A pedagogical guide to cosmic rays and magnetic fields in the universe

Part 1: Magnetic fields

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Leibniz Institute for Astrophysics, Potsdam (AIP)

47th Heidelberg Physics Graduate Days

Heidelberg Graduate School for Physics

- **Magnetic fields** (Christoph Pfrommer)
  * Generation and evolution of magnetic fields
  * Magneto-hydrodynamic (MHD) waves and turbulence
- **Gamma-ray astronomy** (Frank Rieger)
  * Radiative processes
  * Origin of High Energy Radiation
- **Particle acceleration in astrophysics** (FR)
  * Constraints & challenges of particle acceleration
  * Particle acceleration in gaps, via the Fermi process and by reconnection
- **Cosmic rays** (CP)
  * Cosmic ray transport and particle-wave interactions
  * Cosmic ray acceleration in shocks and by turbulence
- **The physics of galaxy formation** (CP)
  * Puzzles in galaxy formation
  * Feedback by stars and active galactic nuclei

Properties of magnetic fields

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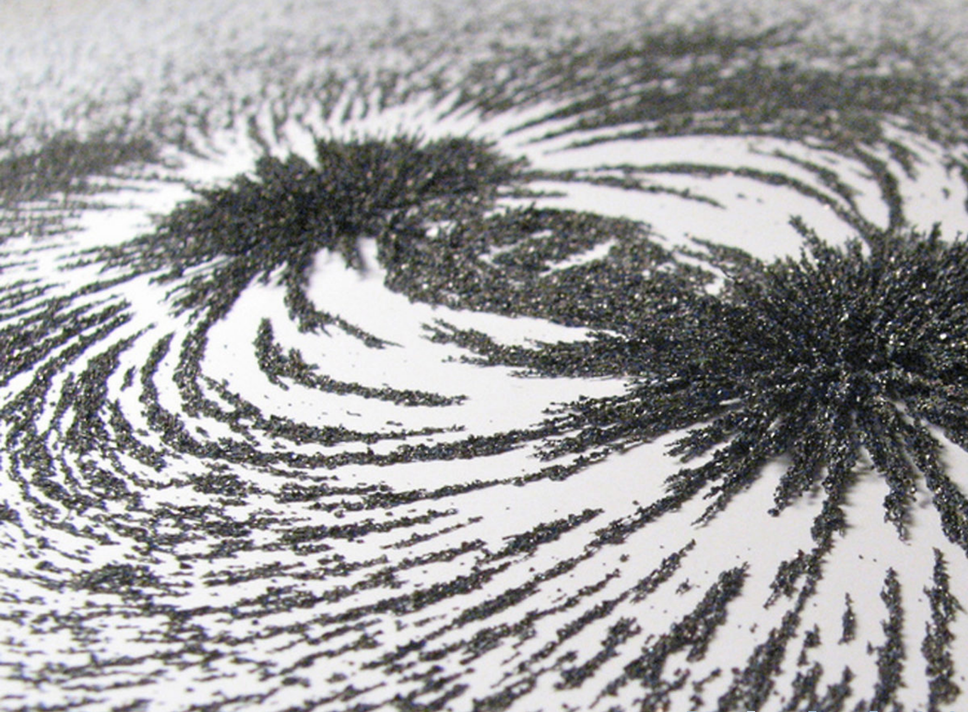
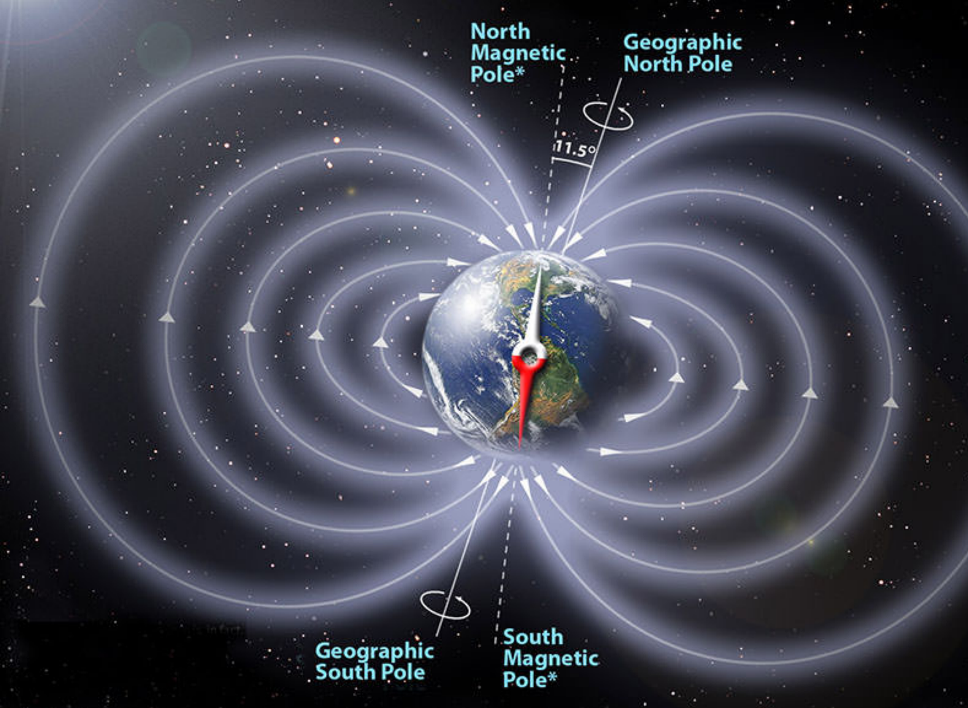
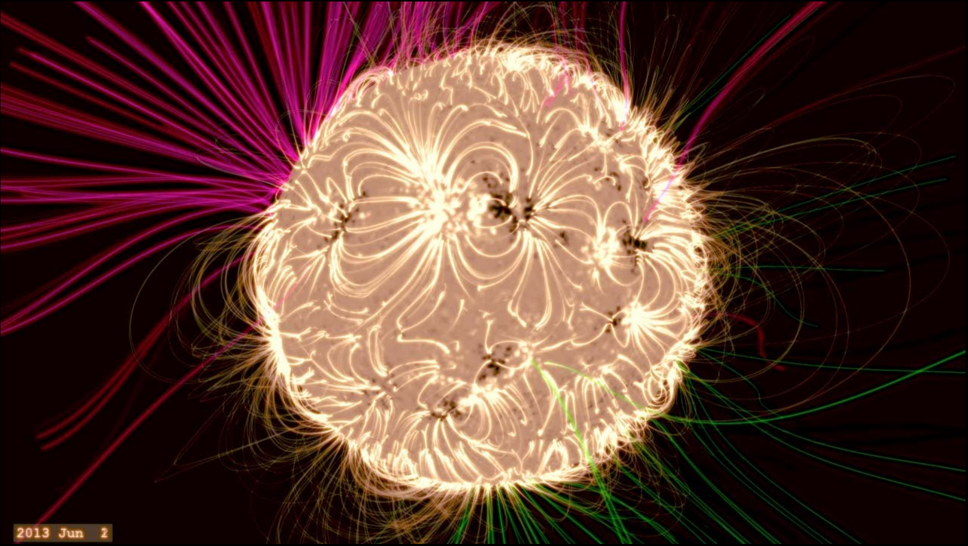
- magnetic fields are essential for accelerating cosmic rays (CRs): diffusive shock acceleration (1$^{\text{st}}$ order Fermi), turbulent MHD interactions with CRs (2$^{\text{nd}}$ order Fermi)
- magnetic fields trace violent high-energy astrophysical processes by illuminating distant CR electron populations through synchrotron emission: structure formation shocks, supernovae, ...

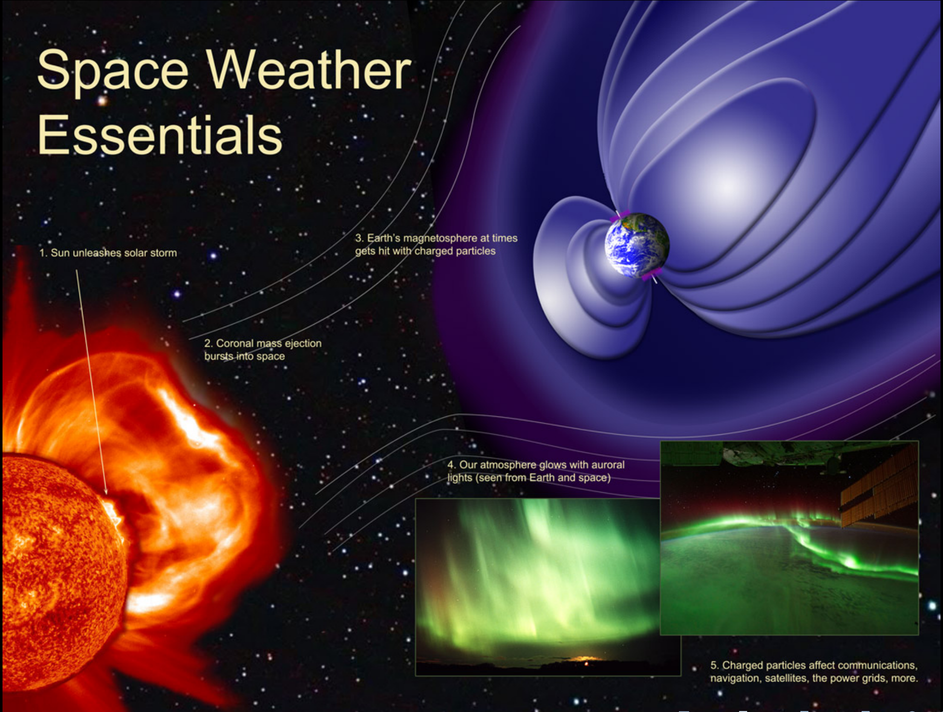



North Magnetic Pole*
Geographic North Pole
11.5°
Geographic South Pole
South Magnetic Pole*

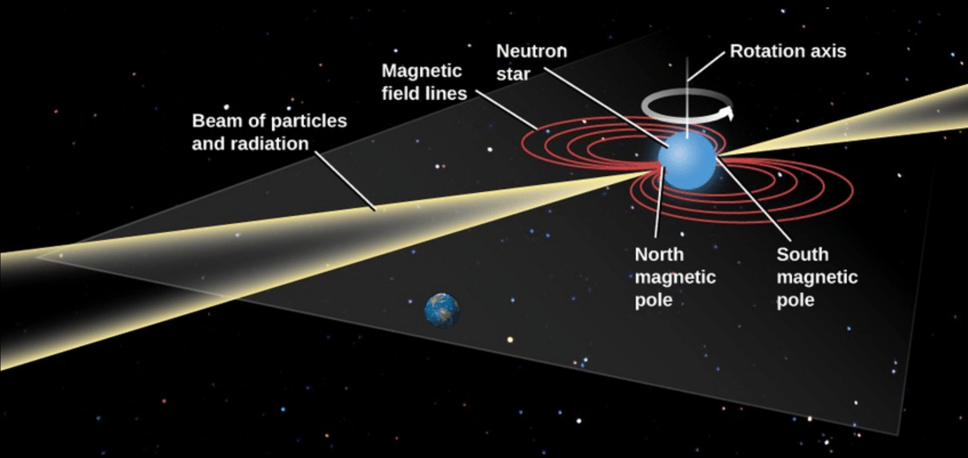
Model of the magnetic field of the Sun



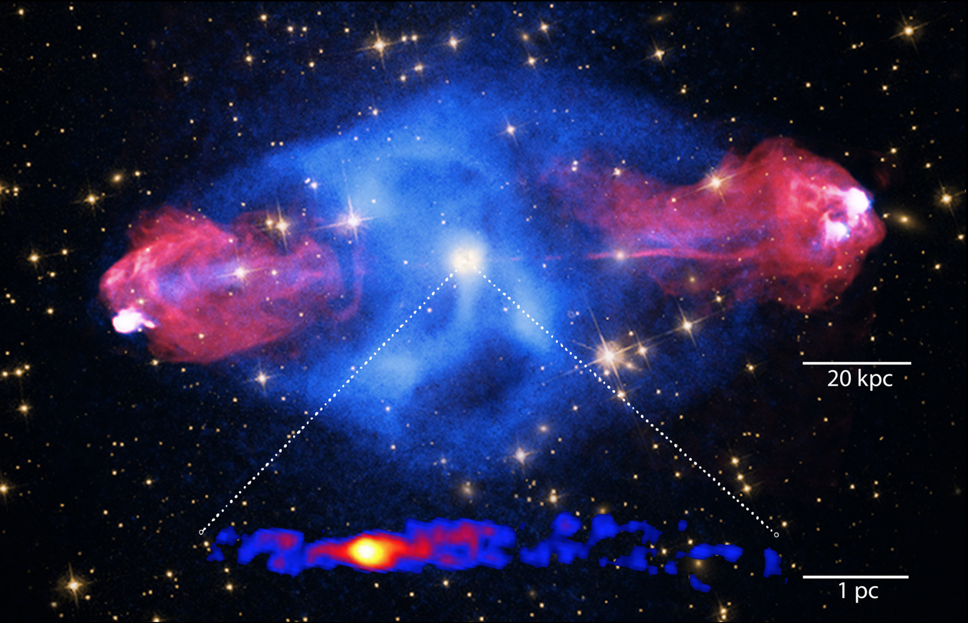
Space Weather Essentials



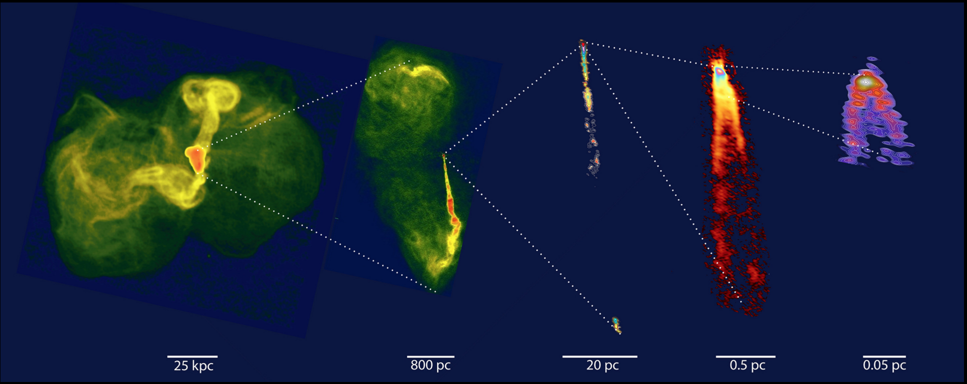
Model of a pulsar or magnetar

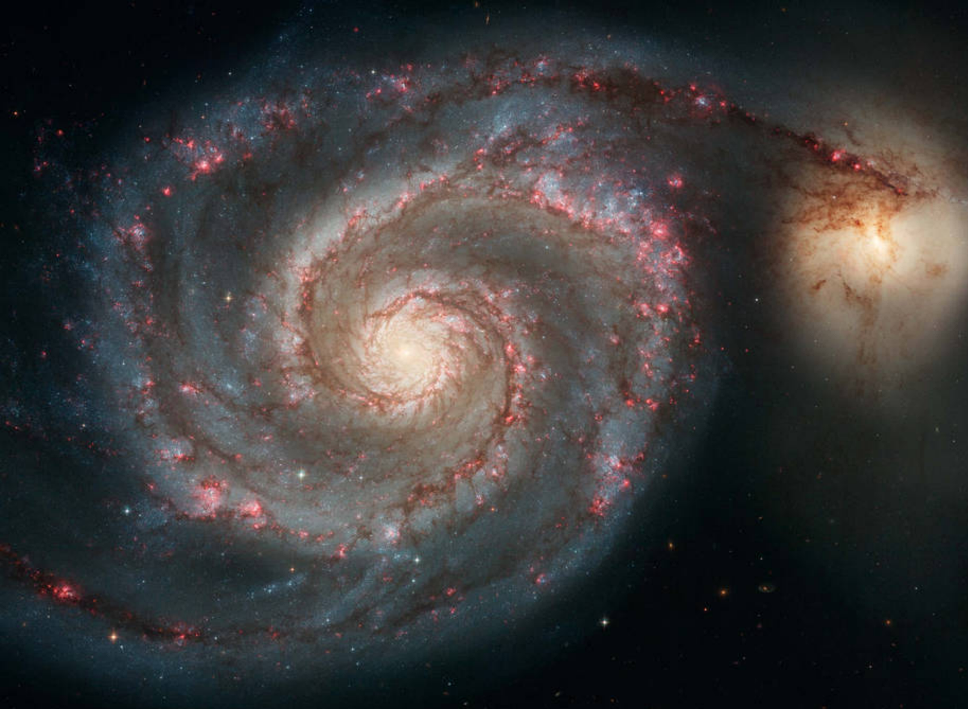


Synchrotron emission of the Cygnus A jet (radio)



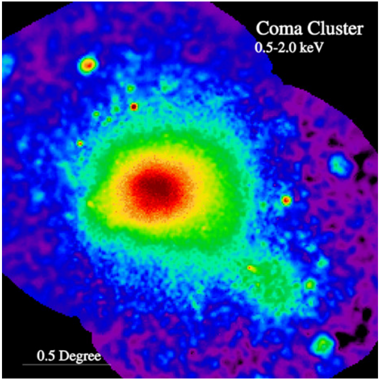
Synchrotron emission of the M87 jet (radio/optical)





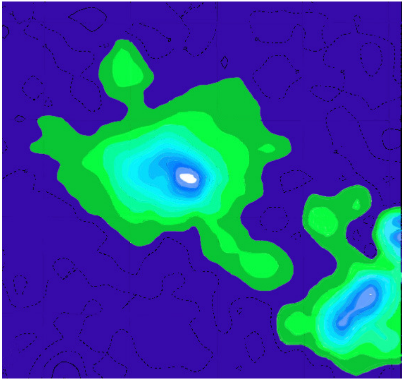
Magnetic field on the largest scales

Giant radio halo in the Coma galaxy cluster



thermal X-ray emission

(Snowden/MPE/ROSAT)



radio synchrotron emission

(Deiss/Effelsberg)

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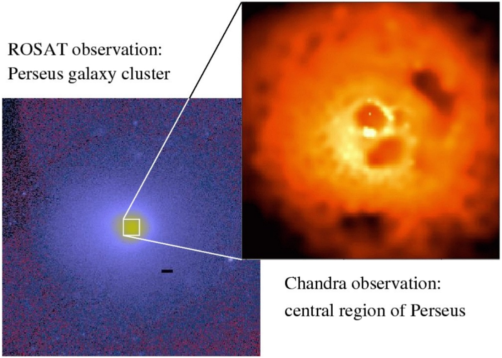
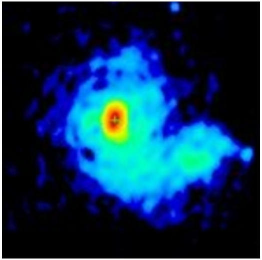
Magnetic field on the largest scales

Radio mini halo in the Perseus galaxy cluster



thermal X-ray emission

(ROSAT; NASA/IoA/A.Fabian et al.)



radio synchrotron emission

(Pedlar/VLA)

Diffuse radio phenomena in galaxy clusters

- radio halos:
  centrally located, more regular morphology, unpolarized
  - giant radio halos: occur in merging clusters, $>$ 1 Mpc-sized, morphology similar to X-rays
  - radio mini halos: occur in cool core clusters, few times 100 kpc in size, emission extends over cool core

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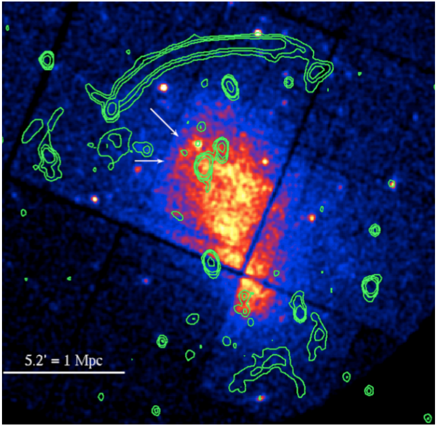
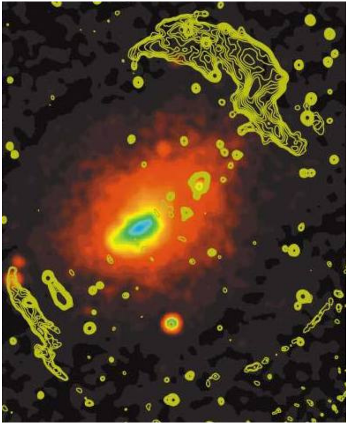
Radio shock: double relic sources



CIZA J2242.8+5301 (“sausage relic”)

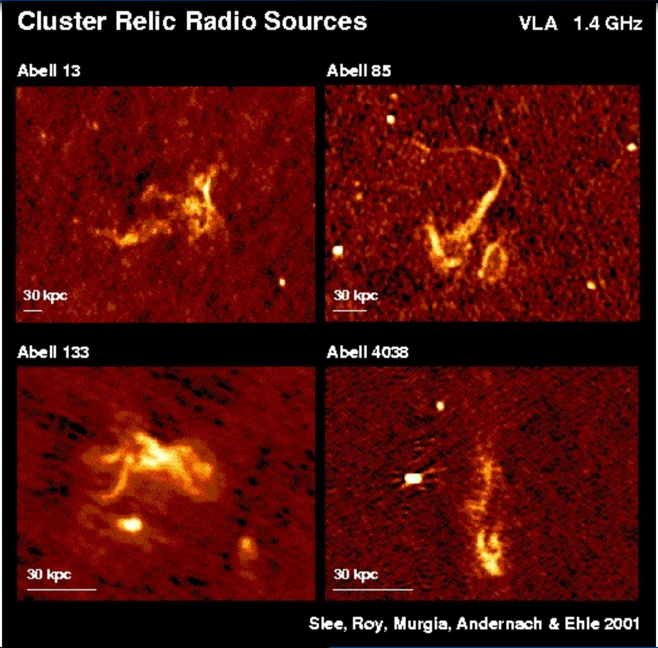
(X-ray: XMM; radio: WSRT; Ogrean+ 2013)



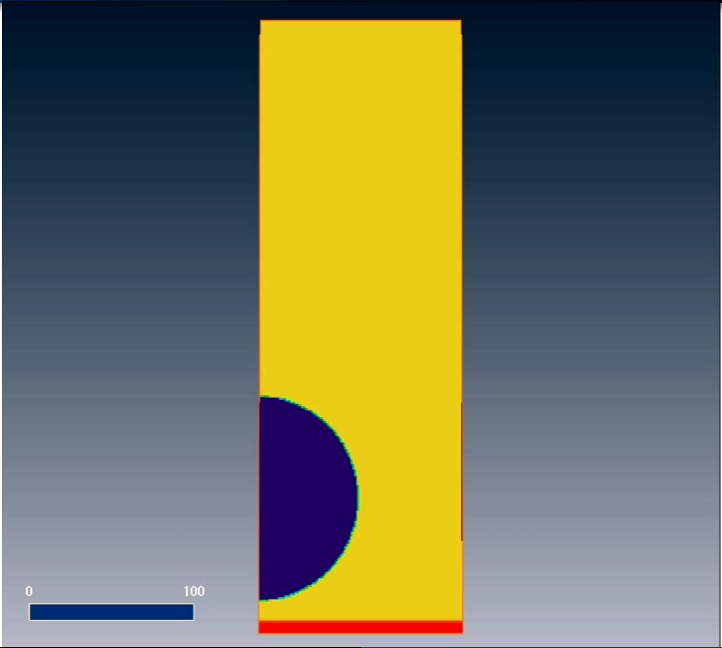
Abell 3667

(radio: Johnston-Hollitt. X-ray: ROSAT/PSPC.)

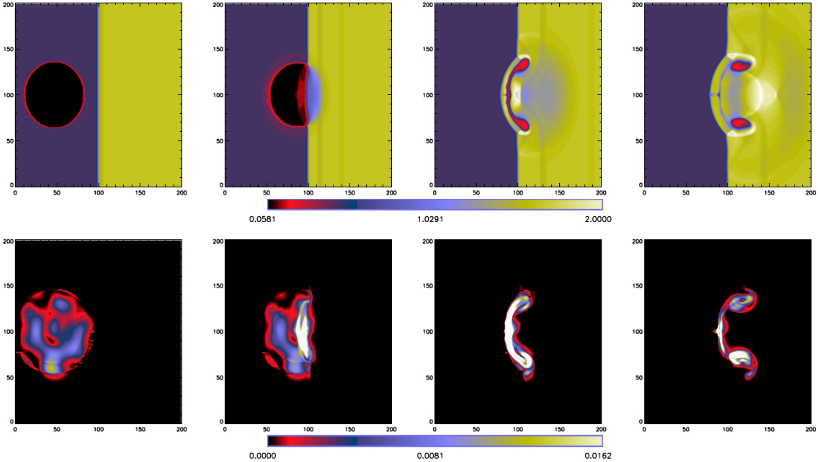
Radio phoenix



Shock overruns an aged radio bubble (Pfrommer & Jones 2011)



Bubble transformation to vortex ring



Enßlin & Brüggen (2002): gas density *(top)* and magnetic energy density *(bottom)*

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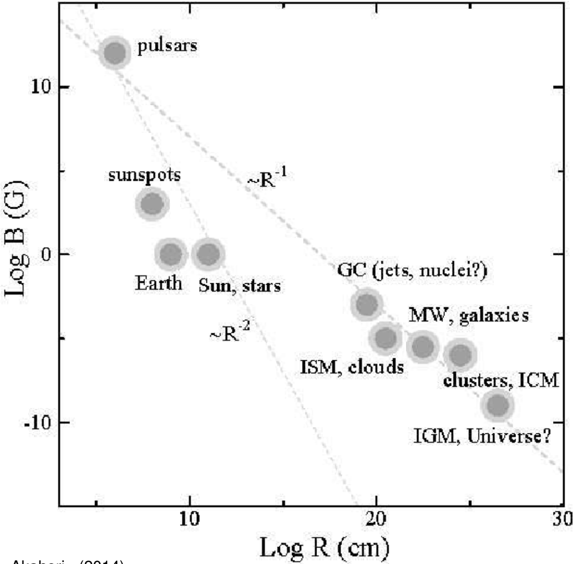
- radio relics:
  irregular morphology, polarized
  - radio shock: at cluster periphery ($<$ Mpc), in some cases coincident with weak X-ray shock, polarized $\rightarrow$ *diffusive shock acceleration (Fermi I)*
  - radio relic bubble: aged radio cocoon, steep spectrum
  - radio phoenix: shock-revived bubble that has already faded out of the radio window $\rightarrow$ *adiabatic compression*

Cosmic magnetic fields

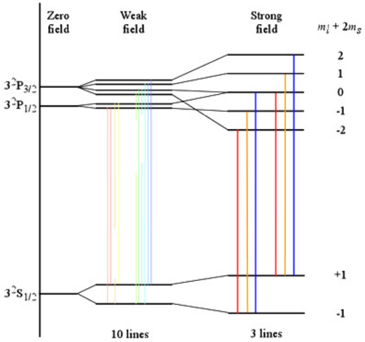
From the strongest to weakest field strengths and from compact to diffuse sources



Akahori+ (2014)

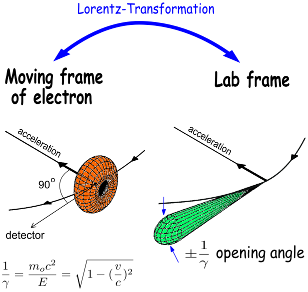
Zeeman effect:

- The Zeeman effect describes the splitting of an atomic level and hence the associated spectral line into several components in the presence of a static magnetic field.
- The amount of splitting depends on the strength of the magnetic field. The splitting is associated with the orbital angular momentum quantum number.
- Detection requires high spectral resolution and sources of high densities (stars, cores of molecular clouds, . . .).



Synchrotron emission:

- Charged particles emit electromagnetic radiation when accelerated, e.g. due to the Lorentz force of a magnetic field.
- This emission is axisymmetric with respect to the acceleration direction in the *particle's rest frame*.
- If the particles move relativistically, then the emission in the *lab frame* is beamed into a forward cone of an opening angle $\theta \sim \gamma^{-1}$ (where γ is the Lorentz factor).



Observing magnetic fields in astrophysics – 2

Synchrotron emission:

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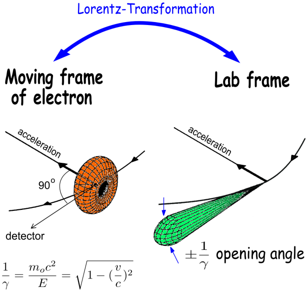
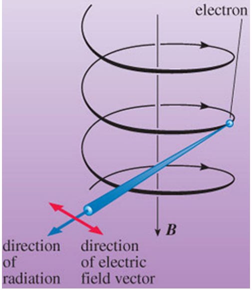
- This emission is axisymmetric with respect to the acceleration direction in the *particle's rest frame*.
- If the particles move relativistically, then the emission in the *lab frame* is beamed into a forward cone of an opening angle $\theta \sim \gamma^{-1}$ (where γ is the Lorentz factor).
- Because the emission (= transverse electromagnetic wave) propagates in a narrow cone, it is linearly polarized.
- The typical synchrotron frequency is

$$\nu_{\text{synch}} = \frac{3eB}{2\pi\, m_e c}\,\gamma^2 \simeq 1\ \text{GHz}\ \frac{B}{\mu\text{G}} \left(\frac{\gamma}{10^4}\right)^2 .$$

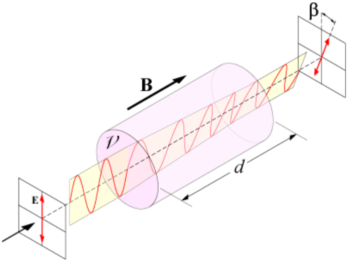
- Power-law cosmic ray electron momentum distributions imply power-law (radio) synchrotron spectra.





Faraday rotation:

- Faraday rotation describes rotation of a linearly polarized electro-magnetic wave in the presence of a line-of-sight magnetic field because of the birefringent property of a plasma.
- This can be seen by splitting the linearly polarized wave into right- and left-hand circularly polarized waves, which propagate at slightly different speeds.
- The observed polarization angle ϕ_{obs} is modified from its intrinsic position angle, $\phi_{\mathrm{intrinsic}}$.



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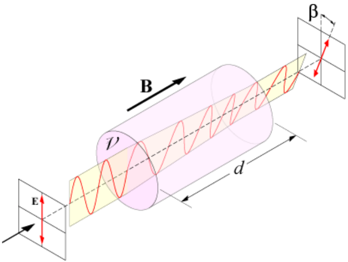
- The observed polarization angle ϕ_{obs} is modified from its intrinsic position angle, $\phi_{\mathrm{intrinsic}}$.
- The rate of rotation scales with the wavelength squared and is given by

$$\phi_{\mathrm{obs}}(\boldsymbol{x}_\perp) = \lambda^2 \mathrm{RM}(\boldsymbol{x}_\perp) + \phi_{\mathrm{intrinsic}}(\boldsymbol{x}_\perp),$$

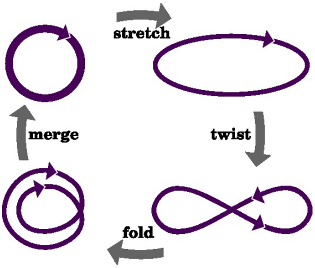
$$\mathrm{RM}(\boldsymbol{x}_\perp) = \frac{e^3}{2\pi\, m_e^2 c^4} \int_0^d n_e(\boldsymbol{x}_\perp, l)\, \boldsymbol{B} \cdot \mathrm{d}\boldsymbol{l}$$

$$= 812\, \frac{\mathrm{rad}}{\mathrm{m}^2}\, \frac{B}{\mu\mathrm{G}}\, \frac{n_e}{10^{-3}\mathrm{cm}^{-3}}\, \frac{d}{\mathrm{Mpc}}.$$



The general picture:

- ***Origin.*** Magnetic fields are generated by 1. electric currents sourced by a phase transition in the early universe or 2. by the Biermann battery.
- ***Growth.*** A small-scale (fluctuating) dynamo is a magneto-hydrodynamical process, in which the kinetic (turbulent) energy is converted into magnetic energy: the mechanism relies on magnetic fields to become stronger when the field lines are stretched.
- ***Saturation.*** Field growth stops at a sizeable fraction of the turbulent energy when magnetic forces become strong enough to resist the stretching and folding motions.



The Biermann battery – 1

- Electron and proton momenta change due to the Lorentz force, the pressure and viscous forces:

$$m_\mathrm{e}\frac{\mathrm{d}\boldsymbol{v}_\mathrm{e}}{\mathrm{d}t} = -e\left(\boldsymbol{E} + \frac{\boldsymbol{v}_\mathrm{e}}{c}\times\boldsymbol{B} + \frac{1}{en_\mathrm{e}}\boldsymbol{\nabla}P_\mathrm{e}\right) - \frac{\nu_\mathrm{visc}m_\mathrm{e}}{n_\mathrm{e}}(\boldsymbol{v}_\mathrm{e} - \boldsymbol{v}_\mathrm{p}),$$

$$m_\mathrm{p}\frac{\mathrm{d}\boldsymbol{v}_\mathrm{p}}{\mathrm{d}t} = e\left(\boldsymbol{E} + \frac{\boldsymbol{v}_\mathrm{p}}{c}\times\boldsymbol{B} + \frac{1}{en_\mathrm{p}}\boldsymbol{\nabla}P_\mathrm{p}\right).$$

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- If $T_\mathrm{p} = T_\mathrm{e}$, we can neglect the proton equation because protons move on average slower than electrons by a factor $\sqrt{m_\mathrm{p}/m_\mathrm{e}}$.
- Viscous forces are very small on large scales: we drop the term $\propto \nu_\mathrm{visc}$.
- We assume a steady state (i.e., $\tau > \omega_\mathrm{pl}^{-1}$, where $\omega_\mathrm{pl}^2 = 4\pi n_\mathrm{e}e^2/m_\mathrm{e}$ is the plasma frequency) and solve for $\boldsymbol{E}$:

$$\boldsymbol{E} = -\frac{\boldsymbol{v}_\mathrm{e}\times\boldsymbol{B}}{c} - \frac{\boldsymbol{\nabla}P_\mathrm{e}}{en_\mathrm{e}}.$$

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- Multiplying this equation by $-c$, taking the curl of it and using Faraday's law, we obtain

$$\frac{\partial\boldsymbol{B}}{\partial t} = -c\boldsymbol{\nabla}\times\boldsymbol{E} = \boldsymbol{\nabla}\times(\boldsymbol{v}_\mathrm{e}\times\boldsymbol{B}) + \frac{c}{e}\boldsymbol{\nabla}\times\left(\frac{\boldsymbol{\nabla}P_\mathrm{e}}{n_\mathrm{e}}\right). \qquad (1)$$

The Biermann battery – 2

- Using $P_\mathrm{e} = n_\mathrm{e} k_\mathrm{B} T_\mathrm{e}$ and the identities $\boldsymbol{\nabla} \times (f \boldsymbol{\nabla} g) \equiv \boldsymbol{\nabla} f \times \boldsymbol{\nabla} g$ and $\boldsymbol{\nabla} \times \boldsymbol{\nabla} f \equiv \mathbf{0}$, we can rewrite the second term of Eq. (1):

$$\begin{aligned}
\frac{1}{k_\mathrm{B}} \boldsymbol{\nabla} \times \left(\frac{\boldsymbol{\nabla} P_\mathrm{e}}{n_\mathrm{e}}\right) &= \boldsymbol{\nabla} \times \left[\frac{1}{n_\mathrm{e}} \boldsymbol{\nabla} (n_\mathrm{e} T_\mathrm{e})\right] = \boldsymbol{\nabla} \times (\boldsymbol{\nabla} T_\mathrm{e}) + \boldsymbol{\nabla} \times \left(\frac{T_\mathrm{e}}{n_\mathrm{e}} \boldsymbol{\nabla} n_\mathrm{e}\right) \\
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- Hence, we obtain the Biermann battery equation,

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\frac{\partial \boldsymbol{B}}{\partial t} = \boldsymbol{\nabla} \times (\boldsymbol{v}_\mathrm{e} \times \boldsymbol{B}) - \frac{c k_\mathrm{B}}{e n_\mathrm{e}} \boldsymbol{\nabla} n_\mathrm{e} \times \boldsymbol{\nabla} T_\mathrm{e}.
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- This equation shows that if there is no magnetic field to start with (i.e., a vanishing first term on the right-hand side), then the magnetic field can be generated by a baroclinic flow with $\boldsymbol{\nabla} n_\mathrm{e} \times \boldsymbol{\nabla} T_\mathrm{e} \neq \mathbf{0}$.

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\frac{1}{k_\mathrm{B}} \boldsymbol{\nabla} \times \left(\frac{\boldsymbol{\nabla} P_\mathrm{e}}{n_\mathrm{e}} \right) &= \boldsymbol{\nabla} \times \left[\frac{1}{n_\mathrm{e}} \boldsymbol{\nabla} (n_\mathrm{e} T_\mathrm{e}) \right] = \boldsymbol{\nabla} \times (\boldsymbol{\nabla} T_\mathrm{e}) + \boldsymbol{\nabla} \times \left(\frac{T_\mathrm{e}}{n_\mathrm{e}} \boldsymbol{\nabla} n_\mathrm{e} \right) \\
&= \boldsymbol{\nabla} \left(\frac{T_\mathrm{e}}{n_\mathrm{e}} \right) \times \boldsymbol{\nabla} n_\mathrm{e} = \frac{1}{n_\mathrm{e}} \boldsymbol{\nabla} T_\mathrm{e} \times \boldsymbol{\nabla} n_\mathrm{e} - \frac{T_\mathrm{e}}{n_\mathrm{e}^2} \boldsymbol{\nabla} n_\mathrm{e} \times \boldsymbol{\nabla} n_\mathrm{e} \\
&= \frac{1}{n_\mathrm{e}} \boldsymbol{\nabla} T_\mathrm{e} \times \boldsymbol{\nabla} n_\mathrm{e}.
\end{aligned}
$$

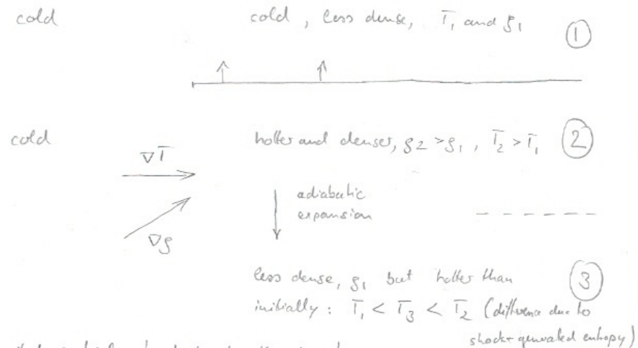
- Hence, we obtain the Biermann battery equation,

$$
\frac{\partial \boldsymbol{B}}{\partial t} = \boldsymbol{\nabla} \times (\boldsymbol{v}_\mathrm{e} \times \boldsymbol{B}) - \frac{c k_\mathrm{B}}{e n_\mathrm{e}} \boldsymbol{\nabla} n_\mathrm{e} \times \boldsymbol{\nabla} T_\mathrm{e}.
$$

- This equation shows that if there is no magnetic field to start with (i.e., a vanishing first term on the right-hand side), then the magnetic field can be generated by a baroclinic flow with $\boldsymbol{\nabla} n_\mathrm{e} \times \boldsymbol{\nabla} T_\mathrm{e} \neq \mathbf{0}$.
- This could be achieved in shocks of the interstellar medium, in ionization fronts, or similar astrophysical sites; in general, baroclinic flows are sourced by rotational motions at shocks of finite extent such as the chaotic collapse of a proto-galaxy.

AIP

- Consider a shock of finite extent that propagates into zero-pressure medium.



→ shocks of limited spatial extend break the barotropic relation $p_e = p_e(n_e)$ which couples p_e to n_e

The Biermann battery – 4

- Magnetic fields generated through this process have very small field strengths: adopting a characteristic density and temperature gradient length of L of a proto-galaxy and assuming gravitational collapse on the free-fall time, $\tau \sim 1/\sqrt{G\rho}$, we obtain

$$B \sim \frac{ck_\mathrm{B}T_\mathrm{e}}{e}\,\frac{\tau}{L^2} \sim \frac{ck_\mathrm{B}T_\mathrm{e}}{e}\,\frac{1}{\sqrt{G\rho}L^2}$$
$$\sim 10^{-20}\mathrm{G}\left(\frac{T_\mathrm{e}}{10^4\,\mathrm{K}}\right)\left(\frac{n}{1\,\mathrm{cm}^{-3}}\right)^{-1/2}\left(\frac{L}{\mathrm{kpc}}\right)^{-2}.$$

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- Naively, going to smaller length scales L should increase B. However, in order to explain the *coherence on scales of several kpcs*, we would have to evoke a process such as a small-scale wind that moves the magnetic fields back to kpc scales and in that process we would have to account for adiabatic losses that accompany the expansion from small to large scales: in the end we would gain nothing from running a Biermann battery on smaller scales.
- This solves the cosmological magneto-genesis problem, but the big challenge remains in growing coherent large-scale magnetic fields from a small-amplitude, small-scale fields: this is a major challenge of dynamo theory!

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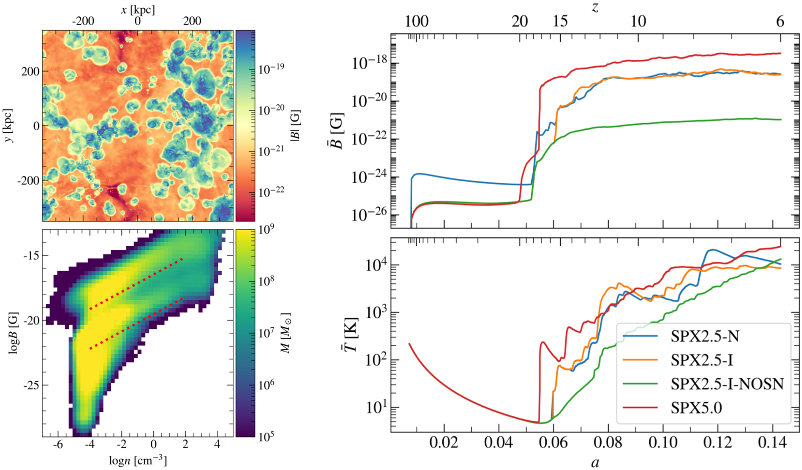
Cosmological magneto-genesis: the Biermann battery



- Cosmological simulations of the Biermann battery during the epoch of reionization with a state-of-the-art galaxy formation model find magnetic field generation at reionization fronts and at supernova-driven outflows (Attia+ 2021)

The induction equation – derivation

- To derive the equations of magneto-hydrodynamics, we need
  1. an equation for the magnetic field evolution, i.e., the induction equation, and
  2. work out the magnetic force and stress.

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- Using Faraday's Law, $\dfrac{\partial \boldsymbol{B}}{\partial t} = -c\,\boldsymbol{\nabla} \times \boldsymbol{E}$, we get

$$\frac{\partial \boldsymbol{B}}{\partial t} = \boldsymbol{\nabla} \times (\boldsymbol{v} \times \boldsymbol{B}) - \boldsymbol{\nabla} \times (c\,\eta\,\boldsymbol{j}).$$

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- Assuming $\eta = \text{const}$, using the identity $\boldsymbol{\nabla} \times (\boldsymbol{\nabla} \times \boldsymbol{B}) \equiv \boldsymbol{\nabla}(\boldsymbol{\nabla} \cdot \boldsymbol{B}) - \boldsymbol{\nabla}^2\boldsymbol{B}$ and the solenoidal condition, $\boldsymbol{\nabla} \cdot \boldsymbol{B} = 0$, we arrive at the *induction equation*:

$$\frac{\partial \boldsymbol{B}}{\partial t} = \boldsymbol{\nabla} \times (\boldsymbol{v} \times \boldsymbol{B}) + D\boldsymbol{\nabla}^2\boldsymbol{B}, \quad \text{where} \quad D = \frac{c^2\,\eta}{4\pi}. \qquad (2)$$

The induction equation – discussion

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- 1$^{\mathrm{st}}$ term: the “convective term” states that the field is frozen into the flow (as we will see momentarily): an important property for astrophysical plasmas!
- 2$^{\mathrm{nd}}$ term: the “diffusive term” represents the diffusive leakage of magnetic field lines across the conducting field, which is important for changing the magnetic topology, e.g. in reconnection.

AIP

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- The "diffusive term" can be neglected for infinite conductivity $\sigma = \eta^{-1}$ or for large magnetic Reynolds numbers $\mathrm{Re_m} \to \infty$:

$$\mathrm{Re_m} = \frac{|\text{convective term}|}{|\text{diffusive term}|} = \frac{L^{-1}\,v\,B}{D\,L^{-2}B} = \frac{L\,v}{D}$$

Magnetic force and stress – 1

Using Ampère's law at low frequencies, $\boldsymbol{\nabla} \times \boldsymbol{B} = 4\pi \boldsymbol{j}/c$, we will now show that the Lorentz force can be written as follows:

$$\boldsymbol{F}_{\mathrm{L}} = \frac{1}{c}\boldsymbol{j} \times \boldsymbol{B} = \frac{1}{4\pi}(\boldsymbol{\nabla} \times \boldsymbol{B}) \times \boldsymbol{B} = -\frac{1}{8\pi}\boldsymbol{\nabla} B^2 + \frac{1}{4\pi}(\boldsymbol{B} \cdot \boldsymbol{\nabla})\boldsymbol{B} = -\boldsymbol{\nabla} \cdot \mathrm{M},$$

where

$$\mathrm{M}_{ij} = \frac{1}{8\pi} B^2 \delta_{ij} - \frac{1}{4\pi} B_i B_j$$

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$$\begin{aligned}(\boldsymbol{\nabla} \times \boldsymbol{B}) \times \boldsymbol{B}\big|_i &= \varepsilon_{ijk}\varepsilon_{jlm}(\partial_l B_m) B_k = \varepsilon_{kij}\varepsilon_{jlm}(\partial_l B_m) B_k \\ &= (\delta_{kl}\delta_{im} - \delta_{km}\delta_{il})(\partial_l B_m) B_k = (\partial_k B_i) B_k - (\partial_i B_k) B_k \\ &= \left[(\boldsymbol{B} \cdot \boldsymbol{\nabla})\boldsymbol{B} - \frac{1}{2}\boldsymbol{\nabla} \boldsymbol{B}^2\right]_i .\end{aligned}$$

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- Because $\boldsymbol{\nabla} \cdot \boldsymbol{B} = 0$, we can rewrite this, yielding

$$\frac{1}{4\pi}\left[(\boldsymbol{B} \cdot \boldsymbol{\nabla})\boldsymbol{B} - \frac{1}{2}\boldsymbol{\nabla}\boldsymbol{B}^2\right]_i = \frac{1}{4\pi}\partial_k\left(B_i B_k - \frac{1}{2}B^2\delta_{ik}\right) = -\partial_k \mathrm{M}_{ik}.$$

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- The first term is the magnetic *curvature or tension force* and the second term is the gradient of the *magnetic pressure* $B^2/8\pi$.

Magnetic force and stress – 2

To get a better understanding, we show that the surface force (per unit area) exerted *by* a bounded volume V *on* its surroundings is given by

$$\boldsymbol{F}_S = \boldsymbol{n} \cdot \mathrm{M} = \frac{1}{8\pi} B^2 \boldsymbol{n} - \frac{1}{4\pi} \boldsymbol{B} B_n,$$

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- The net Lorentz force acting *on* a volume V of fluid can be written as an integral of a magnetic stress vector acting on its surface,

$$\int_V \boldsymbol{F}_{\mathrm{L}} \mathrm{d}V = \int_V \frac{1}{4\pi} (\boldsymbol{\nabla} \times \boldsymbol{B}) \times \boldsymbol{B} \, \mathrm{d}V = -\int_S \boldsymbol{\nabla} \cdot \mathsf{M} \, \mathrm{d}V = -\oint_S \boldsymbol{n} \cdot \mathsf{M} \, \mathrm{d}S.$$

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- To get the force $\boldsymbol{F}_S$ exerted *by* the volume *on* its surroundings, we need to add a minus sign,

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Magnetic force and stress – 3

To understand the meaning of magnetic stress, we take a uniform magnetic field along the z-direction ($\boldsymbol{B} = B\boldsymbol{e}_z$) and compute the surface forces $\boldsymbol{F}_S$ exerted by a rectangular volume that is aligned with the magnetic field (there are 6 different surface elements but symmetry limits the surface forces to two different types). In particular, we ask which magnetic forces (pressure or tension) are contributing to these surface forces:

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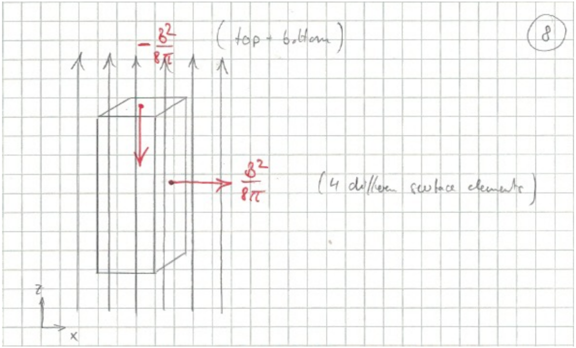

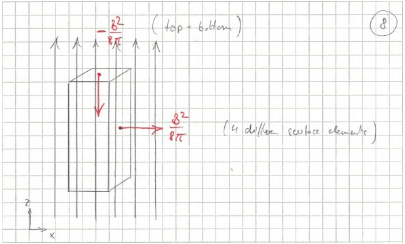

- Take the *surface* perpendicular to the x axis on the right-hand side of the box:

$$\boldsymbol{n} = \boldsymbol{e}_x : \qquad \boldsymbol{F}_{\mathrm{right}} = \boldsymbol{e}_x \cdot \mathrm{M},$$

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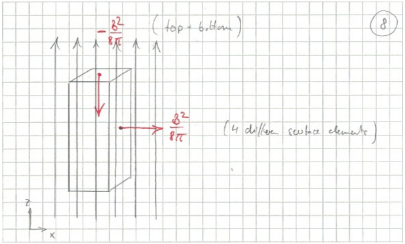
Magnetic force and stress – 4



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- The stress exerted by the magnetic field at the top of the surface element is

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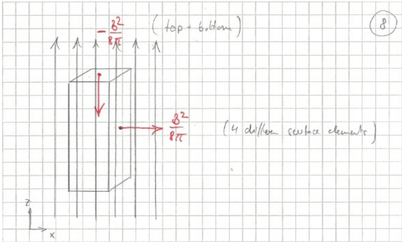
Magnetic force and stress – 4



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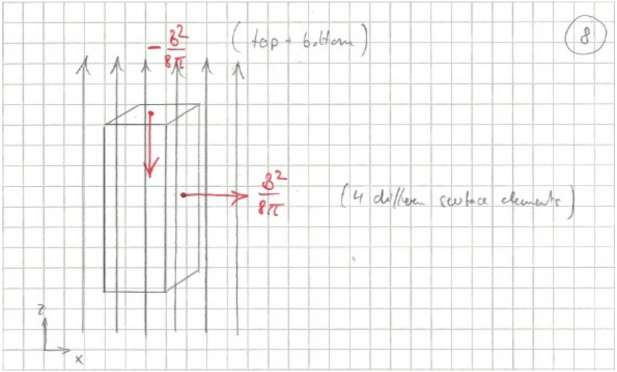
The stress is also perpendicular to the surface and of equal magnitude to that of the magnetic pressure exerted at the vertical surfaces, but of opposite sign!

Magnetic force and stress – 5



Conclusions:

- The magnetic pressure causes the fluid volume to expand in the perpendicular directions to the magnetic field (in *x* and *y* for a field in *z* direction)

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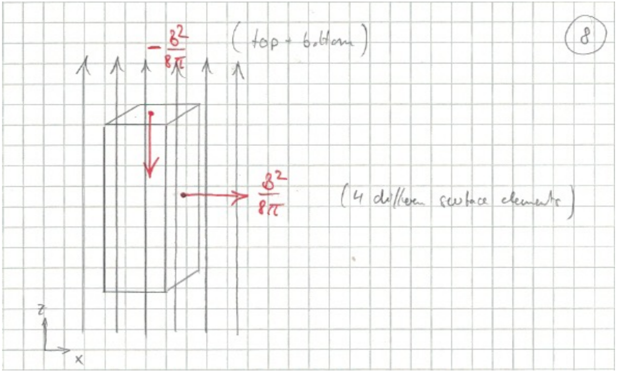

Conclusions:

- The magnetic pressure causes the fluid volume to expand in the perpendicular directions to the magnetic field (in *x* and *y* for a field in *z* direction)
- The magnetic stress in the direction of ***B*** forces the fluid element to contract along the field lines. This acts like a negative pressure similar to a stretched elastic rubber band!

Magneto-hydrodynamics

- For a collisional fluid on scales larger than the particle mean-free path and on time scales longer than the inverse plasma frequency, $\tau > \omega_{\mathrm{pl}}^{-1}$, the evolution of the magnetic vector field $\boldsymbol{B}$ is given by magneto-hydrodynamics (MHD).
- Ideal MHD assumes an inviscid (i.e., no viscosity), ideally conducting fluid.

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- Ideal MHD assumes an inviscid (i.e., no viscosity), ideally conducting fluid.
- To derive MHD, we add the Lorentz force to the momentum evolution equation (the Euler equation) and supplement the system of conservation equations of mass, momentum and entropy by the equation for magnetic induction, Eq. (2) without the diffusion term and obtain the equations of ideal MHD:

$$\frac{\partial \rho}{\partial t} + \boldsymbol{\nabla} \cdot (\rho \boldsymbol{v}) = 0,$$

$$\rho \left(\frac{\partial \boldsymbol{v}}{\partial t} + \boldsymbol{v} \cdot \boldsymbol{\nabla} \boldsymbol{v} \right) = -\boldsymbol{\nabla} P + \boldsymbol{j} \times \boldsymbol{B} = -\boldsymbol{\nabla} \cdot \left[\left(P + \frac{\boldsymbol{B}^2}{8\pi} \right) \bar{\boldsymbol{1}} + \frac{1}{4\pi} \boldsymbol{B}\boldsymbol{B}^{\mathrm{T}} \right],$$

$$\frac{\partial s}{\partial t} + \boldsymbol{v} \cdot \boldsymbol{\nabla} s = 0,$$

$$\frac{\partial \boldsymbol{B}}{\partial t} - \boldsymbol{\nabla} \times (\boldsymbol{v} \times \boldsymbol{B}) = \boldsymbol{0}, \quad \text{subject to the constraint} \quad \boldsymbol{\nabla} \cdot \boldsymbol{B} = 0,$$

where $\rho = \rho(\boldsymbol{x})$, $P = P(\boldsymbol{x})$, $\boldsymbol{v} = \boldsymbol{v}(\boldsymbol{x})$, $\boldsymbol{j} = \boldsymbol{j}(\boldsymbol{x})$, $s = s(\boldsymbol{x})$, and $\boldsymbol{B} = \boldsymbol{B}(\boldsymbol{x})$ are the density, pressure, velocity, electric current, entropy, and magnetic field.

Magnetic flux freezing – 1

- To show that the magnetic flux is "frozen" into the plasma, we start with the induction equation (2) without the diffusion term:

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  which can be rearranged to yield

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- Multiplying this equation by ρ^{-1} and rearranging terms yields

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\boldsymbol{B}}{\rho}\right) = \frac{1}{\rho}\frac{\mathrm{d}\boldsymbol{B}}{\mathrm{d}t} - \frac{\boldsymbol{B}}{\rho^2}\frac{\mathrm{d}\rho}{\mathrm{d}t} = \left(\frac{\boldsymbol{B}}{\rho} \cdot \boldsymbol{\nabla}\right)\boldsymbol{v},$$

  This is the flux-freezing equation of magnetic fields.

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Magnetic flux freezing – 2

Flux freezing condition: $\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\boldsymbol{B}}{\rho}\right) = \left(\frac{\boldsymbol{B}}{\rho}\cdot\boldsymbol{\nabla}\right)\boldsymbol{v}$

- Consider the evolution of $\delta\boldsymbol{x}$ which connects two neighboring points in the fluid:

$$\begin{aligned} \Delta\boldsymbol{x}(t) &= \delta\boldsymbol{x} \\ \Delta\boldsymbol{x}(t+\Delta t) &= \delta\boldsymbol{x} + (\delta\boldsymbol{x}\cdot\boldsymbol{\nabla})\,\boldsymbol{v}\,\Delta t + \mathcal{O}(\Delta t^2) \\ \frac{\mathrm{d}\delta\boldsymbol{x}}{\mathrm{d}t} &= \frac{\Delta\boldsymbol{x}(t+\Delta t) - \Delta\boldsymbol{x}(t)}{\Delta t} = (\delta\boldsymbol{x}\cdot\boldsymbol{\nabla})\,\boldsymbol{v} \end{aligned}$$

- $\boldsymbol{B}/\rho$ and $\delta\boldsymbol{x}$ satisfy the same ODE, hence if initially $\delta\boldsymbol{x} = \varepsilon\boldsymbol{B}/\rho$, the same relation will hold for all times. If $\delta\boldsymbol{x}$ connects two particles on the same field line then they remain on the same field line.

Magnetic flux freezing – 3

Flux freezing condition: $$\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\boldsymbol{B}}{\rho}\right) = \left(\frac{\boldsymbol{B}}{\rho}\cdot\boldsymbol{\nabla}\right)\boldsymbol{v}$$

- What does this flux-freezing condition imply for a uniform contraction/expansion of the plasma?

Magnetic flux freezing – 3

Flux freezing condition: $\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\boldsymbol{B}}{\rho}\right) = \left(\frac{\boldsymbol{B}}{\rho}\cdot\boldsymbol{\nabla}\right)\boldsymbol{v}$

- What does this flux-freezing condition imply for a uniform contraction/expansion of the plasma?
- The plasma resides in a sphere of radius r and conserves mass and magnetic flux $\mathrm{d}\Phi = \boldsymbol{B}\cdot\mathrm{d}\boldsymbol{A}$ (where $\mathrm{d}\boldsymbol{A}$ is the surface element on the sphere). Thus, both ρr^3 and Br^2 are constant and we obtain

$$B \equiv \sqrt{\langle \boldsymbol{B}\rangle} \propto r^{-2} \propto \rho^{\alpha_B}, \quad \alpha_B = \frac{2}{3},$$

for isotropic contraction or expansion, independent of the magnetic topology.

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  for isotropic contraction or expansion, independent of the magnetic topology.
- Note that the scaling exponent α_B depends on the type of symmetry invoked during collapse (whether it is isotropic or not) and can differ for contraction along a homogeneous magnetic field ($\alpha_B = 0$) or perpendicular to it ($\alpha_B = 1$).
- Thus, flux freezing alone predicts a tight relation between B and ρ. Moreover, it has a surprising property called magnetic draping.

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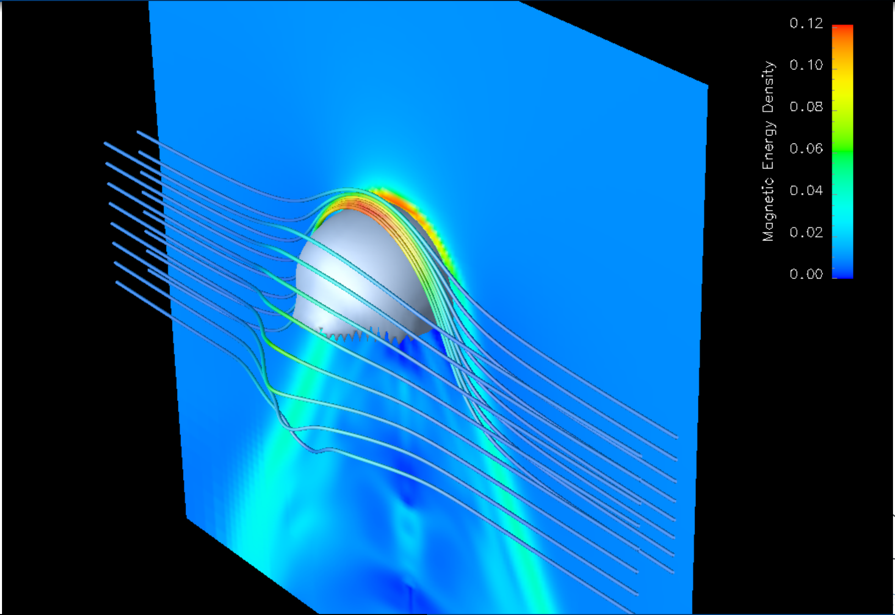
What is magnetic draping?

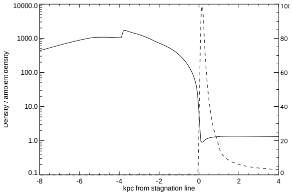
Interaction of an obstacle (Earth, star, galaxy, . . .) with a magnetized plasma

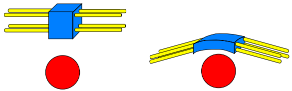


What is magnetic draping?

Interaction of an obstacle (Earth, star, galaxy, ...) with a magnetized plasma

- Is magnetic draping similar to ram pressure compression?
  $\rightarrow$ no, the density is not increased in magnetic draping as shown by ideal MHD simulations *(right)*
- Is magnetic flux still frozen into the plasma?
  yes, but plasma can also move along field lines while field lines get stuck at obstacle





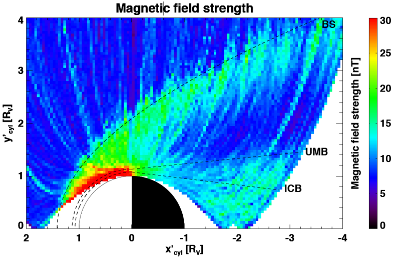
Applications of magnetic draping

- Solar-wind magnetic field is draped around the magnetopause of Earth: this protects Earth from cosmic rays during times of spin flip of the magnetic poles

- draping of solar-wind magnetic field at other moons and planets of the solar system: plasma physics

- hydrodynamic stability of underdense radio bubbles

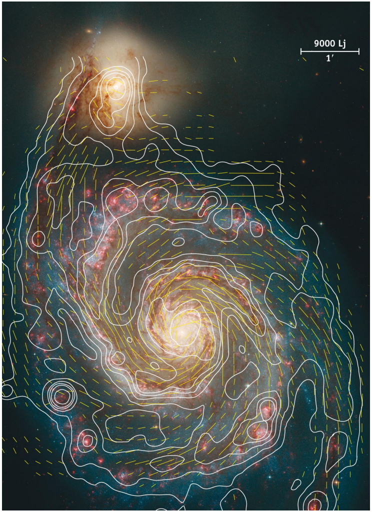
- sharpness (T_e, n_e) of cold fronts: without B, smoothed out by diffusion and heat conduction on $\gtrsim 10^8$ yrs



Guicking et al. (2010): magnetic draping around Venus

- magnetic draping on spiral galaxies in galaxy clusters: method for detecting the orientation of cluster magnetic fields

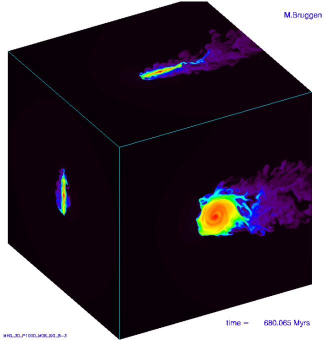
Polarized synchrotron emission in a field spiral: M51



MPIfR Bonn and Hubble Heritage Team

- grand design ‘whirlpool galaxy’ (M51): optical star light superposed on radio contours
- polarized radio intensity follows the spiral pattern and is strongest in between the spiral arms
- the polarization ‘B-vectors’ are aligned with the spiral structure

Ram-pressure stripping of cluster spirals



Brueggen (2008)

- 3D simulations show that the ram-pressure wind quickly strips the low-density gas in between spiral arms (Tonnesen & Bryan 2010)
- being flux-frozen into this dilute plasma, the large scale magnetic field will also be stripped

  $\rightarrow$ resulting radio emission should be unpolarized

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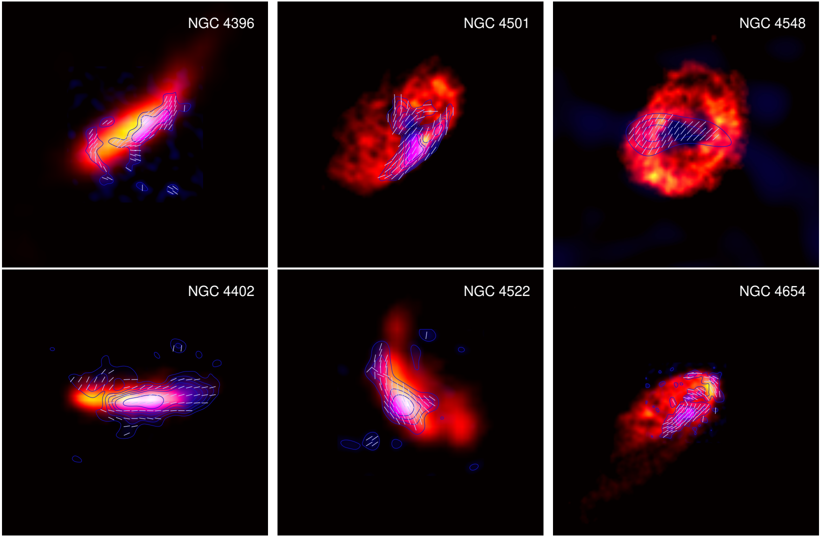
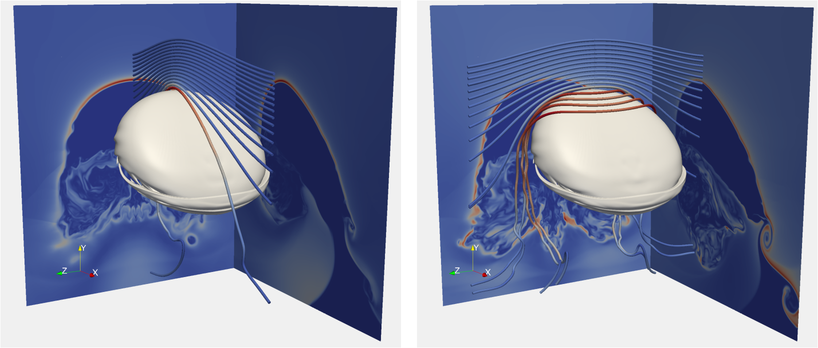
Polarized synchrotron ridges in Virgo spirals



Vollmer et al. (2007): 6 cm PI (contours) + B-vectors; Chung et al. (2009): HI (red)

Magnetic draping around a spiral galaxy



Athena simulations of spiral galaxies interacting with a uniform cluster magnetic field. There is a sheath of strong field draped around the leading edge (shown in red). CP & Dursi (2010, Nature Phys.)

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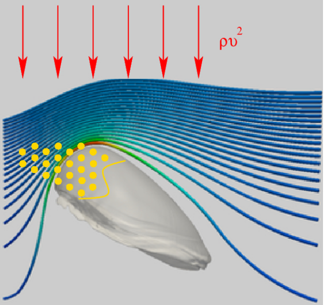
Magnetic draping around a spiral galaxy – physics



- the galactic ISM is pushed back by the ram pressure wind $\sim \rho v^2$
- the stars are largely unaffected and lead the gas
- the draping sheath is formed at the contact of galaxy/cluster wind
- as stars become SN, their remnants accelerate CRes that populate the field lines in the draping layer

- CRes are transported diffusively (along field lines) and advectively as field lines slip over the galaxy
- CRes emit radio synchrotron radiation in the draped region, tracing out the field lines there $\rightarrow$ coherent polarized emission at the galaxies' leading edges

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Modeling the electron population

- cooling time scale of synchrotron emitting electrons (CRe):

$$\nu_{\rm sync} = \frac{3eB}{2\pi\, m_{\rm e} c}\,\gamma^2 \simeq 5\text{ GHz}\left(\frac{B}{7\,\mu\text{G}}\right)\left(\frac{\gamma}{10^4}\right)^2,$$

$$\tau_{\rm sync} = \frac{E}{\dot{E}} = \frac{6\pi\, m_{\rm e} c}{\sigma_{\rm T} B^2 \gamma} = 5\times 10^7\text{ yr}\left(\frac{\gamma}{10^4}\right)^{-1}\left(\frac{B}{7\,\mu\text{G}}\right)^{-2}$$

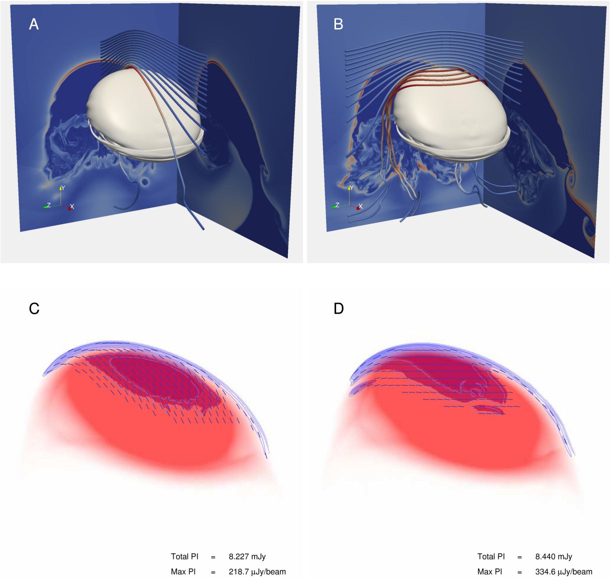
- typical SN rates imply a homogeneous CRe distribution
- FIR-radio correlation of Virgo spirals show comparable values to the solar circle: take MW CRe distribution inside our galaxies,

$$n_{\rm cre} = C_0\, e^{-(R-R_\odot)/h_R}\, e^{-|z|/h_z}$$

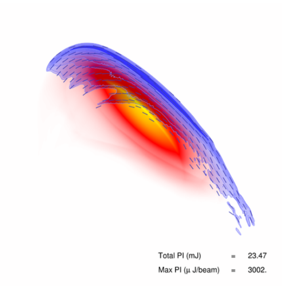
  with normalization $C_0 \simeq 10^{-4}\,\text{cm}^{-3}$ as well as scale heights $h_R \simeq 8$ kpc and $h_z \simeq 1$ kpc, normalized at Solar position
- truncate at contact of ISM-ICM, attach CRe distribution $\perp$ to contact surface with $h_\perp \simeq 150$ pc phase)

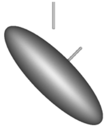
Magnetic draping and polarized synchrotron emission

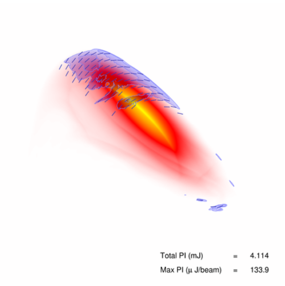
Synchrotron B-vectors reflect the upstream orientation of cluster magnetic fields



Simulated polarized synchrotron emission

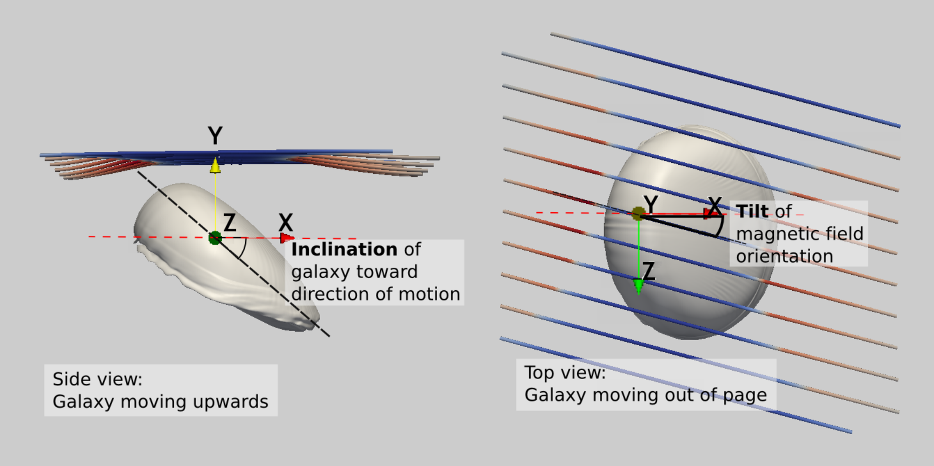




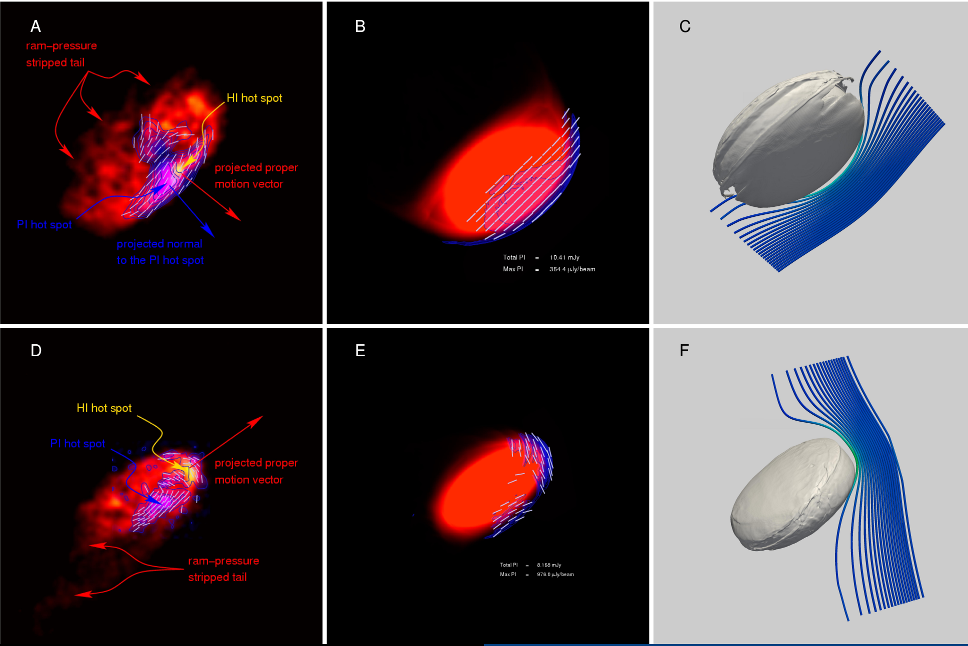


Movie of the simulated polarized synchrotron radiation viewed from various angles and with two field orientations.

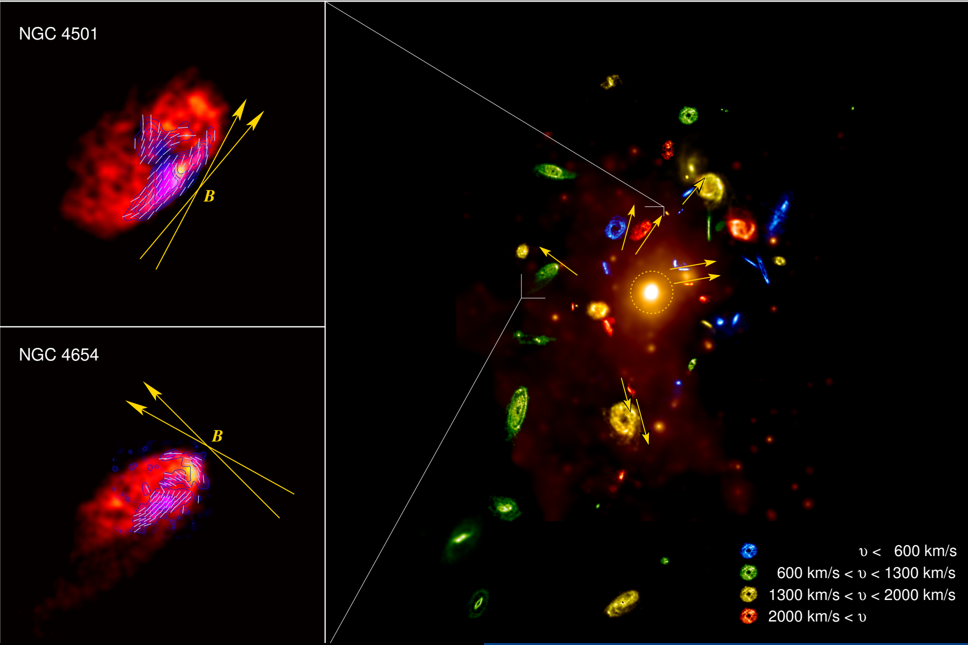
Varying galaxy inclination and magnetic tilt



Observations versus simulations



Mapping out the magnetic field in Virgo



Discussion of radial field geometry

- The alignment of the field in the plane of the sky is significantly more radial than expected from random chance. Considering the sum of deviations from radial alignment gives a chance coincidence of less than 1.7% ($\sim 2.2\,\sigma$).
- For the three nearby galaxy pairs in the data set, all have very similar field orientations.

$\rightarrow$ Which effect causes this field geometry?

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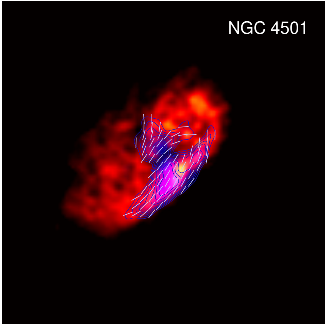
$\rightarrow$ Which effect causes this field geometry?

Perhaps this is a residual of radial infall of gas into the galaxy cluster

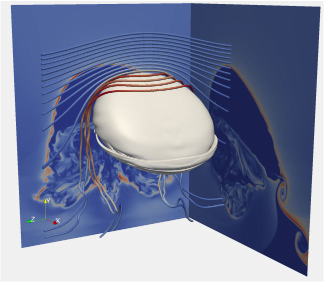
(Ruszkowski+2010)

Conclusions on magnetic draping around galaxies



- draping of cluster magnetic fields naturally explains polarization ridges at Virgo spirals

Conclusions on magnetic draping around galaxies



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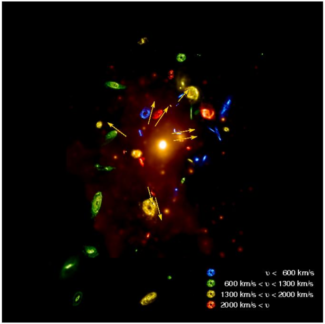

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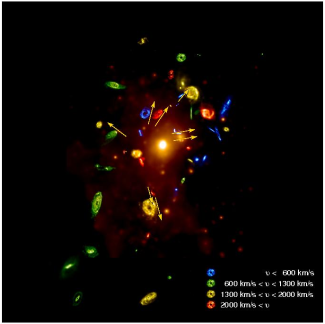

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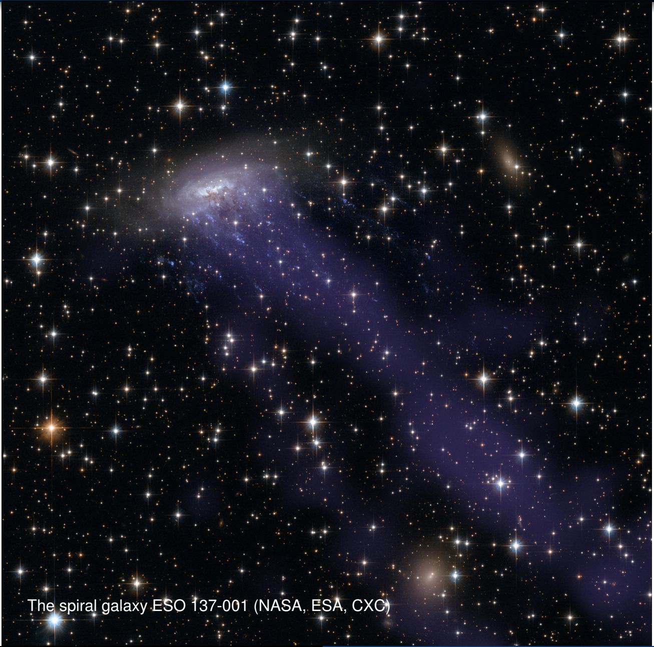
- this finding has consequences for thermal conduction across clusters if there is a residual radial field component
- important implications for thermal cluster history $\rightarrow$ galaxy cluster cosmology

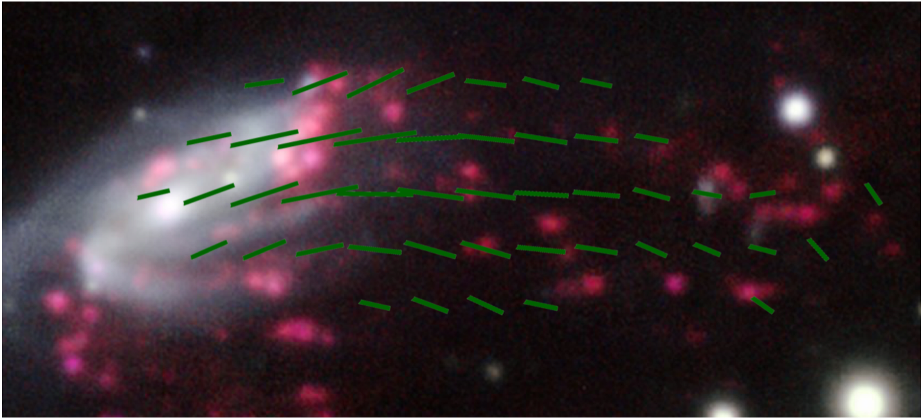
Jellyfish galaxies in clusters



The spiral galaxy ESO 137-001 (NASA, ESA, CXC)

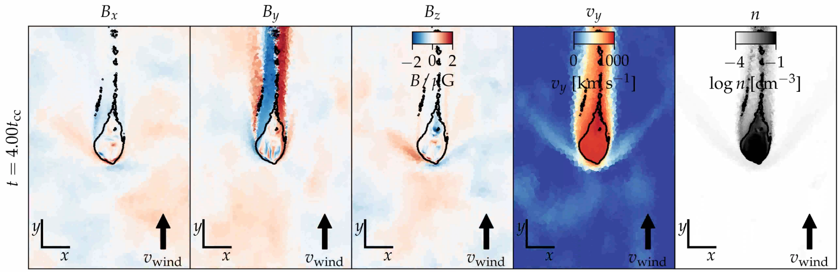
Protective layer: magnetic field of a jellyfish galaxy

Observations of aligned *B* polarization vectors along the tail of galaxy JO206



Müller+ (2021, Nature Astronomy)

Simulating a jellyfish galaxy



Müller+ (2021, Nature Astronomy), Sparre, CP+ (2020)

- A jellyfish galaxy experiences ram-pressure stripping as a result of its fast motion in the intracluster medium.
- The turbulent wind magnetic field is wrapped around the galaxy and stretched in the wake by shear motions as well as cooling of thermally unstable mixed wind material.
- The magnetic field facilitates the formation of long gaseous filaments.

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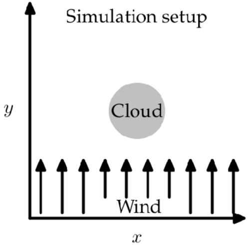
Interaction of a cold cloud with a hot wind

Magnetic tension and pressure modify the dynamics of the interaction

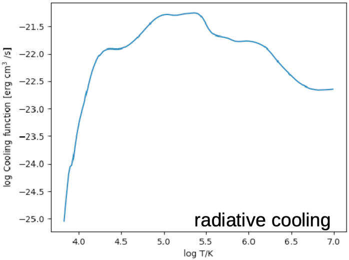


Sparre, CP, Ehlert (2020)

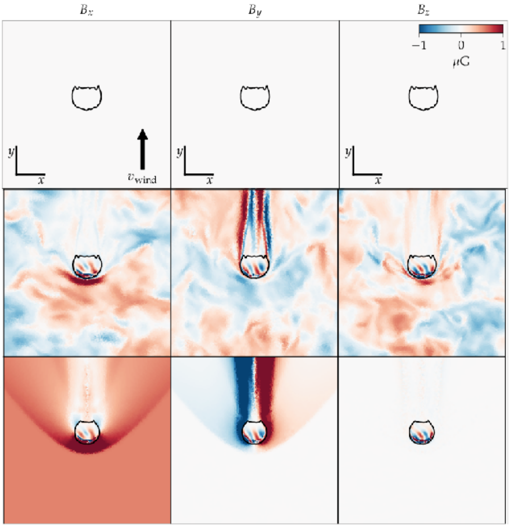
$T_{\text{cloud}} = 10^4$ K
$n_{\text{cloud}} = 0.1$ cm^{-3}
$R = 25$ pc

$T_{\text{wind}} = 5.0 \times 10^6$ K
$n_{\text{wind}} = 2 \times 10^{-4}$ cm^{-3}
$M = 1.5$

$\beta = 10$ or ∞ ($\beta \equiv P_{\text{th}}/P_B$)



Magnetic field configurations



no magnetic field

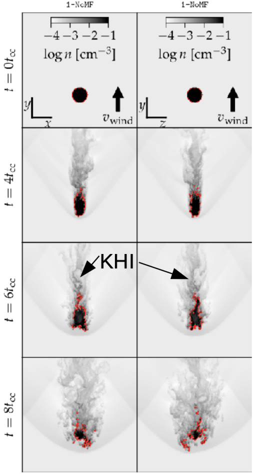
turbulent ***B***

uniform ***B***

Sparre, CP, Ehlert (2020)

Magnetic field alters dynamics of cloud shattering

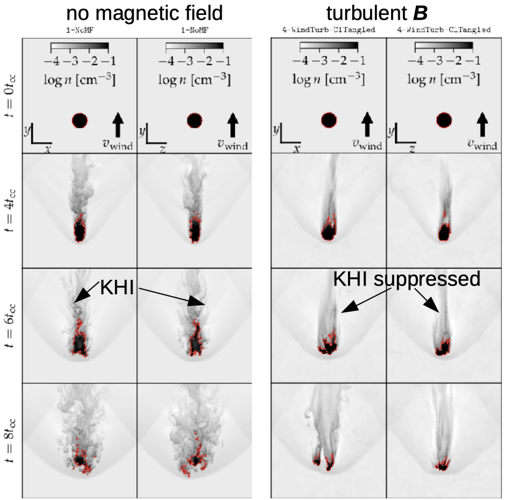
no magnetic field



Sparre, CP, Ehlert (2020)

KHI = Kelvin Helmholtz instability

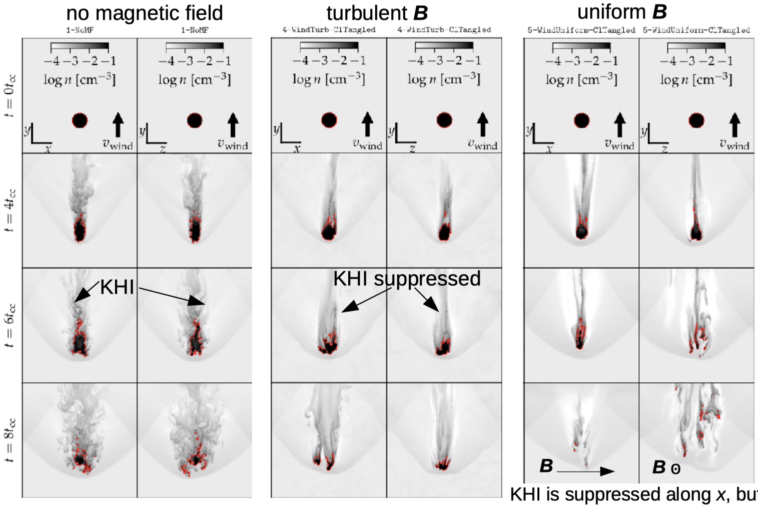
Magnetic field alters dynamics of cloud shattering



Sparre, CP, Ehlert (2020)

A magnetic field suppresses the Kelvin-Helmholtz instability (KHI) in the wake of the cloud

Magnetic field alters dynamics of cloud shattering

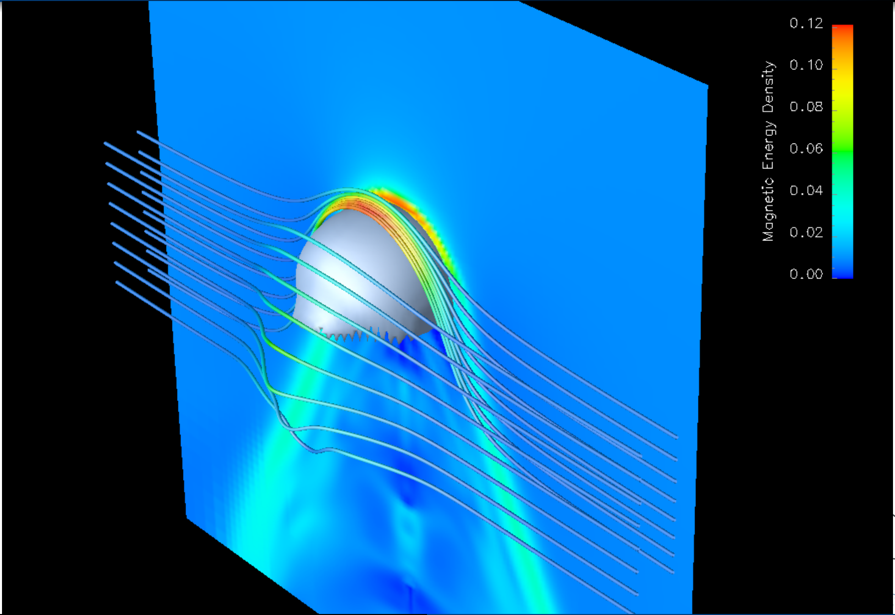


KHI is suppressed along x, but not z

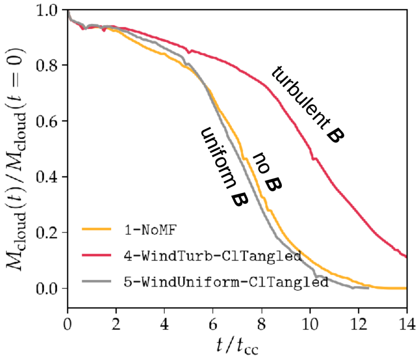
Sparre, CP, Ehlert (2020)

A magnetic draping layer protects against instabilities

Magnetic pressure and tension forces alter the dynamics of the interaction

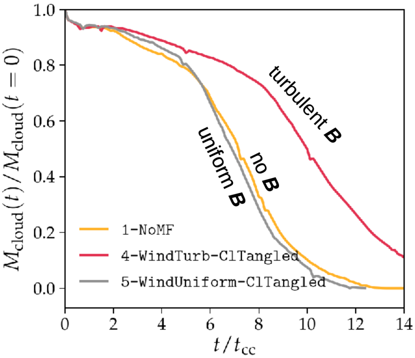


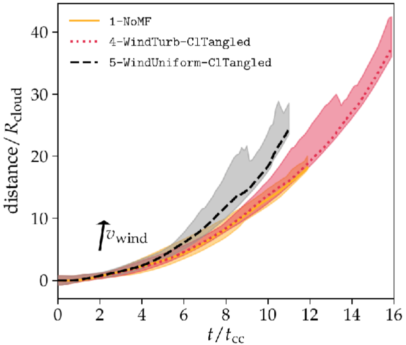
A turbulent *B* field extends cloud's lifetime



$$t_{\rm cc} \equiv \frac{R_{\rm cloud}}{v_{\rm wind}} \sqrt{\frac{\rho_{\rm cloud}}{\rho_{\rm wind}}}.$$

A uniform $\boldsymbol{B}$ field initially accelerates cloud more





- KHI instability shatters a **small cloud** into small pieces that mix with and dissolve into the hot wind
- magnetic field protects against instabilities and increases survival time by 30%, but does not halter the cloud's destruction

(Sparre, CP, Ehlert 2020)

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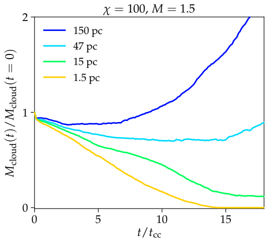
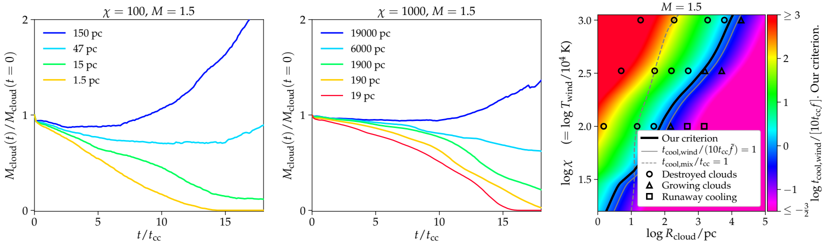
The growth regime



- ram-pressure stripped gas from a **large cloud** mixes with the hot wind to intermediate temperatures
- thermal instability causes further cooling and **net accretion of hot gas to the cold tail** (Armillotta+ 2017, Gronke & Oh 2018, 2019, Li+ 2019, Sparre+ 2020, Kanjilal+ 2020)
- **momentum transfer from hot wind to cooling accreting material** implies formation of long gaseous tail of the jellyfish galaxy!

AIP



- **hot-wind cooling time sets transition radius** and not the mixed-phase cooling time ⇒ cloud growth criterion (Sparre+ 2020):

$$\frac{t_{\rm cool,wind}}{t_{\rm cc}} < 10 f(M, R_{\rm cloud}, n_{\rm wind}, v_{\rm wind})$$

Conclusions on magnetic fields dynamics

Interaction of a cold cloud with a hot wind:

- magnetic field provides tension on a moving object and decelerates it
- magnetic field protects against instabilities and increases the survival time
- destruction regime: transport of dense gas to several kpcs hard to explain because cloud shatters and dissolves in the wind
- growth regime: momentum transfer from hot wind to the cooling and accreting material implies formation of long gaseous tail of the jellyfish galaxy

Sound Waves – 1

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- The dispersion relation for sound waves is derived by combining the first four equations (for mass and momentum). We perturb the fluid, split the dynamical quantities into background values (that do not depend on time) and small perturbations: $\rho = \rho_0 + \delta\rho$, $\boldsymbol{v} = \delta\boldsymbol{v}$ (note: $\boldsymbol{v}_0 = \mathbf{0}$), $P = P_0 + \delta P$. The constraint equation for the background reads

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- The perturbed mass and momentum equations are to first order (after using Eq. 3):

$$\begin{aligned} \frac{\partial \delta\rho}{\partial t} + \boldsymbol{\nabla} \cdot (\rho_0 \delta\boldsymbol{v} + \cancel{\delta\rho \boldsymbol{v}_0}) &= 0, \\ \frac{\partial}{\partial t}(\rho_0 \boldsymbol{v}) + \boldsymbol{\nabla} \cdot (\cancel{\delta\rho \boldsymbol{v}_0 \boldsymbol{v}_0^{\mathrm{T}}} + \cancel{\rho_0 \delta\boldsymbol{v}\boldsymbol{v}_0^{\mathrm{T}}} + \bar{\mathbf{1}}\delta P) &= \mathbf{0}. \end{aligned}$$

AIP

Sound Waves – 2

- We recap the perturbed mass and momentum equations to first order:

$$\begin{aligned} \frac{\partial \delta\rho}{\partial t} + \boldsymbol{\nabla}\cdot(\rho_0 \delta \boldsymbol{v}) &= 0 && \Big|\partial_t(\,\cdot\,) \\ \frac{\partial}{\partial t}(\rho_0 \delta \boldsymbol{v}) + \boldsymbol{\nabla}\cdot(\bar{\boldsymbol{1}}\delta P) &= \boldsymbol{0} && \Big|\boldsymbol{\nabla}\cdot(\,\cdot\,) \end{aligned}$$

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- Using the Fourier transformation convention

$$\delta\rho(\boldsymbol{x}, t) = \int \frac{\mathrm{d}^3 k}{(2\pi)^3}\, \delta\hat{\rho}(\boldsymbol{k}, \omega)\, \mathrm{e}^{-\mathrm{i}\omega t + \mathrm{i}\boldsymbol{k}\cdot\boldsymbol{x}},$$

we decompose Eq. (4) into plane waves to obtain the dispersion relation for sound waves

$$\begin{aligned} &-\omega^2 \delta\hat{\rho} + k^2 \delta\hat{P} = 0, \\ &\omega^2 = \frac{\delta\hat{P}}{\delta\hat{\rho}} k^2 \qquad \Longrightarrow \qquad \omega = \pm\sqrt{\frac{\delta\hat{P}}{\delta\hat{\rho}}}\, k, \end{aligned} \tag{5}$$

where only the positive root has a physical meaning and $k = |\boldsymbol{k}|$.

AIP

Sound Waves – 3

- We recap the dispersion relation for sound waves

$$\omega = \sqrt{\frac{\delta \hat{P}}{\delta \hat{\rho}}} k.$$

- Adopting the equation of state $P = P(\rho, s)$, we can relate Fourier to configuration space quantities:

$$\frac{\delta \hat{P}}{\delta \hat{\rho}} = \left.\frac{\partial P}{\partial \rho}\right|_s .$$

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- Sound waves are longitudinal perturbations of the pressure that propagate with c_s and have $v_\mathrm{gr} = v_\mathrm{ph}$.

AIP

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  i.e., the sound wave is a degenerate solution and accounts for four eigenvalues.
- Perturbing the entropy equation yields to first order in Fourier space

$$\mathrm{i}\omega \delta \hat{s} - \delta \hat{\boldsymbol{v}} \cdot \boldsymbol{\nabla} s_0 = 0$$

$$\Longrightarrow \quad \omega = 0 \quad \text{and} \quad s_0 = \text{const}.$$

  The entropy mode is a compressible zero-frequency mode with eigenfunctions $\delta P = \delta \boldsymbol{v} = \delta \boldsymbol{B} = 0$ and $\delta T / T = -\delta \rho / \rho = 2 \delta s / 5$.

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Magneto-hydrodynamic waves – 1

- Add magnetic fields to the system in the ideal MHD approximation. How many equations and eigenvalues do you have now?

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$$\frac{\partial s}{\partial t} + \boldsymbol{v} \cdot \boldsymbol{\nabla} s = 0,$$

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where $\rho = \rho(\boldsymbol{x})$, $P = P(\boldsymbol{x})$, $\boldsymbol{v} = \boldsymbol{v}(\boldsymbol{x})$, $\boldsymbol{j} = \boldsymbol{j}(\boldsymbol{x})$, $s = s(\boldsymbol{x})$, and $\boldsymbol{B} = \boldsymbol{B}(\boldsymbol{x})$ are the density, pressure, velocity, electric current, entropy, and magnetic field.

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- There are a total of 8 equations: 5 hydrodynamics equations plus 3 components of the induction equation. However, the constraint equation, $\boldsymbol{\nabla} \cdot \boldsymbol{B} = 0$, reduces the dimensionality to seven degrees of freedom.

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In a magnetized plasma, there are seven different wave modes:

- The 2 polarization states of shear Alfvén modes are polarized transverse to the unperturbed magnetic field; the restoring force of these transverse magnetic perturbations is the tension force; the group velocity is along the mean magnetic field with $v_{\mathrm{ph}} = v_{\mathrm{A}} = B/\sqrt{4\pi\,\rho}$; Alfvén modes are incompressible, i.e., $\delta\rho = 0$, and $\delta v_{\mathrm{A}} \propto \delta B$.
- The 2 polarization states of fast magnetosonic modes are equivalent to sound waves in high-β plasmas, where $\beta = P_{\mathrm{th}}/P_B = 2c_{\mathrm{s}}/v_{\mathrm{A}}$; the restoring force of these longitudinal (compressible) magnetic perturbations is the magnetic pressure force; fast modes do not interact with Alfvén modes.
- There are 2 polarization states of slow magnetosonic modes. In a high-β plasma, they represent a compressible Alfvén mode.
- The entropy mode: zero-frequency wave with fluctuations in n and T such that the thermal pressure $P = \mathrm{const}$.

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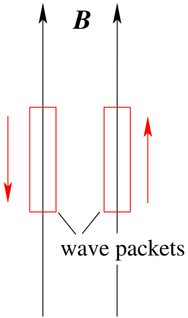
Alfvénic turbulence – the picture



Interacting Alfvén wave packets.

- Alfvénic turbulence is incompressible:

$$\frac{\delta v_A}{v_A} = \frac{\delta B}{B}$$

- What happens when the two wave packets are interacting?
- The down-going packet causes field line wandering such that the upward going packet is broken apart after a distance $L_{||}(\lambda)$.
- In other words, the travel time across this wave package in the direction of the mean magnetic field equals the eddy turn-over time in the perpendicular direction.
- This gives rise to the critical balance condition of Alfvénic turbulence

(Goldreich & Shridhar 95, 97, Lithwick & Goldreich 01)

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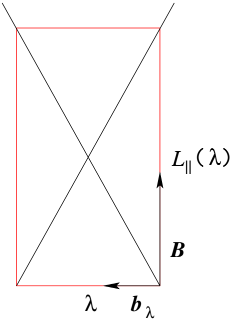
Alfvénic turbulence - the scaling



Geometrical interpretation of the "critical balance" condition.

- The critical balance condition reads:
$$L_{\|} = \frac{\lambda B}{b_\lambda}$$

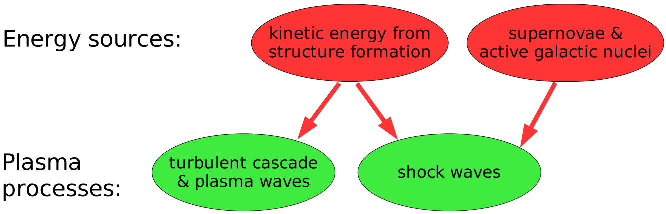
- In Kolmogorov turbulence, the energy flux of the fluctuating field at scale λ is constant, $b_\lambda^2/t_\lambda = \text{const}$. Equating the wave travel time along $\boldsymbol{B}$, $t_{\|}$, with the eddy turn-over time in the perpendicular direction, t_λ, we get
$$t_{\|} = \frac{L_{\|}}{v_{\mathrm{A}}} = \frac{\lambda\, B}{v_{\mathrm{A}}\, b_\lambda} = t_\lambda \propto b_\lambda^2,$$

- Because $B \propto v_{\mathrm{A}} = \text{const.}$ in incompressible turbulence, we obtain the scaling of Alfvénic turbulence:
$$b_\lambda \propto \lambda^{1/3} \quad \text{or} \quad L_{\|} \propto \lambda^{2/3}\, L_{\mathrm{MHD}}^{1/3}$$

$\Rightarrow$ the smaller the scale λ, the more anisotropic is the turbulent scaling and the more elongated are the eddies ($L_{\|}/\lambda \propto \lambda^{-1/3}$) whose long axis is aligned with the local $\langle B\rangle$!

Multi messenger approach for non-thermal processes

Relativistic populations and radiative processes in clusters:



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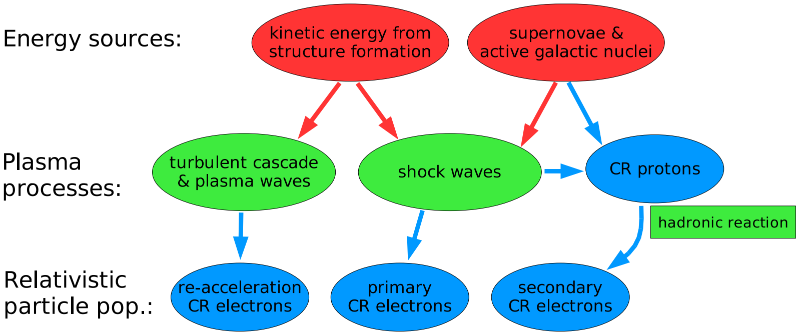
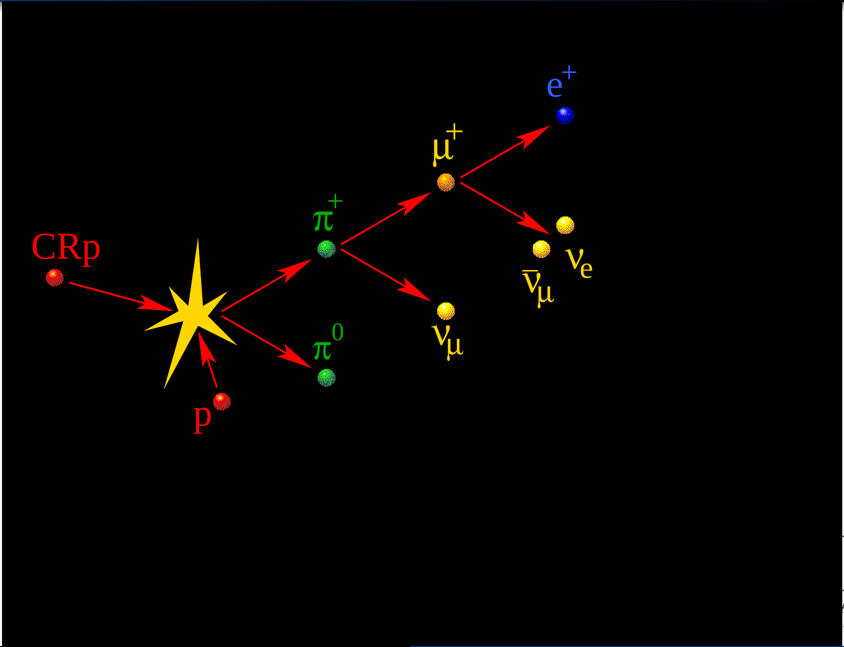


Hadronic cosmic ray proton interaction



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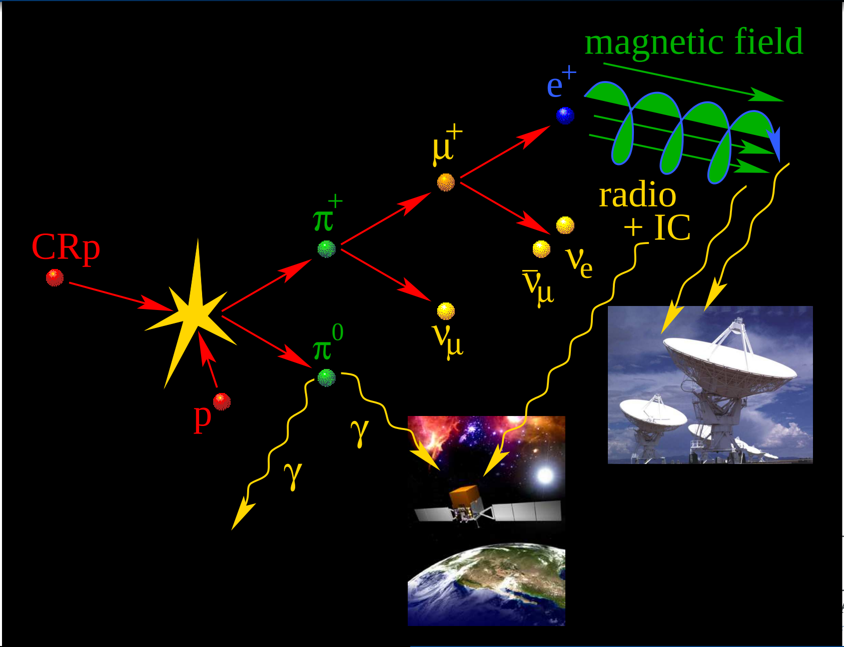

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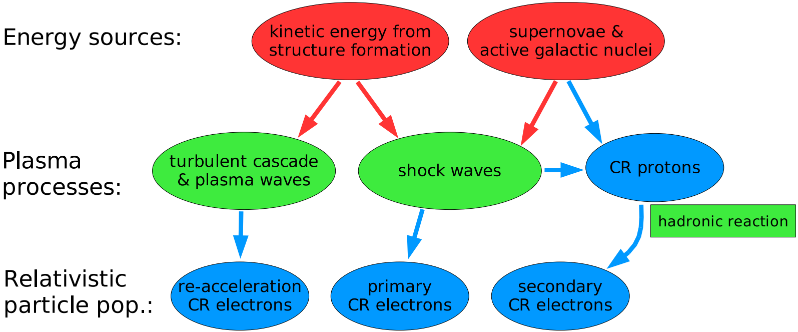

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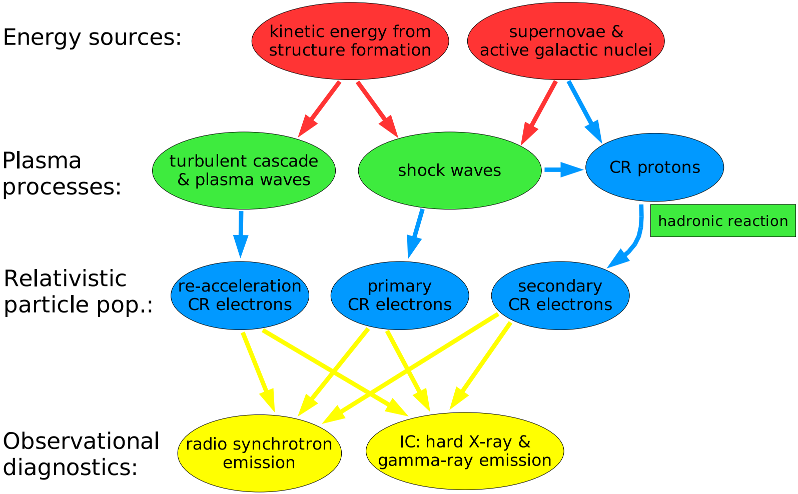

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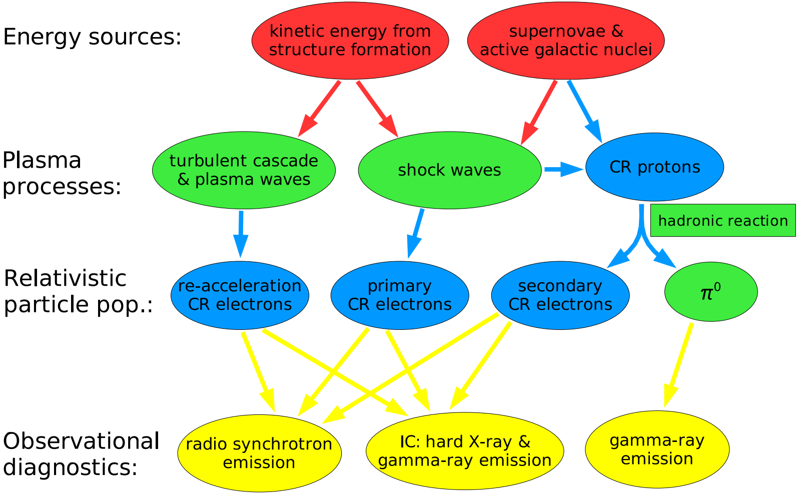
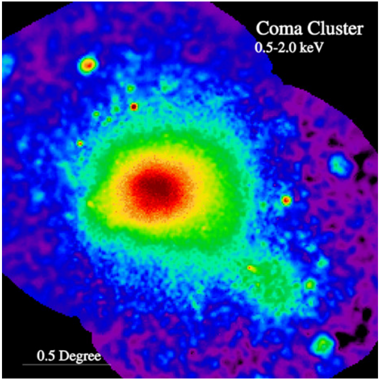


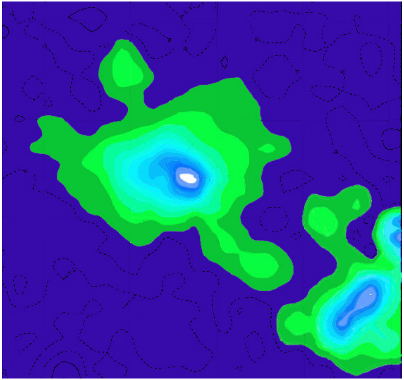
Magnetic fields in galaxy clusters

Giant radio halo in the Coma galaxy cluster



thermal X-ray emission

(Snowden/MPE/ROSAT)



radio synchrotron emission

(Deiss/Effelsberg)

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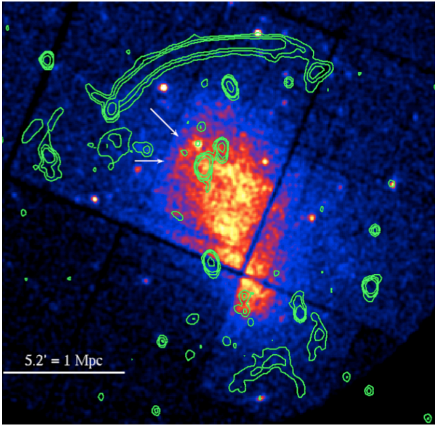
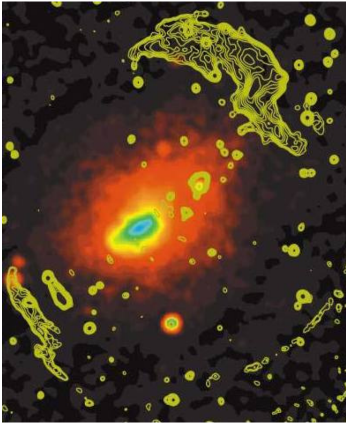
Magnetic fields in galaxy clusters

Radio shock: double relic sources





CIZA J2242.8+5301 (“sausage relic”)

(X-ray: XMM; radio: WSRT; Ogrean+ (2013))

Abell 3667

(radio: Johnston-Hollitt. X-ray: ROSAT/PSPC.)

Cosmic rays and magnetic fields in clusters – 1

The energy loss rate of a relativistic electron of energy $E_\mathrm{e} = \gamma m_\mathrm{e} c^2$ is given by

$$\dot{E}_\mathrm{e} = \frac{\sigma_\mathrm{T} c}{6\pi} \left(B_\mathrm{cmb}^2 + B^2\right)\gamma^2,$$

where σ_T is the Thomson cross section, m_e is the electron rest mass, c is the light speed, γ is the Lorentz factor, B is the magnetic field strength and $B_\mathrm{cmb} \simeq 3.2\mu\mathrm{G}$ is the equivalent field of the cosmic microwave background (cmb) energy density today.

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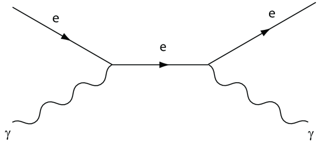
- The first term in the parenthesis $\propto B_\mathrm{cmb}^2$ describes energy loss due to inverse Compton (IC) scattering off of CMB photons, while the second term in the parenthesis $\propto B^2$ describes energy loss due to synchrotron emission. The structural similarity of the formulae is not a coincidence but caused by the same Feynman diagram of the scattering process: while IC emission evokes real photons, synchrotron emission borrows a virtual photon from the magnetic field.



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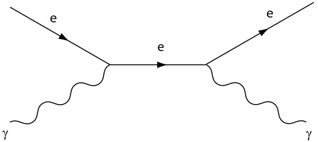

- The cooling time $t_\mathrm{cool} = E_\mathrm{e}/\dot{E}_\mathrm{e}$ of a relativistic electron is given by

$$t_\mathrm{cool} = \frac{E_\mathrm{e}}{\dot{E}_\mathrm{e}} = \frac{6\pi m_\mathrm{e} c}{\sigma_\mathrm{T}\ (B_\mathrm{cmb}^2 + B^2)\ \gamma} \approx 200\,\mathrm{Myr},$$

for $B = 1\ \mu\mathrm{G}$ and $\gamma = 10^4$.

Cosmic rays and magnetic fields in clusters – 2

- Recall the cooling time $t_{\mathrm{cool}} = E_{\mathrm{e}}/\dot{E}_{\mathrm{e}}$ of a relativistic electron,

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  for $B = 1\ \mu\mathrm{G}$ and $\gamma = 10^4$.
- The synchrotron frequency in the monochromatic approximation is given by

$$\nu_{\mathrm{synch}} = \frac{3eB}{2\pi\, m_{\mathrm{e}} c}\,\gamma^2 \simeq 1\ \mathrm{GHz}\,\frac{B}{\mu\mathrm{G}}\left(\frac{\gamma}{10^4}\right)^2 .$$

AIP

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$$\nu_\mathrm{synch} \quad = \quad \frac{3eB}{2\pi\, m_\mathrm{e} c}\,\gamma^2 \simeq 1\ \mathrm{GHz}\,\frac{B}{\mu\mathrm{G}}\,\left(\frac{\gamma}{10^4}\right)^2.$$

- Combining both equations by eliminating the Lorentz factor γ yields the cooling time of electrons that emit at frequency ν_syn,

$$t_\mathrm{cool} = \frac{\sqrt{54\pi m_e c\, eB\nu_\mathrm{syn}^{-1}}}{\sigma_\mathrm{T}\,(B_\mathrm{cmb}^2 + B^2)} \lesssim 190\,\left(\frac{\nu_\mathrm{syn}}{1.4\,\mathrm{GHz}}\right)^{-1/2}\ \mathrm{Myr},$$

The cooling time t_cool is then bound from above and attains its maximum cooling time at $B = B_\mathrm{cmb,0}/\sqrt{3} \simeq 1.8\,\mu\mathrm{G}$, independent of the magnetic field.

Giant radio relics in galaxy clusters

- Recall the cooling time of electrons that emit at frequency ν_syn,

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Giant radio relics in galaxy clusters

- Recall the cooling time of electrons that emit at frequency $\nu_{\rm syn}$,

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- We assume that the relativistic electrons are accelerated at a strong cluster merger shock and are advected with the post-shock gas. Assuming that the incoming gas had a pre-shock velocity of $v_1 = 1200$ km/s in the shock frame, we get a post-shock velocity

$$v_2 = \frac{\rho_1}{\rho_2}\, v_1 = \frac{(\gamma-1)\mathcal{M}_1^2 + 2}{(\gamma+1)\mathcal{M}_1^2}\, v_1 = 400 \left(\frac{v_1}{1200\,{\rm km\,s}^{-1}}\right) {\rm km\,s}^{-1}$$

for a shock Mach number of $\mathcal{M}_1 = 3$.

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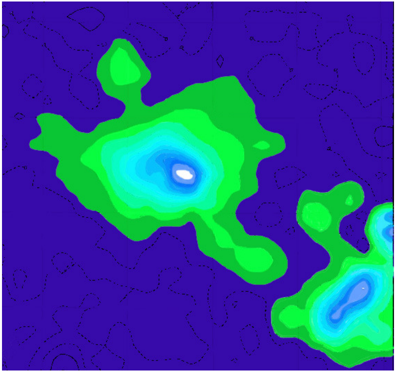
- This implies a maximum cooling length $L_{\mathrm{cool,\,max}} = v_2 t_{\mathrm{cool,\,max}} = 80$ kpc, which decreases for larger magnetic field strengths to assume a value of

$$L_{\mathrm{cool}} = v_2 t_{\mathrm{cool}} = \frac{v_2\sqrt{54\pi m_e c\, eB\nu_{\mathrm{syn}}^{-1}}}{\sigma_{\mathrm{T}}\,(B_{\mathrm{cmb}}^2 + B^2)} \approx 30 \left(\frac{\nu_{\mathrm{syn}}}{1.4\,\mathrm{GHz}}\right)^{-1/2} \mathrm{kpc}$$

  for 5 μG. Typical radial extends of radio relics are of that size. Hence, one can use the relic geometry to estimate magnetic field strengths (projection effects!).

Giant radio halos in galaxy clusters

- The maximum cooling length is $L_{\text{cool, max}} = v_2 t_{\text{cool, max}} = 80$ kpc at 1.4 GHz.
- The spatial extend of giant radio halos is ~ 2 Mpc and the emission is not polarized.



radio synchrotron emission (Deiss/Effelsberg)

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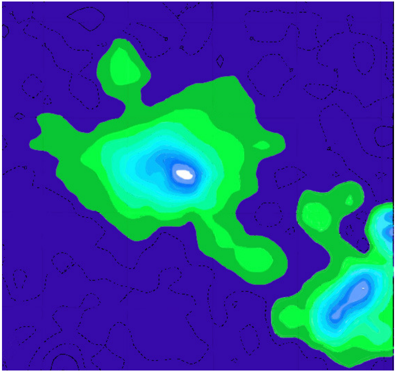

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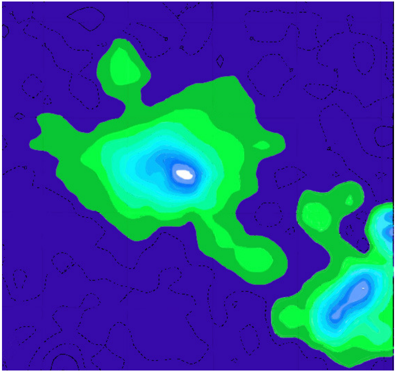
- Because $L_{\text{halo}} \approx 25 L_{\text{cool, max}}$ there must be a volume filling acceleration process of relativistic electrons.



radio synchrotron emission (Deiss/Effelsberg)

AIP

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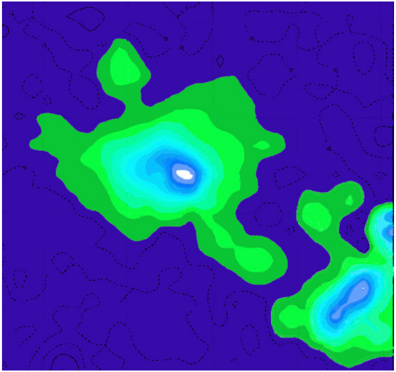

radio synchrotron emission (Deiss/Effelsberg)

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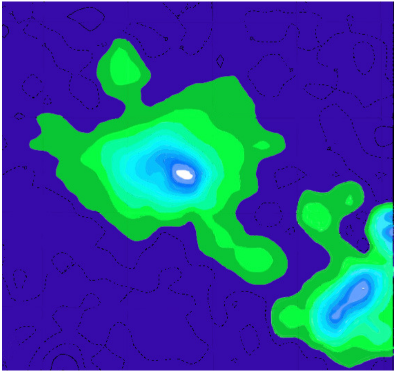

radio synchrotron emission (Deiss/Effelsberg)

- Hadronic model: relativistic protons interact hadronically with gas protons and produce secondary electrons/positrons that emit in the radio.
- Reacceleration model: fossil or secondary electrons interact with turbulent magneto-hydrodynamic waves and experience Fermi-II acceleration that makes them visible at radio wave lengths.

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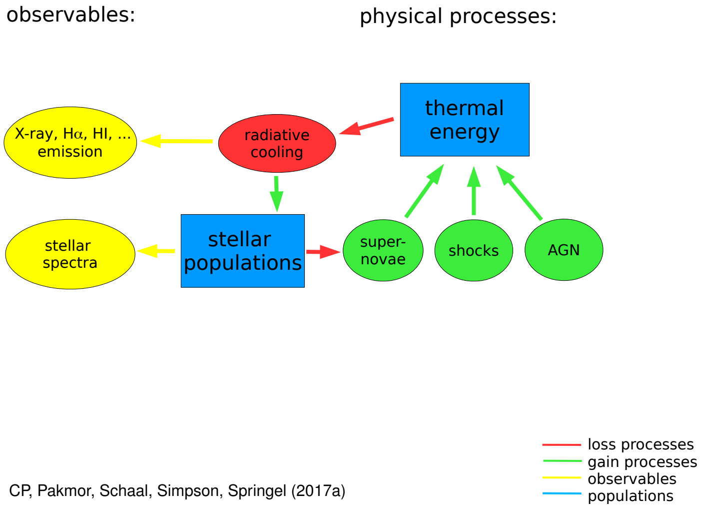
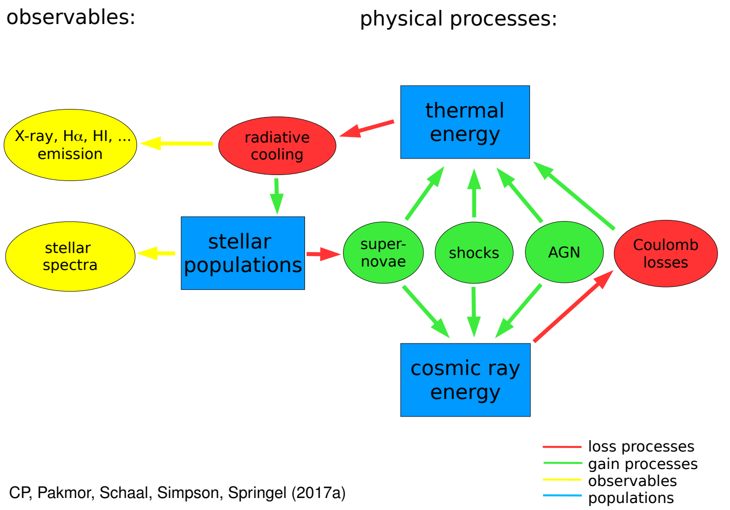


CP, Pakmor, Schaal, Simpson, Springel (2017a)

Simulations with cosmic ray physics



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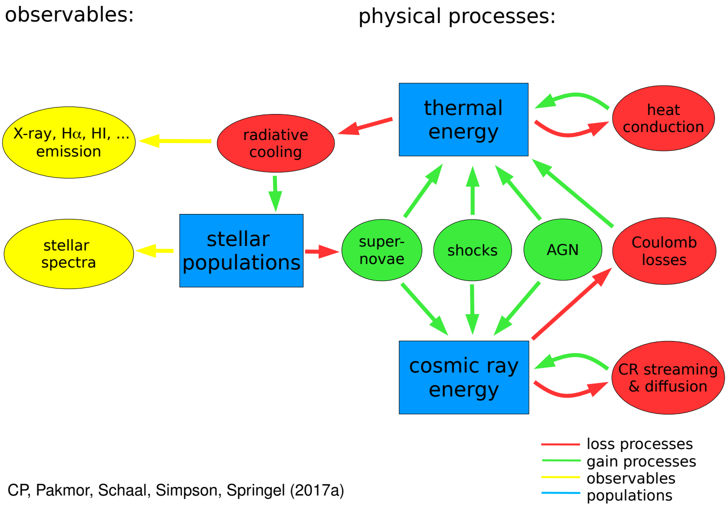

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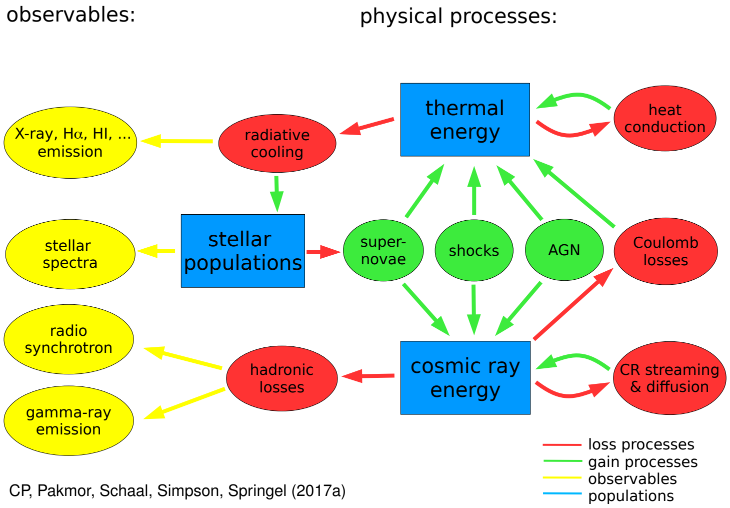
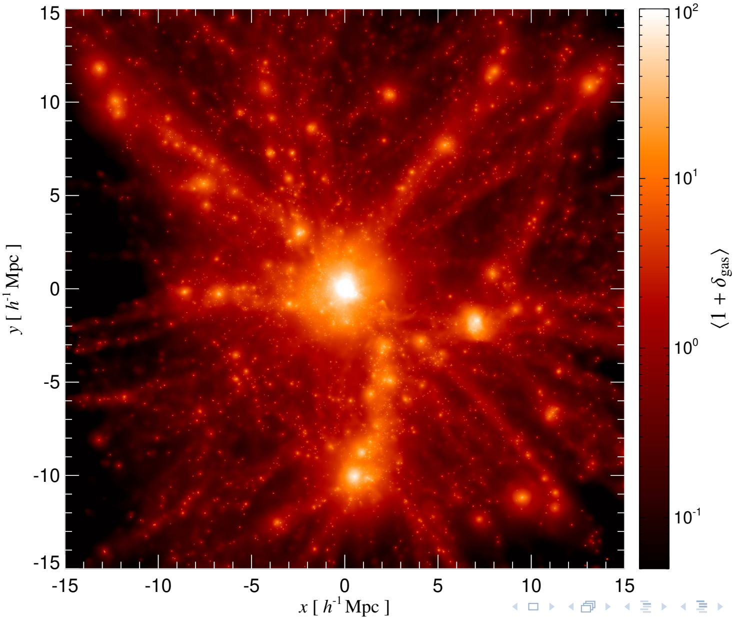


CP, Pakmor, Schaal, Simpson, Springel (2017a)

Cluster simulation: gas density



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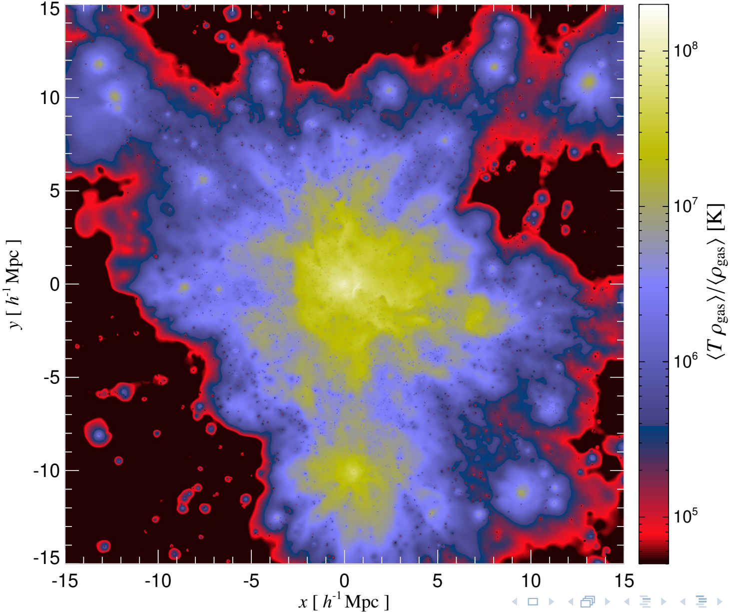
Mass weighted temperature



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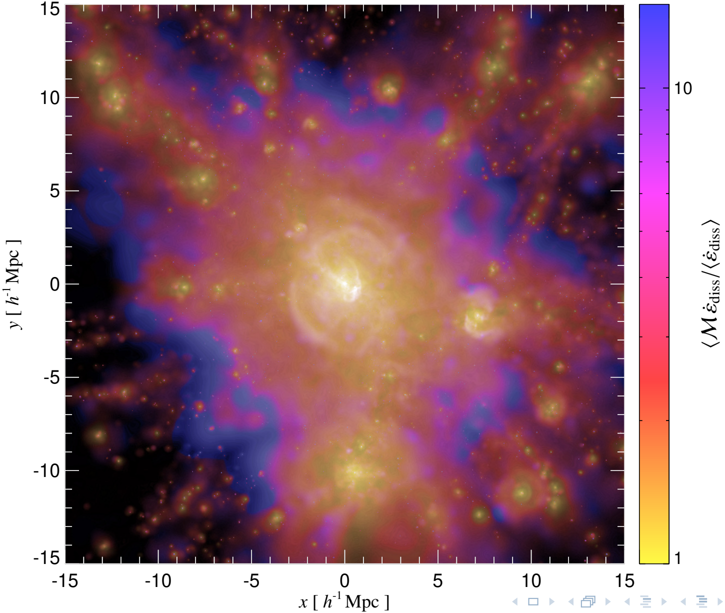
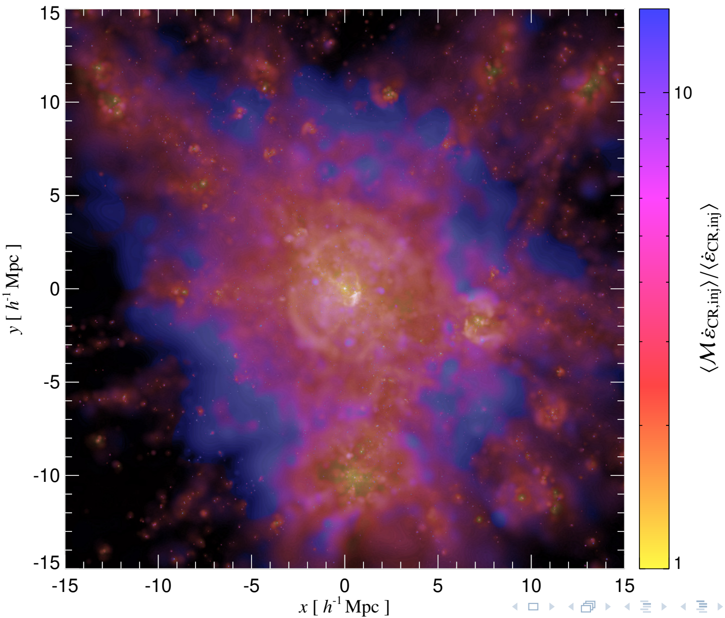
Mach number distribution weighted by $\varepsilon_{\mathrm{diss}}$



Mach number distribution weighted by $\varepsilon_{\mathrm{CR,inj}}$



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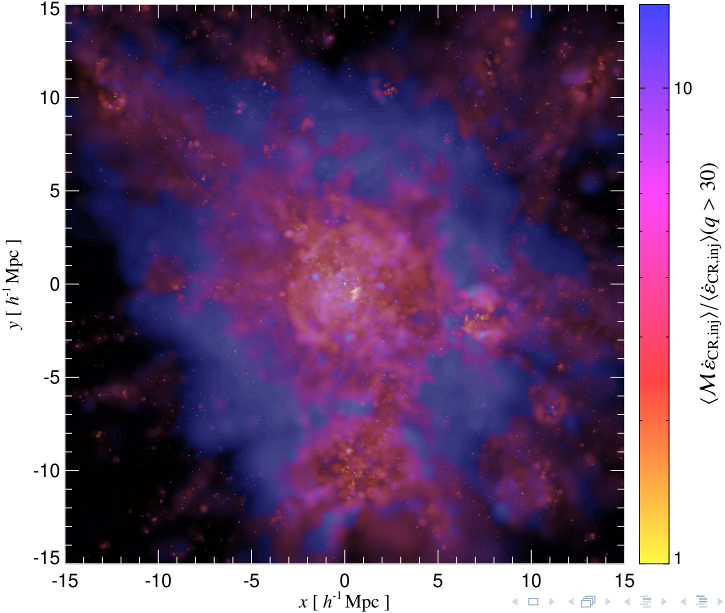
Mach number distribution weighted by $\varepsilon_{\mathrm{CR,inj}}(q > 30)$



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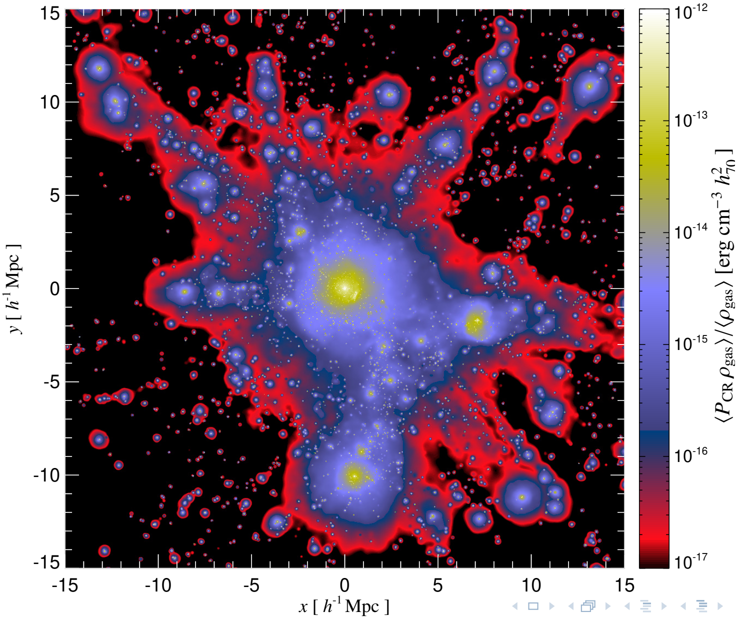
CR pressure $P_{\rm CR}$



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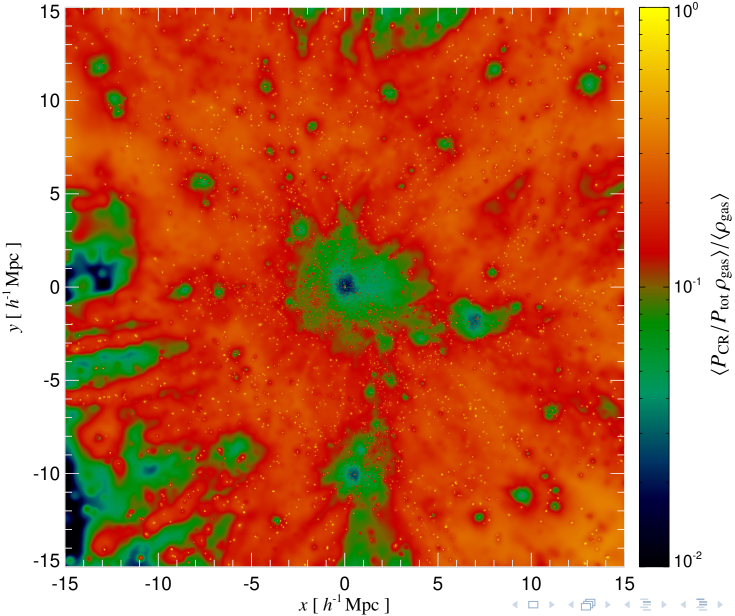
Relative CR pressure P_{CR}/P_{total}



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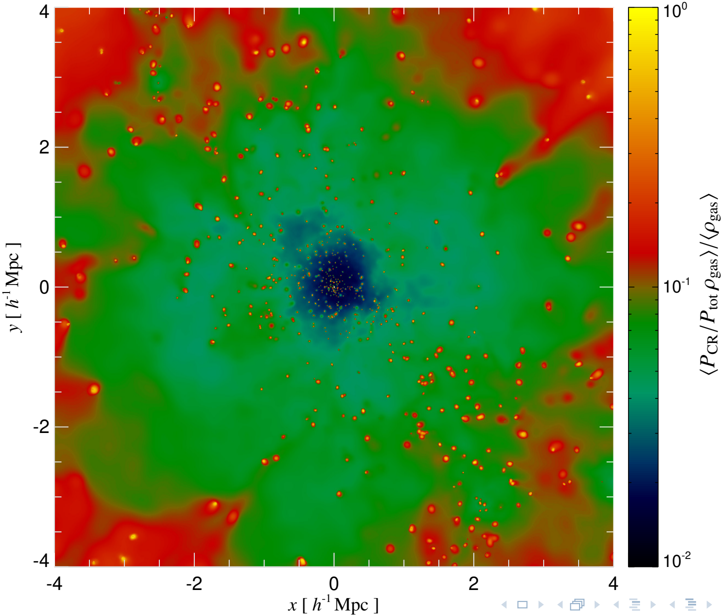
Relative CR pressure $P_{\mathrm{CR}}/P_{\mathrm{total}}$



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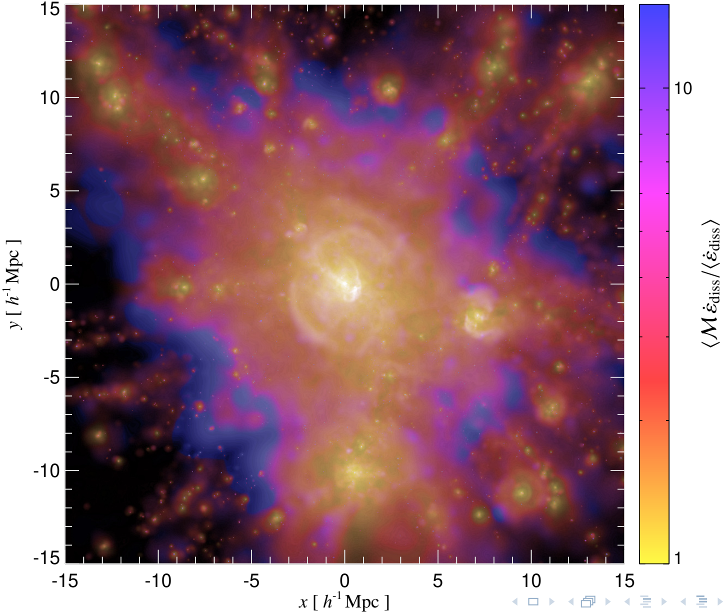
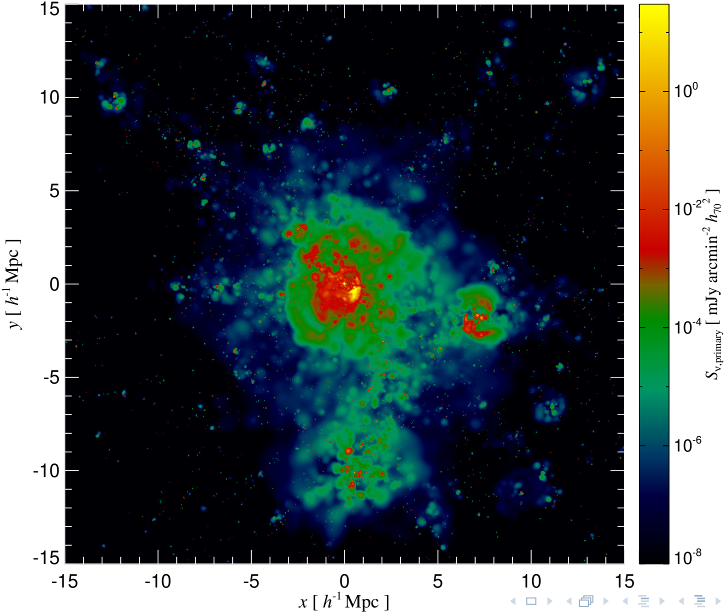


15
10
5
0
-5
-10
-15
y [h^{-1} Mpc]
-15
-10
-5
0
5
10
15
x [h^{-1} Mpc]
10
1
$\langle \mathcal{M} \dot{\varepsilon}_{\rm diss} / \langle \dot{\varepsilon}_{\rm diss} \rangle$

Radio web: primary CRe (1.4 GHz)



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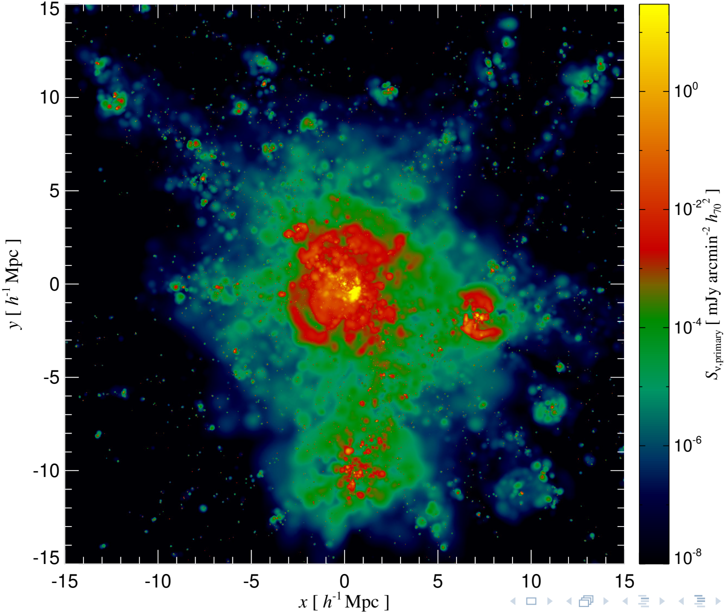
Radio web: primary CRe (150 MHz)



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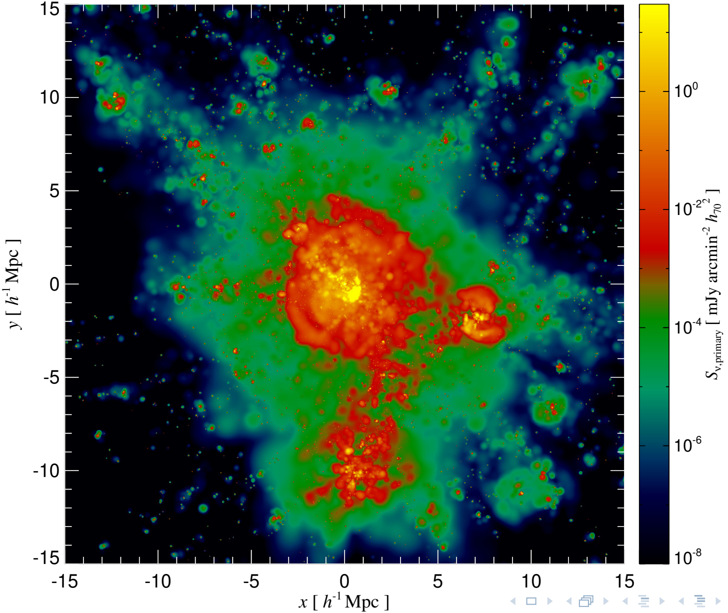
Radio web: primary CRe (15 MHz)



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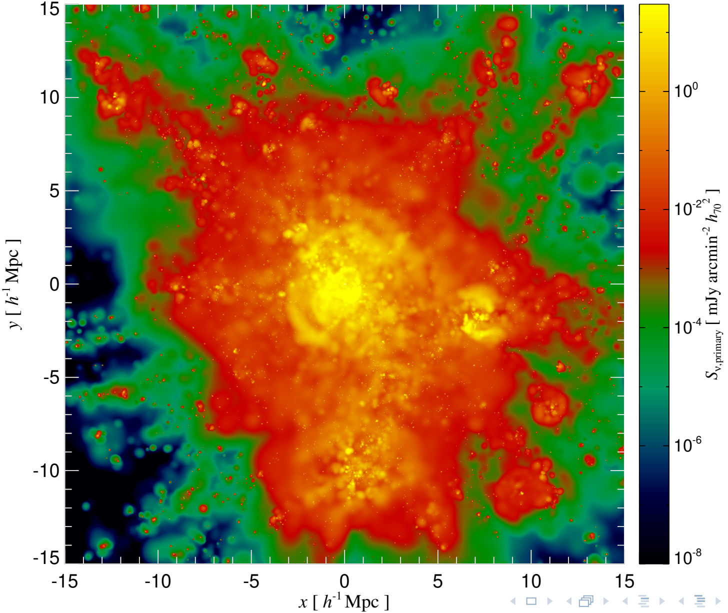
Radio web: primary CRe (15 MHz), slower magnetic decline



Magnetic fields in the universe – 1

Recap of today's lecture

- **Properties of astrophysical magnetic fields:**
  * magnetic fields exist in all astrophysical objects on scales from km to several Mpcs and show field strengths from 10^{-9} G to 10^{15} G
  * magnetized objects include planets, stars, pulsars/magnetars, black-hole accretion discs and jets, galaxies, galaxy clusters
  * magnetic observables: Zeeman effect, synchrotron intensity & polarization, Faraday rotation

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- **Magnetic field evolution:**
  * Biermann battery can generate $\boldsymbol{B}$ field from a baroclinic flow without $\boldsymbol{B}_0$
  * the magnetic dynamo stretches, folds, twists, and merges the field so that it grows exponentially fast until saturation
  * the magnetic flux is frozen into the thermal plasma

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- **Magnetic force and stress:**
  * magnetic pressure causes the fluid to expand perpendicular to the mean magnetic field if $P_B = B^2/8\pi > P_{\rm th}$
  * magnetic stress forces the fluid element to contract along the field lines if $P_B > P_{\rm th}$: analogy of a stretched elastic rubber band!

- **Magneto-hydrodynamic waves and turbulence:**
  * MHD supports 7 modes: two polarization states of Alfvén waves, slow- and fast magnetosonic waves each, and the zero-frequency entropy mode
  * MHD turbulence has an anisotropic cascade where eddies become more elongated towards smaller scales and locally align with $\langle \boldsymbol{B} \rangle$

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- **Magnetic draping:**
  * an object moving (super-Alfvénically) through a magnetized medium drapes a dynamically strong magnetic sheath around it
  * magnetic draping suppresses interface instabilities and modifies dynamics
  * polarized radio emission from draping sheath allows to infer upstream magnetic field orientation

Magnetic fields in the universe – 2

Recap of today's lecture

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- **Non-thermal processes in clusters:**
  * radio relics and halos prove the existence of volume-filling magnetic fields and relativistic electrons in the ICM
  * the radial extent (short axis) of radio relics that propagate on the sky enables to estimate the magnetic field strength via a cooling length argument
  * what powers radio halos? hadronic interactions or Fermi-II reacceleration?

AIP

Literature

There are many excellent texts on magnetic fields in the universe. If I had to select three I would probably pick these ones that range from a basic introduction to numerical modeling to a solid review:

- **Introductory text to magneto-hydrodynamics (MHD):**
  Essential Magnetohydrodynamics for Astrophysics, Spruit,
  `https://arxiv.org/abs/1301.5572`
- **Review of numerical techniques for ideal and non-ideal MHD, applied to the context of star formation simulations:**
  Numerical Methods for Simulating Star Formation, Teyssier & Commercon, 2019, FrASS, 6, 51
  `https://arxiv.org/abs/1907.08542`
- **Review of astrophysical magnetic fields with a focus on their generation and maintenance by turbulence:**
  Astrophysical magnetic fields and nonlinear dynamo theory, Brandenburg & Subramanian, 2005, PhR, 417, 1
  `https://arxiv.org/abs/astro-ph/0405052`

If you want to refresh your memory on the derivation of the hydrodynamic equations, of shock waves and hydrodynamic turbulence, I suggest to read Section 3.1 of my

- **Lecture notes that cover many topics in theoretical astrophysics:**
  The Physics of Galaxy Clusters, Pfrommer,
  `https://pages.aip.de/pfrommer/Lectures/clusters.pdf`