Exercises for
Cosmology & Galaxies (Summer Term 2018)

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Exercise sheet 6

Due: July 3, 2018, 16:00

To be emailed as a pdf or scanned hand-written document (via e-mail to kehlert@aip.de). Remember to put your names on the document and number your pages; you can work in groups of ≤ 2 but every student should hand in his/her own solution sheet and indicate clearly who contributed.
A total of 40 points can be achieved when completing this sheet of which 30 points will count as the maximum attainable.

1. **General questions** (8 points)

   Please provide a brief and concise answer to the following questions (max. 3 sentences per question):

   (a) To familiarize yourself a bit with galaxy clusters; list typical numbers of member galaxies, masses/sizes of clusters (in gas M_{g}, stars M_{star} and dark matter M_{DM}, separately) and temperatures/densities of the gas in clusters. Compare these values to the size/mass of the Milky Way and the ranges in temperature/density of the interstellar medium.

   (b) State and briefly explain three different ways of discovering galaxy clusters in different regimes of the electromagnetic spectrum, respectively.

   (c) List six important sources of the solar system elements throughout the history of the universe.

   (d) Briefly summarize the stages of the matter cycle, which explains the enrichment of the interstellar medium with heavy elements through stellar evolution. What is the related "G-Dwarf problem"?

2. **Basic Galactic Chemical Evolution and the G-Dwarf Problem** (13 points)

   (a) Recapitulate the assumptions and fundamental equations going into the so-called Simple Closed Box (SCB) model.

   (b) Explain (with words, not just mathematically) the physical meaning of the relation $\mathrm{d}(Z)/\mathrm{d}t = (y - Z)\mathrm{d}s/\mathrm{d}t$, where $g(t)$ ($s(t)$) is the mass in gas (stars), $Z(t)$ is the mass fraction of metals in the gas phase at time t and y is the mass fraction of heavy elements returned to the ISM by the stars.

   (c) Assuming $y = \text{const.}$, derive the relation $s(t)/M_0 = 1 - \exp[-(Z(t) - Z_0)/y]$, where M_0 corresponds to the total mass of the system. This equation describes the mass fraction of stars with metallicities $Z' < Z(t)$. Calculate s/M_0 for different choices of $Z(t)$, assuming reasonable values for Z_0 and y.

   (d) Convert the data from the Geneva-Copenhagen Survey (GCS) given in the table in Fig. 1 into a normalised cumulative distribution that you can compare with the prediction of the SCB model. Assume the data values to be zero outside the lowest and highest metallicities given in the table. Use $\log_{10}(Z) = [\mathrm{Fe/H}]$ to convert between the different definitions of metallicity.

(e) Prepare a graphic comparing the GCS data and your obtained values from the SCB model.

(f) Change the value of y (the yields) in the above formula and recompute the fractions. Which value of y fits the data best?

(g) Change the value of Z_0 and recompute $s(t)/M_0$. Can you now get the model into agreement with the observations? What does changing this value imply physically?

(h) Finally, compare your values obtained with the SCB model with the observed metallicity distribution function of the Galactic Halo, also given in the table in Fig. 1. Comment on the results.

$[\mathrm{Fe/H}]_{\mathrm{GCS}}$	fraction	$[\mathrm{Fe/H}]_{\mathrm{Halo}}$	fraction
-0.7	0.0003	-2.95	0.0
-0.65	0.0006	-2.85	0.0022
-0.6	0.0009	-2.75	0.0053
-0.55	0.0031	-2.65	0.0136
-0.5	0.0043	-2.55	0.0158
-0.45	0.0065	-2.45	0.0241
-0.4	0.0112	-2.35	0.0232
-0.35	0.0167	-2.25	0.0241
-0.3	0.0217	-2.15	0.0324
-0.25	0.0329	-2.05	0.0372
-0.2	0.053	-1.95	0.0534
-0.15	0.0878	-1.85	0.0666
-0.1	0.1057	-1.75	0.0788
-0.05	0.1206	-1.65	0.0757
0.0	0.124	-1.55	0.0727
0.05	0.1175	-1.45	0.1081
0.1	0.0927	-1.35	0.0959
0.15	0.0726	-1.25	0.0911
0.2	0.0493	-1.15	0.0709
0.25	0.0335	-1.05	0.0451
0.3	0.0214	-0.95	0.0394
0.35	0.0068	-0.85	0.0149
0.4	0.004	-0.75	0.0092
0.45	0.0009	-0.65	0.0004

Figure 1: Observed mass fractions of stars in stated metallicity bins from the Geneva-Copenhagen Survey (GCS) of the Solar neighbourhood in the galactic disk (left) and the observational metallicity distribution of the galactic halo (right). NOTE, the fractions are not normalized yet.

3. **The Accreting Box Model** (7 points)

This exercise is similar to the previous one, but instead of assuming a closed box, we now feed our system with pristine (metal-free) gas. The consequence of this assumption is that the relation from the closed box model $\mathrm{d}g/\mathrm{d}t = -\mathrm{d}s/\mathrm{d}t$ does not hold any more, as the total mass $M(t)$ increases over time. In addition, we will make the assumption that the total gas mass is constant, which is often referred to as the extreme infall model.

(a) The equation $\mathrm{d}(Zg)/\mathrm{d}t = (y - Z)\mathrm{d}s/\mathrm{d}t$ still holds. Assuming $Z(0) = 0$, rewrite and integrate the equation to find $Z(t) = y(1 - \exp[1 - M(t)/g])$.

(b) Calculate again the cumulative distribution of stellar metallicities and compare the prediction of this model with the observations of (i) the solar neighbourhood (GCS) and (ii) the Galactic halo, using the same table as previously in Fig. 1.

(c) Discuss the results. Which model performs better for the solar neighbourhood, which performs better for the Galactic halo? Comment on the reasons for your findings.

4. **Gas in a NFW Halo** (12 points)

Simulations of the evolution of cold dark matter in the universe find self-similar dark matter density profiles which can be described as a so-called Navarro-Frenk-White (NFW) profile. Consider such a dark-matter halo with NFW density profile, i.e.,

$$\rho(r) = \frac{\rho_\mathrm{s}}{x(1+x)^2} \quad \text{with} \quad x = \frac{r}{r_\mathrm{s}}, \tag{1}$$

where ρ_s describes the central density and r_s a characteristic scaling radius beyond which the density profile falls off and within which the density profile flattens significantly. This density profile diverges in the center, i.e., for $x \to 0$.

(a) Confirm that the gravitational potential of an NFW halo is given by

$$\Phi(r) = -\frac{GM_\mathrm{s}}{r_\mathrm{s}} \frac{\ln(1+x)}{x}. \tag{2}$$

Gas filled in the halo's gravitational potential Φ satisfies Euler's equation

$$\frac{\nabla p_\mathrm{gas}}{\rho_\mathrm{gas}} = -\nabla\Phi(r), \tag{3}$$

where p_gas is the gas pressure.

(b) Assuming an isothermal and ideal gas, show that the gas density profile is

$$\rho_\mathrm{gas} = A \exp\left(-\frac{\bar{m}\Phi}{kT}\right), \tag{4}$$

where T is the temperature, k is Boltzmann's constant, $\bar{m}$ is the mean particle mass, and A is a constant.

(c) Using Equation (2), show that

$$-\frac{\bar{m}\Phi}{kT} = 3\frac{\ln(1+x)}{x} \tag{5}$$

if the gas is in equilibrium with the gravitational potential (virialized system).

(d) Is the gas density finite in the halo's center? Compare the density profiles of gas and dark matter and explain the differences.

(e) What happens to the gas-to-dark matter mass density ratio ρ_gas/ρ at large radii? Is this a realistic behavior and if not, what would have to be changed in the model to make it more realistic?