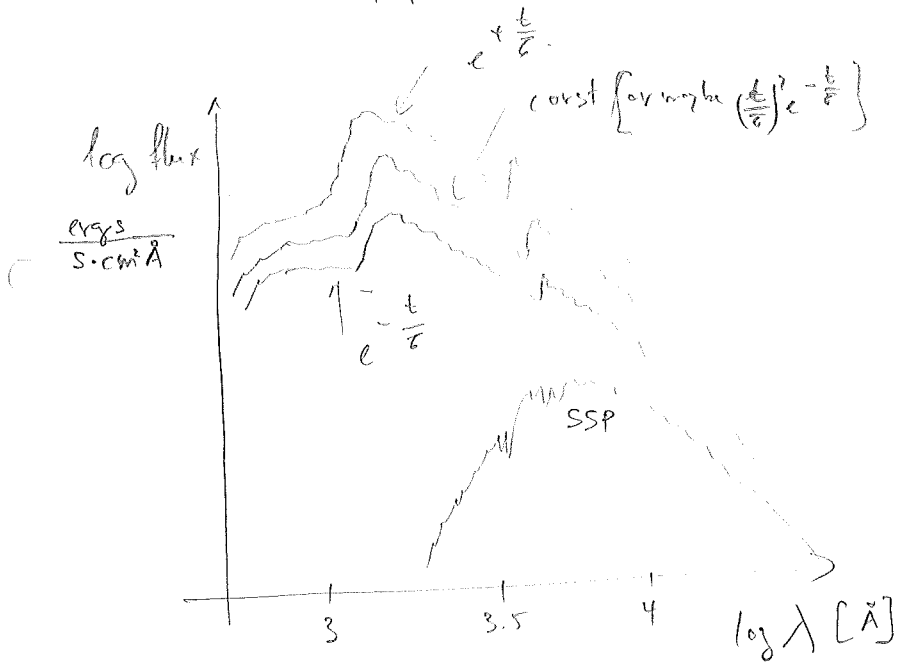
- a) 1 Myr: all massive stars are still on MS; featureless SED, Ly break
- b) 10 Myr: most massive stars have already died, first Red Supergiants (R) appear
- c) 100 Myr: development of the Balmer break + Balmer lines  
-> top of main sequence is made of A stars
- d) 300 Myr: fading of IR light (peak at 10 Myr by red supergiants) is stopped, due to the appearance of TP-AGB stars
- e) 1 Gyr: - TP-AGB stars at strongest contribution, molecular bands in UV a possible trace of R stars due to post-AGB phase, maybe HB
- f) 10 Gyr: RGB contributes the most to NIR, gradual fading

Evolution of spectra of different SFH

CASES: SSP: 1 Gyr old

CSP: made of the same SSP but with different SF laws

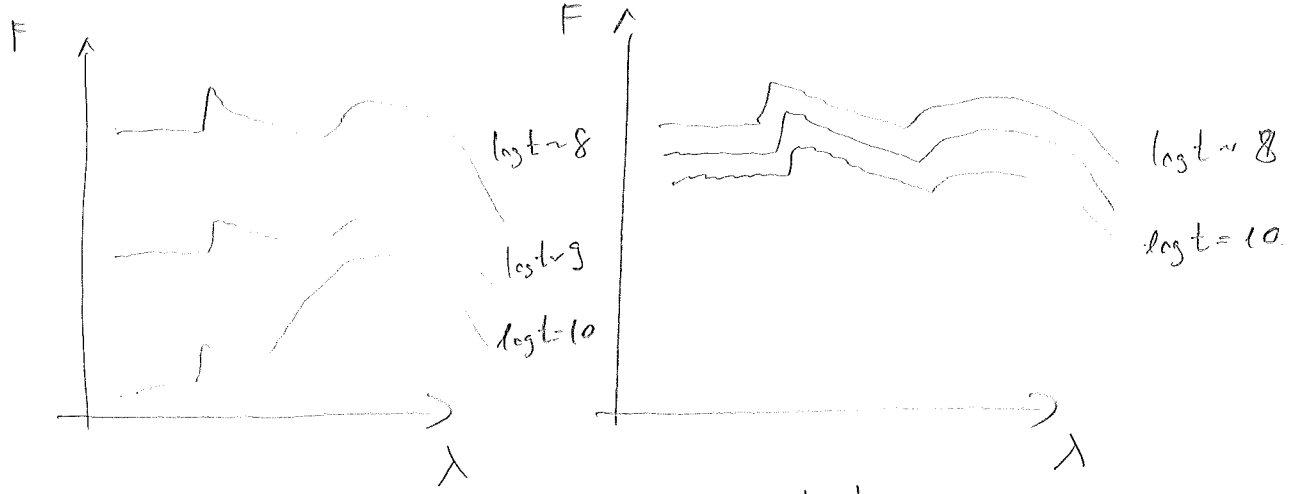
=> SED of a population that is forming stars for 1 Gyr given a SF law  
 => very similar as long as  $\tau \geq 0.5$  Gyr (not very short SF)



show by 5.4 from  
 Greggio & Renzini 2011

- as a comparison, a SSP of 1 Gyr is very different  
 => all stars were created instantaneously, 1 Gyr ago, and are possibly evolving since  
 (why upper SED are from populations that still make stars)

1. assume  $SFR \sim e^{-t/\tau}$        $\tau = 1 \text{ Gyr}$  ;  $\tau = 4 \text{ Gyr}$   
 => how will SED look at different ages



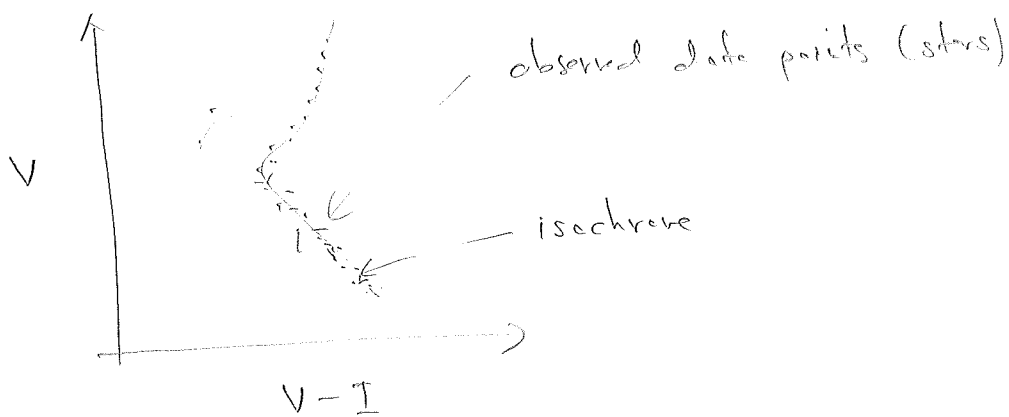
- much faster SED evolution when  $\tau$  is short  
 - these kind of models are never "quenched" - never stop SF (exponential decay)

## Determining ages & metallicities

- spectral synthesis resulted in construction of SSP & CSP (given a SFH)
- how can we use this to determine AGE, METALLICITY (and MASS) in galaxies or stellar systems?

### a) Resolved stellar populations

- observations come from photometry: different filters.
- CMD - colour-magnitude diagram  $\Leftarrow$  observational HR diagram



- make use of isochrones
- $\Rightarrow$  going from Hertzsprung-Russell diagram (theory) to CMD (observers)
- $\Rightarrow$  from  $(T_{\text{eff}}, \log g, L_{\text{bol}}) \rightarrow V, B-V$  or similar
- $\Rightarrow$  in practice, have to convert spectra of stars in a CSP (given SFR & or SSPs) into colours taking into account properties of the instrument (filter)



SED of CSP Stellar populations (25)

flux in a "k"-band

$$F_k = \int d\lambda R_{k,\lambda} \cdot F_\lambda$$

Response function of a filter for "k"-band

SFR

spectrum of SSP

$$F_\lambda = \int_0^t dt' \psi(t') f_{SSP}(\lambda, t-t')$$

colour index  $M_i - M_j$  (or colour centered around wavelengths  $\lambda_i, \lambda_j$ )

$$M_i - M_j = \text{zeropoint} - 2.5 \log \frac{F_i}{F_j}$$

↳ depends on the photometric system

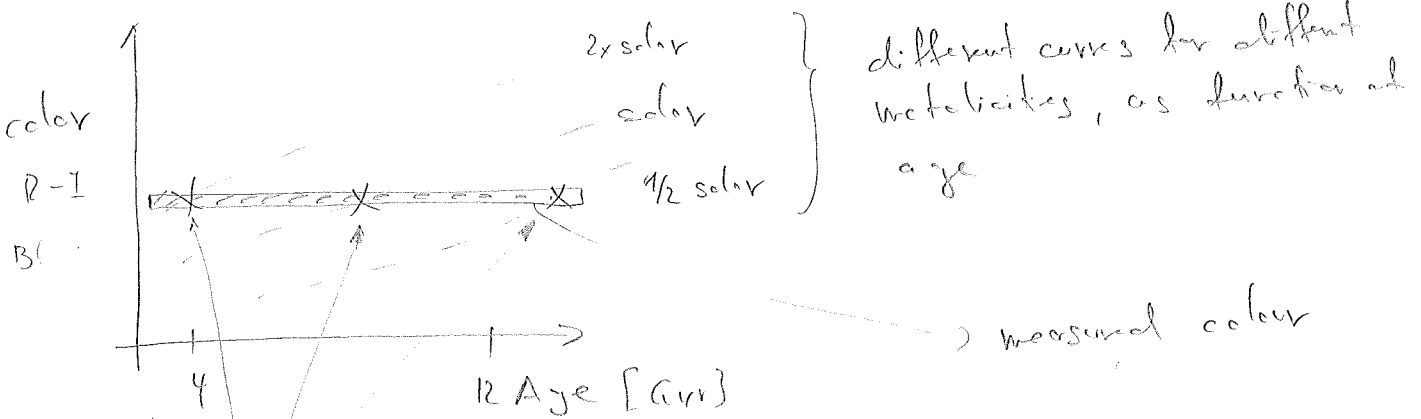
- lets assume we can plot an isochrone in CMD  
 => what is needed to fit it (show slide)

- deep CMD - below MS turn-off
- age - position of the turnoff
- metallicity - position & shape of RGB
- distance to the system: overall magnitude match
- redening of the system (dust): overall colour

b) Unresolved stellar populations

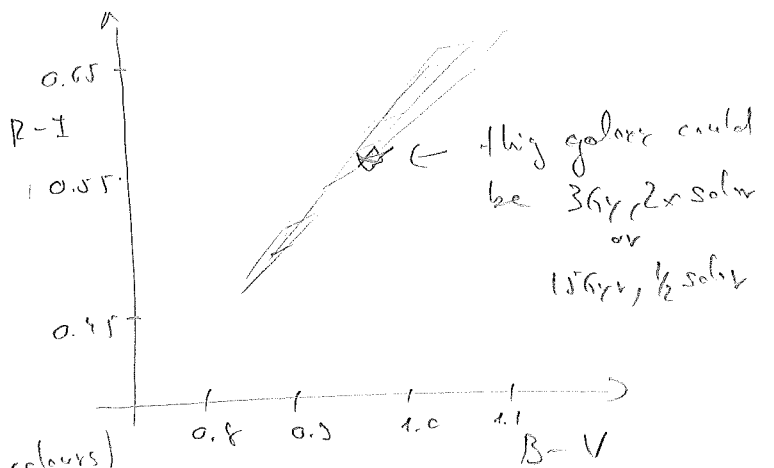
- if we can't resolve individual stars (to make CMD of stellar pop.), we can try to image regions of galaxies (all the full galaxy) and see how these colours are predicted from theory

⇒ construction of model grids



- 1 colour doesn't give sufficient info to determine age and metallicity ⇒ multiple possibilities

- 2 colour are not much better
- a bit better in NIR



⇒ AGE - METALLICITY DEGENERACY

- can't distinguish between (when using colours)

- old - but - metal - poor

- young - but - metal - rich

- both age & metallicity cause a SSP to redden (same effect on overall colour)

- to break/lift the age-z degeneracy one should use spectra (even then there are difficulties, as systems are not SSP)

c) Spectra: Breaking the age- $Z$  degeneracy

- spectra of good signal-to-noise can show differences between age & metallicity, as some absorption lines are sensitive to age, some to metallicity

=> [show slide SSP  $t=10$ ,  $Z=0.4$ ,  $t=2.5$   $Z=0.0$ ]

=> difference in Balmer lines (age), Fe lines (metallicity)

- two approaches

a) use "absorption indices"

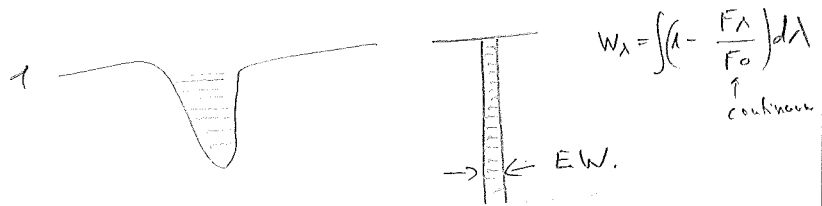
b) fit full spectrum to recover SFH

a) degeneracy breaking power in spectra is localised to some features

=> define "indices" which isolate these features

1) define a pseudo-continuum (real continuum not known)

2) express absorbed flux as an EQUIVALENT WIDTH (EW)



EW is the width of a rectangular, completely absorbed segment of the spectrum with the same integrated absorption (AREA) as the line itself.

3) calibrate by measuring some features on models (SSP) exactly as for the observed spectra (instrumental effects!)

or maybe stars of known  $Z, t$

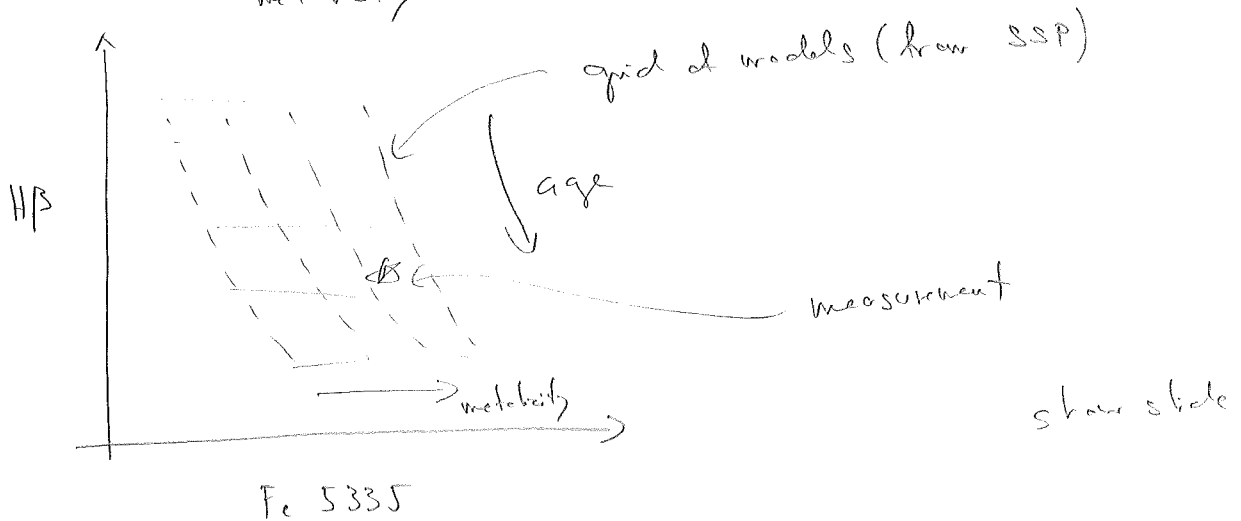
Breaking degeneracy age-Z continued



H $\beta$ , H $\gamma$ , Fe 5335, ...

historically important:  
Lick Observatory  
Stellar Library  
(Worthey + 1994)

measured indices are plotted on a grid of models;  
need to choose right indices that follow mostly only age or metallicity



- how to get model predictions: (two ways)

- 1) create a SSP (sum stellar library along an isochrone) and measure the same indices on this synthetic spectrum
- 2) measure indices on each stellar spectrum in the library, compute luminosity-weighted average index along the isochrone

b) full spectrum fit

- use a set of models to fit the spectrum
- > estimate the average (mass-weighted) age & metallicity from those spectra that were used in the fit (more about it soon: mass estimate + luminosities)

## How to estimate the stellar mass?

- assume: we observe a galaxy with  $L = 10^{10} L_{\text{sun}}$
- what is it?
  - an old giant galaxy with  $M \sim 10^{10} M_{\odot}$
  - a young dwarf that is making stars  $M \sim 10^8 M_{\odot}$   
(show 1st slide with galaxies)
- conversion between  $M$  &  $L \equiv$  mass-to-light ratio  $M/L$
- - each star has a  $M/L$ ; sun  $1 L_{\odot}$  for  $1 M_{\odot}$
- SSP: a  $M/L$  is obtained by summing all stellar masses over all stellar luminosities of stars along an isochrone  
 $M/L$  depends - IMF, age, metallicity
- CSP:  $M/L$  depends on SFH, IMF, age, metallicity

What mass is measured? given by SSP or CSP, and used as a number

- multiple options:
  - baryonic mass that made stars ( $M_i$ ) initial mass
  - actual mass in stars & stellar remnants  $M_i$  - mass return
  - actual mass in living stars

=> for comparison with dynamical models: living stars + remnants but no gas from mass loss

$$M_{\#} = \int_{t_0}^t \Psi(t') dt' - R(t)$$

stellar mass = integral of SFR - mass return

$$R(t) = \int_{t_0}^t dt' \int_{M_{\tau_0}(t'-t_0)}^{M_{\tau_0}} dM \phi(M) [1 - \Pi_{\#}(M)] \Psi[t - t_0 - \delta(M)]$$

mass return  $\phi(M)$  mass loss  $\Psi$  SFR

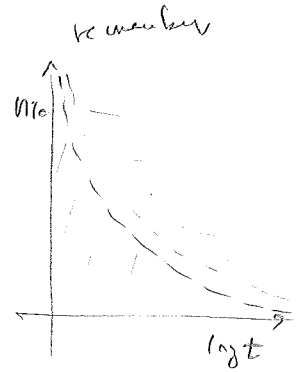
live mass  $\rightarrow$  lifetime of stars of mass  $M$

=> present  $\Pi(t)$ :

$$\Pi(t) = \Pi_{in} \text{ for } \Pi_{in} < \Pi_{\tau_0}(t)$$

$$\Pi(t) = \Pi_{rem} \text{ for } \Pi_{in} > \Pi_{\tau_0}(t)$$

$\uparrow$  remnant mass

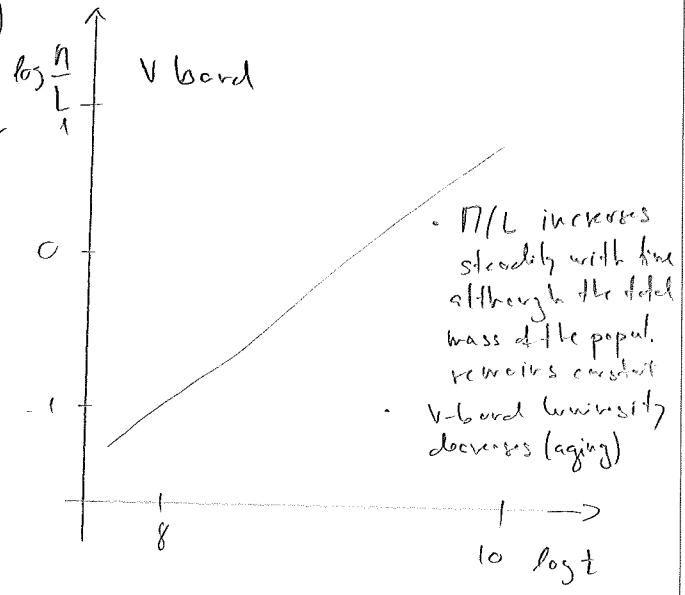
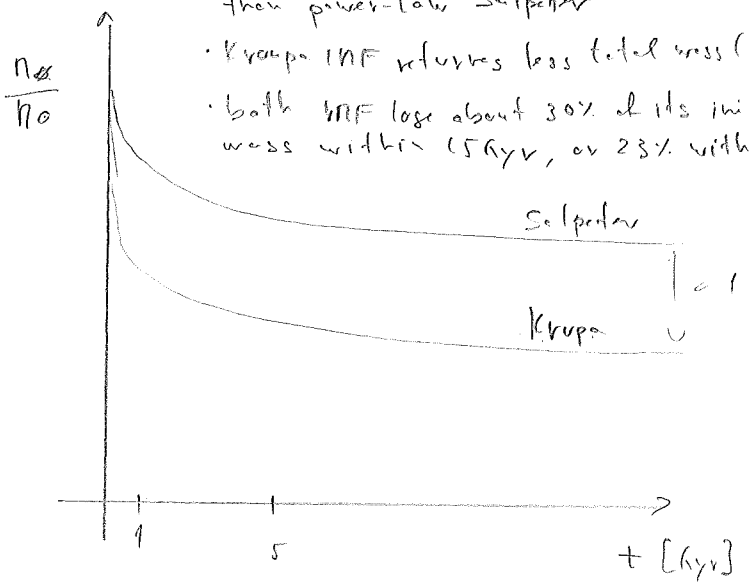


=>  $\Pi_{rem}$ :

- $\Pi_{in} < 8.5 \Pi_0 \Rightarrow$  white dwarf  $\Pi_{rem} = 0.077 \Pi_{in} + 0.48$
- $8.5 \leq \Pi_{in} < 40 \Pi_0 \Rightarrow$  neutron star  $\Pi_{rem} = 1.4 \Pi_0$
- $\Pi_{in} > 40 \Pi_0 \Rightarrow$  BH  $\Pi_{rem} = 0.5 \Pi_{in}$

(over prescription)

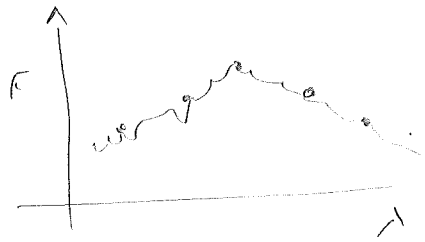
- Kruppa IMF has less low mass stars than power-law Salpeter
- Kruppa IMF returns less total mass (16% less)
- both IMF lose about 30% of its initial mass within 5 Myr, or 23% within 1 Myr



How to derive mass/light?

- ① use colour, say B, V: B-V
  - $\eta/L$  depends on age/SFH, metallicity of a galaxy
  - colour also depends on age & metallicity

- ② have multiple-colour <sup>or</sup> and fit a CSP (with SFH) to it



- ③ fit the full spectrum by a CSP, or look for specific features
  - e.g. 4000Å break and H $\delta$ 
    - ↳ varies with age smoothly
    - ↳ recent SF (100%)

THIS IS REPEATED LATER: SKID IT NOW!

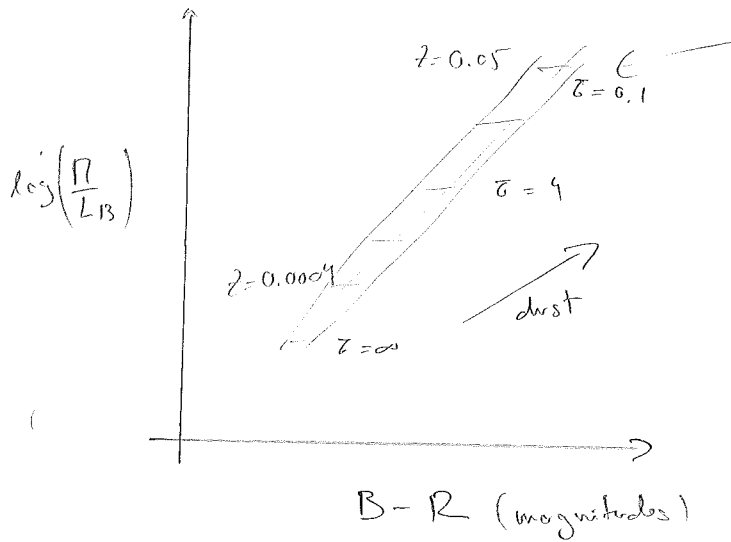
$\Rightarrow$  final  $\eta/L$  is a combination of  $\eta/L$  of individual SSP, but not literally  
 ↳ for a chosen IMF

$\Rightarrow$  luminosity weighting:

$$\langle \frac{\eta}{L} \rangle = \frac{\sum_{i=1}^N w_i \frac{\eta_i}{L_i} \cdot L_i}{\sum_{i=1}^N w_i L_i} = \frac{\sum_i w_i \eta_i}{\sum_i w_i L_i}$$

① Broad band colours - easy stellar masses

- $\Pi/L$  depends on age/SFH, metallicity of galaxy
- colours also depend on age & metallicity



models for a given

- IMF (constant for all models)
- $z$  (varying) [but age-metallicity degeneracy helps here]
- SFR  $z$

$$\log\left(\frac{\Pi}{L_B}\right) = a_B + b_B (B-R)$$

e.g. Bell et al. (2003)

$\Rightarrow$  B-R colour is a good predictor of  $\Pi/L_B$

- $\rightarrow$  dust not a big problem
- $\rightarrow$  it reddens the galaxy, but makes it dimmer
- $\Rightarrow$  approximately preserves colour- $\Pi/L$  relation

$\Rightarrow$  cheap  $\Pi/L$  (only observations in 2 bands)

$\Rightarrow$  uncertainties:

- SFR  $\rightarrow$  works well only for SFR  $\sim z^{-\frac{1}{6}}$
- $\rightarrow$  large errors for other types of SFH

crucial for understanding results

$\Rightarrow$  general expression

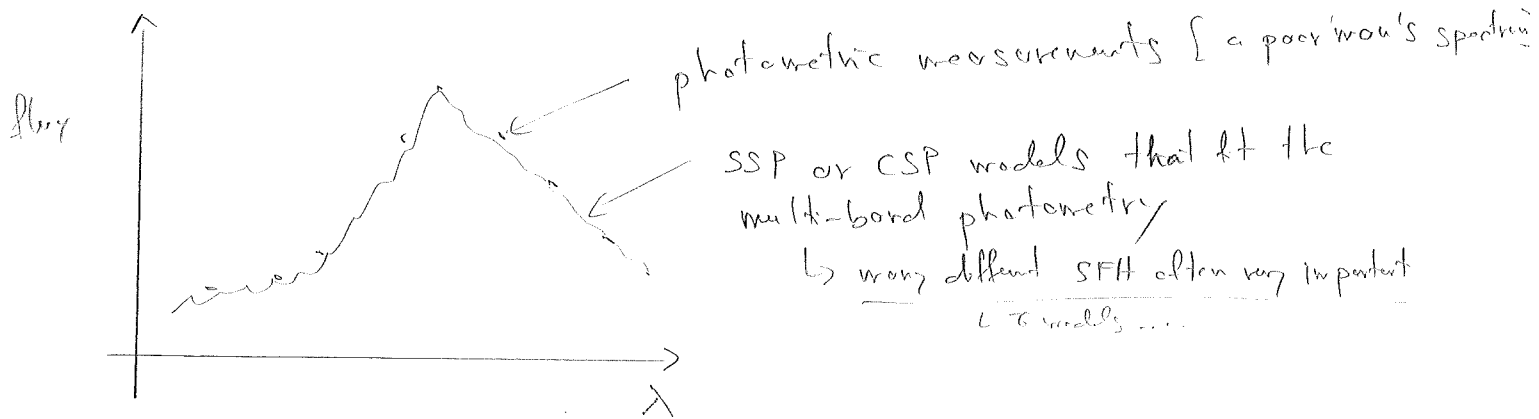
$$\log \frac{\Pi}{L} \Big|_i = a_i + b_i \underbrace{(m_j - m_k)}_{\text{colour (e.g. B-R)}}$$

$i, j, k$ : can all be different (B, V, K), or some can be similar  $i=j$

$\Rightarrow a_i, b_i$  depend on the choice of colour and are obtained from models



## ② Stellar masses from SED fitting



- photometry from many bands: UV/optical/NIR

⇒ method

- find SFH compatible with the observed flux
- $D/L$  from the best fit model [will show later the formula]

⇒ caveat

- we don't know the redshift of the source
- has to be fitted simultaneously or determined by other means (e.g. spectra)

⇒ photo-z

⇒ precision:

- comparable to colour
- better handle on SFH
- a poor-man's spectrum

### ③ Stellar masses from spectroscopy

i) looking for some special features in the spectra

e.g. 4000 Å break and H $\beta$  line index

↓  
varies smoothly with

$L_{\text{cut}}$  SF (1.00  $\Pi_{\text{H}}$ )

$\Rightarrow$  determining the population of stars (e.g. SSP that best represents that) and get its mass

ii) fitting the full spectrum (in a non-parametric way)

- real galaxies don't respect our SFR (smooth) parametrizations

how to do it

- similar to kinematics

- use a library of SSP models

- use a similar code as for kinematics

- find a combination of stars that best reproduce the observations

- need some sort of tuning or smoothing of the results

$\Rightarrow$  show example  
Shetty & Cappellari +16

- result: a set of SSP templates, each with  $L_i$  and  $\Pi_i$

- what is the final  $\Pi/L$

$$\Pi/L_{\text{R}} = \frac{\sum_{j=1}^N w_j \Pi_j}{\sum_{j=1}^N w_j L_j}$$

$\rightarrow$  pre-chosen IMF

$$\Pi/L_{\text{R}} = \frac{\sum_j w_j \frac{\Pi_j}{L_j} \cdot L_j}{\sum_j w_j L_j}$$