Exercises for
Cosmology & Galaxies (Summer Term 2018)

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Exercise sheet 4

Due: June 5, 2018, 16:00

To be emailed as a pdf or scanned hand-written document (via e-mail to kehlert@aip.de). Remember to put your names on the document and number your pages; you can work in groups of ≤ 2 but every student should hand in his/her own solution sheet and indicate clearly who contributed.
A total of 40 points can be achieved when completing this sheet of which 30 points will count as the maximum attainable.

1. **General questions** (5 points) Please provide a brief and concise answer to the following questions (max. 3 sentences per question):

   (a) Describe the concept behind *hierarchical structure formation*.

   (b) List and describe three major problems of Big-Bang cosmology that can be solved with a phase of rapid inflation.

   (c) What do astronomers describe as the *finger of gods* and why do they appear?

   (d) What do astronomers mean when they talk about SDSS and what limitations should be considered when calculating number densities of galaxy populations?

   (e) Why does the luminosity distance increase with redshift whereas the angular diameter decreases after a certain redshift?

2. **Optical Depth due to Reionization** (9 points)

   After recombination the Universe eventually became *reionized*, which allows CMB photons to once again scatter off electrons. The Thomson scattering optical depth from an observer at $z = 0$ out to a redshift of z can be written as

$$\tau(z) = \int_{D_{\mathrm{prop}}(0)}^{D_{\mathrm{prop}}(z)} \sigma_{\mathrm{T}} n_{\mathrm{e}} \mathrm{d}D_{\mathrm{prop}} = \int_0^z \sigma_{\mathrm{T}} n_{\mathrm{e}}(z') \frac{c}{(1+z')H(z')} \,\mathrm{d}z', \qquad (1)$$

   where σ_{T} is the Thomson scattering cross-section and $n_{\mathrm{e}}(z')$ is the physical electron number density at a redshift $z = z'$. One effect of this scattering is to suppress the size of any CMB fluctuations by mixing together photons from cold and warm regions. To a first approximation, the amplitudes of the perturbations are multiplied by a factor $e^{-\tau}$, with τ given by the formula above.

   Assume that the Universe reionizes instantly at a redshift z_{reion} and that the intergalactic medium has a primordial composition (mass fractions of $X = 0.76$ for hydrogen and $1 - X = 0.24$ for helium). Compute $e^{-\tau}$ as a function of z_{reion} using the canonical cosmological model of $(\Omega_{\mathrm{b},0}, \Omega_{\mathrm{m},0}, \Omega_{\Lambda,0}, h) = (0.045, 0.3, 0.7, 0.7)$. You may use a suitable approximation in your final integral. What does the fact that we see *any* CMB fluctuations imply regarding the range of possible values for z_{reion}?

3. **Inflation with a Quadratic Potential** (11 points)

In the slow-roll approximation, the evolution equations governing our inflaton, ϕ, can be written as

$$H^2 = \frac{8\pi G}{3} V(\phi), \qquad 3H\dot{\phi} = -\frac{\mathrm{d}V}{\mathrm{d}\phi}, \tag{2}$$

where $V(\phi)$ is the potential. Consider a model for inflation where

$$V(\phi) = \frac{1}{2} m^2 \phi^2. \tag{3}$$

(a) Solve the evolution equations for this case, and determine the time evolution of the scale factor a with the initial conditions $a = a_\mathrm{i}$ and $\phi = \phi_\mathrm{i}$ at $t = 0$.

(b) For what range of ϕ values is the solution inflationary?

(c) What condition must be obeyed by ϕ_i to ensure that an expansion of at least 10^{30} takes place?

4. **Luminosity function** (9 points)

Consider the luminosity function $\phi(L)\mathrm{d}L$, which describes the number density of galaxies with luminosities in the range $L \pm \mathrm{d}L/2$ in the form of the Schechter function:

$$\phi(L)\mathrm{d}L = \phi^* \left(\frac{L}{L^*}\right)^\alpha \exp(-L/L^*)\mathrm{d}L, \tag{4}$$

where ϕ^* is the normalization factor and L^* corresponds to the luminosity above which the power-law form of $\phi(L)$ drops off exponentially.

(a) Convert the luminosity function $\phi(L)\mathrm{d}L$ to bolometric magnitudes $\phi(M)\mathrm{d}M$ which is the preferred unit system of observers.

(b) Determine the average luminosity density using the Schechter function. Show that the average luminosity is finite for a slope $\alpha > -2$ and diverges for $\alpha < -2$.

(c) Estimate the expected distance between Milky Way-like galaxies with b-band luminosity $L_\mathrm{b} = 1.7 \times 10^{10}\ L_\odot$? Typical values for the parameters of the Schechter function derived from b-band measurements are $\phi_b^* = 1.6 \times 10^{-2} h^{-3}$ Mpc^{-3}, $\alpha_b = -1$ and $L_\mathrm{b}^* = 1.2 \times 10^{10} h^{-2}\ L_\odot$. Assume a cosmology where $h = 0.7$. Compare your results to the distance to the nearby Andromeda galaxy of 780 kpc.

5. **Two-point correlation function** (6 points)

The spatial distribution of galaxies can be expressed by a two-point correlation function $\xi(r)$, where r corresponds to the distance between galaxies. This function can be given in real space as a power law:

$$\xi(r) = \left(\frac{r}{r_0}\right)^{-\gamma},$$

where r_0 is the correlation length and γ sets the slope. However, the real space correlation function is influenced by redshift-space distortions and is also a function of apparent line-of-sight seperation in redshift-space, π. An approach to circumvent these distortions is to consider the *projected* two-point correlation function, w_p, instead. The projected function relates to the real space, as:

$$w_p(r_p) = \int_{-\infty}^{+\infty} \xi(r, \pi) \mathrm{d}\pi,$$

Or alternatively in terms of the true line-of-sight seperation, r_3

$$w_p(r_p) = \int_{-\infty}^{+\infty} \xi\left(\sqrt{r_p^2 + r_3^2}\right) \mathrm{d}r_3,$$

where r_p describes the distance between galaxies in the projected plane (see Fig. 1). Show that if the real-space two-point correlation function is a power law then w_p is also a power law with a slope $-\gamma + 1$.

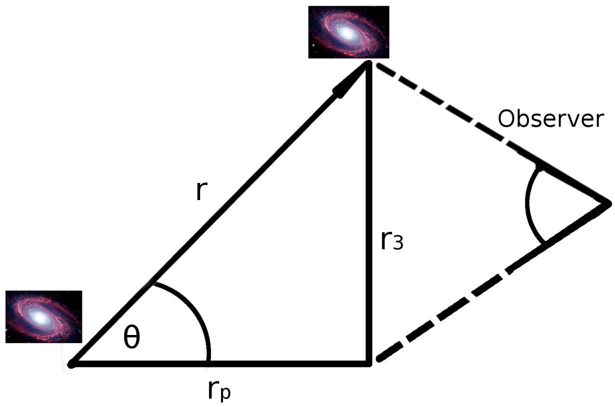

Figure 1: Explanation of the used coordinate system.