Exercises for
Cosmology & Galaxies (Summer Term 2018)

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Exercise sheet 5

Due: June 19, 2018, 16:00

To be emailed as a pdf or scanned hand-written document (via e-mail to kehlert@aip.de). Remember to put your names on the document and number your pages; you can work in groups of ≤ 2 but every student should hand in his/her own solution sheet and indicate clearly who contributed.
A total of 40 points can be achieved when completing this sheet of which 30 points will count as the maximum attainable.

1. **General questions** (10 points) Please provide a brief and concise answer to the following questions (max. 3 sentences per question):

   (a) What is the so called *initial mass function* (IMF) and how can one constrain the IMF observationally?

   (b) List and describe two different characteristics of the two main types of supernova (SN), i.e. SN Ia and SN II in the context of galaxy formation.

   (c) What are four ingredients you should consider for your model when you want to do stellar population synthesis of a composite stellar population?

   (d) What is the main degeneracy faced when attempting stellar population synthesis of a galaxy, in which stars cannot be resolved individually. Explain a way to overcome this degeneracy.

   (e) Name three different observations that will allow you to constrain the mass to light ratio of a galaxy. Comment on their applicability to large galaxy surveys.

2. **SSP for the zero-age main sequence** (10 points)
   Suppose a gas cloudconverts instantly into stars following a Salpeter Initial Mass Function with power-law index -2.35. We ignore very high mass and very low mass stars to avoid divergence of the mass integral and stochastic sampling of the IMF. We now explore the properties of such a simple stellar population (SSP) of age zero.

   (a) Derive an analytical relation for the main sequence, using the well-known scaling relations for main sequence stars relating mass M, radius R, luminosity L and effective temperature T_{eff}, $L \sim M^{3.5}$, $R \sim M^{0.75}$ as well as the total power radiated by a black body $L = 4\pi R^2 \sigma_{\text{SB}} T_{\text{eff}}^4$ where σ_{SB} corresponds to the Stefan-Boltzmann constant. To get there, construct relations $T_{\text{eff}}(M)$ and $L(M)$ where M is the mass of the star, and then combine the two into one relation $L(T_{\text{eff}})$.

   (b) Use (a) to construct "minimal" isochrones for various ages, only to the turnoff point (i.e. ignoring all the red giant evolutionary phases). Plot these isochrones into a Hertzsprung-Russell diagram and indicate the turnoff points for different ages. You may find the following equation useful which approximates the main-sequence lifetime of a star $\tau_{\rm ms}$ as,

$$\tau_{\rm ms} \sim \frac{M}{M_\odot}\frac{L_\odot}{L} 10 \text{ Gyr}. \tag{1}$$

Derive an approximate analytical relation for the effective temperature at the turnoff point as a function of time.

(c) If there are 10000 stars with masses between 0.5 and 8 $M_\odot$ in this SSP, how many stars have masses between 0.5-1 $M_\odot$ (and 1-2, 2-4, 4-8 $M_\odot$)?

3. **Initial Mass Function** (10 points)
Assume the following two parameterizations of the initial mass function with masses in the range 100 $M_\odot > M > 0.01$ $M_\odot$:

   i. Salpeter with single power-law index $\alpha = -2.3$

   ii. Kroupa (broken power-law) with the power-law index α given by
   $\alpha = -2.3$ for $0.5 < M/M_\odot < 100$
   $\alpha = -1.3$ for $0.08 < M/M_\odot < 0.5$
   $\alpha = -0.3$ for $0.01 < M/M_\odot < 0.08$

   (a) For each IMF calculate the fractional number of stars (relative to the total number) between the following mass intervals (expressed in solar masses):
      i. $0.01 < M < 0.08$
      ii. $0.08 < M < 0.5$
      iii. $0.5 < M < 4$
      iv. $M > 4$

   Download an isochrone table from the website: http://stev.oapd.inaf.it/cgi-bin/cmd for a simple stellar population with age 5 Gyrs (age of the sun), solar metallicity and with a Salpeter IMF (can be selected in the web interface). **Choose version PARSEC version 1.2S (instead of the default PARSEC v1.2S + COLIBRI PR16)**. The other parameters can be left at their default values.

   (b) Find the mass below which 50% of the mass fraction of the population resides.
   (c) Repeat the analysis for a Kroupa IMF.
   (d) What is the relative difference in the total mass of these stellar populations assuming a Salpeter or Kroupa IMF.

4. **Evolution of the mass-to-light ratios** (10 points)
As discussed in the lecture, the mass-to-light ratio of a galaxy M/L is essentially the total mass divided by the total luminosity of the galaxy. However, only the luminosity L can be observed directly, while the mass M has to be inferred. One way to estimate the mass of a system is to use the predictions of stellar population models to derive the M_p/L_p ratio of a population of stars p with the same age and metallicity. Through

colour	a_B	b_B	a_V	b_V	a_K	b_K
$B-V$	-0.942	1.737	-0.628	1.305	-0.206	0.135

Table 1: Stellar M/L as a function of colour.

spectral fitting one can determine the relevant populations, which constitute this galaxy. Thus, one knows the mass-to-light ratio of the individual populations M_p/L_p. Finally, one measures the absolute luminosity L of the system and determines the total mass M through calculating the weighted average of the determined populations as $M = \frac{\sum_p^{N_{\rm pop}} w_p M_p}{\sum_p^{N_{\rm pop}} w_p L_p} L$, where $N_{\rm pop}$ is the number of populations and w_p correspond to the weights.

The colour of a galaxy is influenced by the main properties of the main stellar populations making up this galaxy, i.e. age, metallicity,... Consequently, the information provided by the colour is sufficient to constrain M/L of the galaxy in the form:

$$\log(M/L_i) = a_i + b_i \times (m_j - m_k) \tag{2}$$

where i, j, k are different filter bands, m_j and m_k are magnitudes (so that m_j - m_k is a colour). Assume we observe a galaxy in the B, V, and K band. Then we can define:

$$\begin{aligned} \log(M/L_B) &= a_B + b_B \times (B-V) \\ \log(M/L_V) &= a_V + b_V \times (B-V) \\ \log(M/L_K) &= a_K + b_K \times (B-V). \end{aligned} \tag{3}$$

Bell et al. (2003, ApJS, 149, 289), used a large sample of galaxies from SDSS and the Two Micron All Sky Survey (2MASS) with a similar method as described above to determine the fit parameters listed in Table 1.

Here, instead of looking at galaxies, we will apply the equations 3 to a simple stellar populations where all stars share the same age in a given population. You will compare how the estimates of M/L depend on the different color bands and on the ages as well as metallicities of the populations. Table 2. lists the B and V magnitudes of these stellar populations of varying ages and metallicities.

(a) Plot the $B-V$ colour as a function of age for both metallicities.

(b) Plot the mass to light ratio for the V, B and K bands and for the given metallicities as a function of time.

(c) Fit the evolution of M/L as a function of time t to a simple power law (i.e., determine α in $M/L \sim t^{\alpha}$) for the three different bands and the two given metallicities.

(d) Discuss the results. Briefly state the similarities and possible differences in the temporal evolution of M/L for the three different bands. What is the effect of varying the metallicity?

[Fe/H]	age [Gyr]	B	V	[Fe/H]	age [Gyr]	B	V
0.0	1	−9.25	−9.84	+0.3	1	−8.90	−9.63
0.0	1.5	−8.81	−9.57	+0.3	1.5	−8.44	−9.29
0.0	2	−8.53	−9.37	+0.3	2	−8.14	−9.08
0.0	2.5	−8.33	−9.21	+0.3	2.5	−7.92	−8.88
0.0	3	−8.17	−9.08	+0.3	3	−7.74	−8.72
0.0	3.5	−8.01	−8.92	+0.3	3.5	−7.61	−8.61
0.0	4	−7.87	−8.79	+0.3	4.5	−7.49	−8.51
0.0	4.5	−7.75	−8.67	+0.3	5	−7.39	−8.42
0.0	5	−7.64	−8.56	+0.3	5	−7.30	−8.34
0.0	5.5	−7.55	−8.48	+0.3	5.5	−7.21	−8.26
0.0	6	−7.46	−8.41	+0.3	6	−7.12	−8.18
0.0	6.5	−7.39	−8.35	+0.3	6.5	−7.05	−8.11
0.0	7	−7.32	−8.29	+0.3	7	−6.98	−8.05
0.0	7.5	−7.25	−8.23	+0.3	7.5	−6.91	−7.99
0.0	8	−7.19	−8.18	+0.3	8	−6.78	−7.87
0.0	8.5	−7.13	−8.12	+0.3	8.5	−6.78	−7.87
0.0	9	−7.07	−8.07	+0.3	9	−6.72	−7.82
0.0	9.5	−7.02	−8.02	+0.3	9.5	−6.67	−7.77
0.0	10	−6.97	−7.97	+0.3	10	−6.61	−7.72

Table 2: B and V magnitudes for individual stellar populations as a function of age and metallicity.