$$D_{\mu\mu} = \frac{\Omega^2 (1-\mu^2)}{2B_0^2} \, Re \int_0^\infty dt \, e^{i(k_\parallel v_\parallel t + n\epsilon\Omega t - \omega t)} \qquad \delta(k_\parallel v_\parallel - \omega + n\epsilon\Omega)$$

$$\begin{bmatrix} 1 \\ m^2c^2 \end{bmatrix} \left\{ \begin{matrix} P_{RR} \\ R_{RR} \end{matrix} (\vec{k},t) J_{n+1}^2(W) + \begin{matrix} P_{LL} \\ R_{LL} \end{matrix} (\vec{k},t) J_{n-1}^2(W) - J_{n+1}(W) J_{n-1}(W) \left[\begin{matrix} P_{RL} \\ R_{RL} \end{matrix} (\vec{k},t) e^{2i\psi} + \begin{matrix} P_{LR} \\ R_{LR} \end{matrix} (\vec{k},t) e^{-2i\psi} \right] \right\}$$

$P_{ij}(\vec{k},t) \to P_{ij}(\vec{k})$ for steady state turbulence.

We need correlation tensor $\langle \delta B_i \, \delta B_j^* \rangle$ rather than the energy spectrum.

$$S_{ij} = \frac{B_0^2}{6\pi L^{\frac{1}{3}}} \left(\delta_{ij} - \frac{k_i k_j}{k^2} \right) k_\perp^{-10/3} \exp\left(- \frac{|k_\parallel| L^{1/3}}{k_\perp^{2/3}} \right)$$

$$Tr(S_{ij}) = E_k^{3D}.$$

Dimensionless tensors

$$M_{ij}(\vec{k}) = \frac{\langle B_i(\vec{k}) B_j^*(\vec{k}) \rangle}{B_0^2} \qquad K_{ij}(\vec{k}) = \frac{\langle v_i(\vec{k}) v_j^*(\vec{k}) \rangle}{V_A^2}$$

Homework 2a:

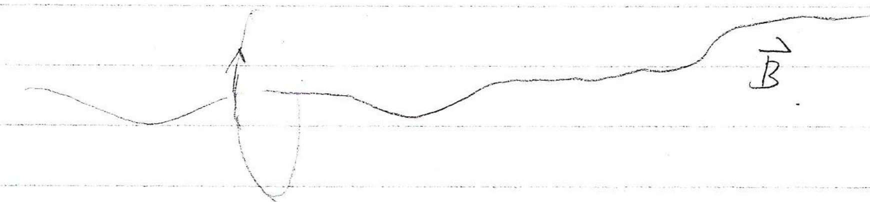
$$\begin{matrix} D_{\mu\mu} \\ D_{pp} \end{matrix} = \frac{\pi \Omega^2 (1-\mu^2)}{2} \int_{k_{min}}^{k_{max}} dk^3 \, \delta(k_\parallel v_\parallel - \omega \pm \epsilon\Omega)$$

$$\begin{bmatrix} 1 \\ m^2 V_A^2 \end{bmatrix} \left\{ (J_2^2 + J_0^2) \begin{bmatrix} M_{RR}(\vec{k}) + M_{LL}(\vec{k}) \\ K_{RR}(\vec{k}) + K_{LL}(\vec{k}) \end{bmatrix} - 2 J_2 J_0 \left(e^{i2\psi} \begin{bmatrix} M_{RL} \\ K_{RL} \end{bmatrix} + e^{-2i\psi} \begin{bmatrix} M_{LR}(\vec{k}) \\ K_{LR}(\vec{k}) \end{bmatrix} \right\}.$$

⑩ Gyroresonance:

$\omega - k_{\parallel} v_{\parallel} = n \varepsilon \Omega$. $\quad v_A \ll v_{\parallel}$ $\quad n = \pm 1 \cdots \infty$.

$\Rightarrow \quad k_{\parallel} \sim \dfrac{\Omega}{v_{\parallel}}$ bumpy road.



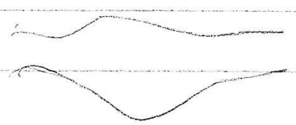
Homework 2b. derive the Fokker-Planck coefficients

$$\begin{bmatrix} D_{\mu\mu} \\ D_{pp} \end{bmatrix} = \frac{v^{2.5} \mu^{3.5}}{\Omega^{1.5} L^{2.5} (1-\mu^2)^{0.5}} \Gamma\left[6.5,\ k_{max}^{-2/3} k_{\parallel res} L^{1/3}\right] \begin{bmatrix} 1 \\ m^2 V_A^2 \end{bmatrix}.$$

Transit time damping (TTD)

$\omega = k_{\parallel} v_{\parallel}$ $\quad n = 0$ (Landau resonance)

$v_{\parallel} = V_A / \cos\theta$,



mirror for $-\left(\dfrac{m V_\perp^2}{2B}\right) \nabla_{\parallel} B$. $\Rightarrow$

only applies to compressible modes (fast, slow).

No resonant scale, all scales contribute!

(11)

Scattering by fast modes

Damping affects cascade, cascade is truncated at a scale where $\tau_{cas} \sim \tau_{damp}$

Collisionless damping (due to TTD with electrons)

$$\Gamma_L = \frac{\sqrt{\pi\beta}}{4}\frac{\omega \sin^2\theta}{\cos\theta}\left[\sqrt{\frac{m_e}{m_H}}\exp\left(-\frac{m_e}{m_H\beta\cos^2\theta}\right)+5\exp\left(-\frac{1}{\beta\cos^2\theta}\right)\right]$$

Viscous damping

$$\Gamma_{vis} = \begin{cases} k_\perp^2\eta_0/6\rho_i & \beta \ll 1 \\ k^2\eta_0(1-3\cos^2\theta)/6\rho_i & \beta \gg 1 \end{cases}$$

$$\frac{D_{\mu\mu}}{D_{pp}} = \frac{\pi(\Omega v\mu)^{0.5}(1-\mu^2)}{2L^{0.5}}\begin{cases}\frac{1}{7}\left[1-(\tan^2\theta_c+1)^{-7/4}\right] \\ \frac{1}{3}\left[1-(\tan^2\theta_c+1)^{-3/4}\right]m^2V_A^2\end{cases} \quad \tan\theta_c = \frac{k_{\perp,c}}{k_{\parallel,res}}$$

$$A(E) = \frac{\partial[vp^2D(p)]}{4p^2\partial p}, \quad D(p) = \frac{1}{2}\int_{-1}^{1}D_{pp}\,d\mu$$

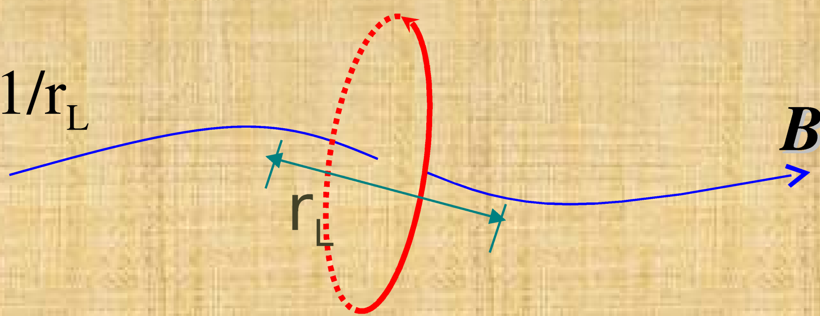
Resonance mechanism

Gyroresonance

$\omega - k_{\parallel} v_{\parallel} = n\Omega \, (n = \pm 1, \pm 2 \ldots)$,

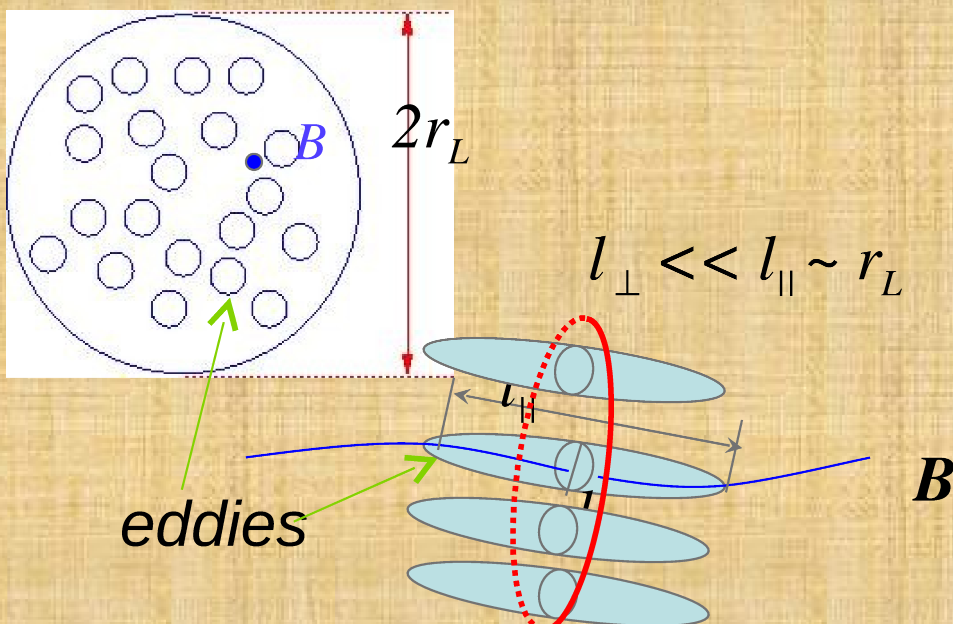
Which states that the MHD wave frequency (Doppler shifted) is a multiple of gyrofrequency of particles ($v_{\parallel}$ is particle speed parallel to **B**).

So, $k_{\parallel,res} \sim \Omega/v = 1/r_L$



Transport in Alfvenic turbulence

1. "random walk"



scattering efficiency is reduced

2. "steep spectrum"

$$E(k_\perp) \sim k_\perp^{-5/3},\ k_\perp \sim L^{1/3} k_{||}^{3/2}$$

$$\Rightarrow E(k_{||}) \sim k_{||}^{-2}$$

steeper than Kolmogorov!

Less energy on resonant scale

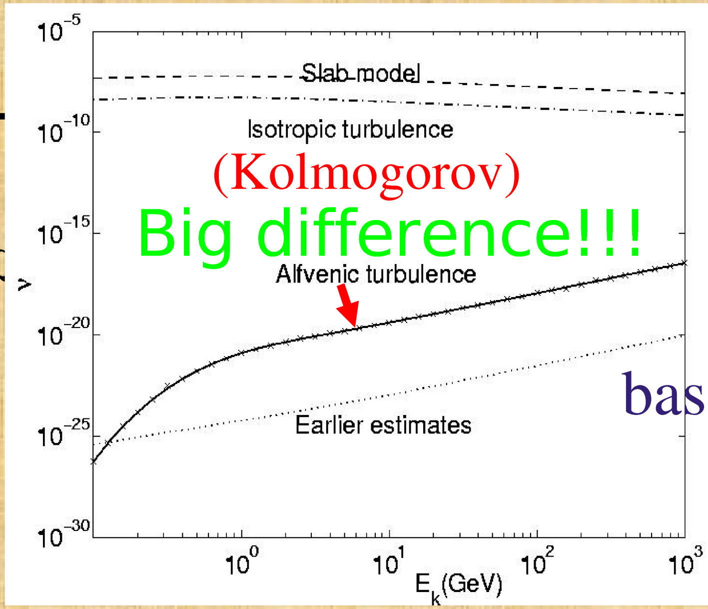
Alfven modes contribute marginally to particle acceleration and scattering if energy is injected from large scale!

(Chandran 00, Yan & Lazarian 02, 04

What induces scattering?

Scattering frequency

Alfven modes

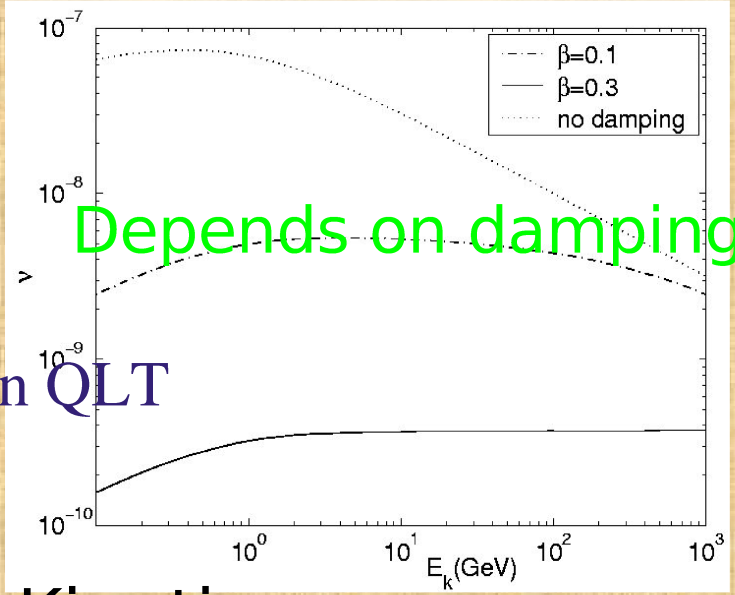


(Kolmogorov)

Big difference!!!

Kinetic energy

Fast modes



Depends on damping

based on QLT

Kinetic energy

Alfven modes are inefficient. Fast modes dominate CR scattering in spite of damping (Yan & Lazarian 02,04).

Observed abundance of secondary elements in CRs

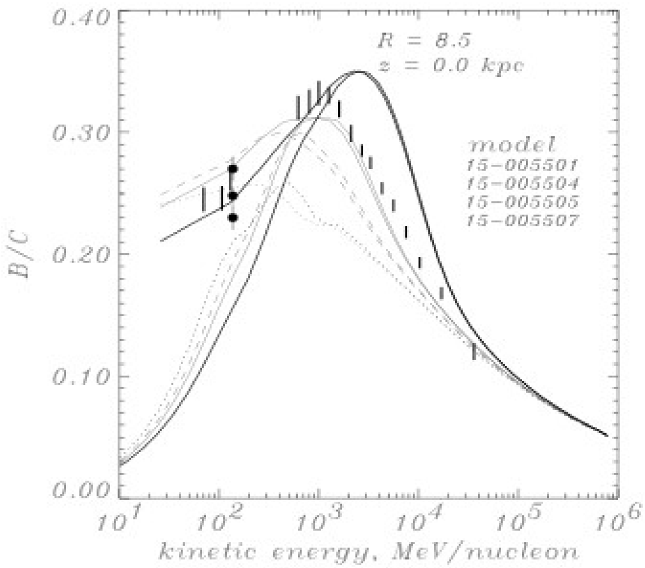
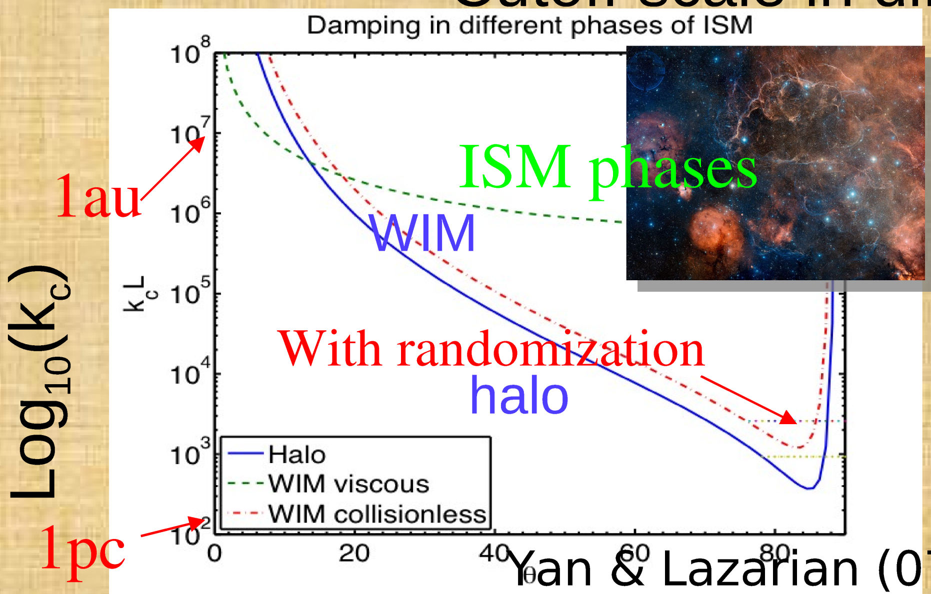


Figure 1. B/C ratio for diffusive reacceleration models with z_h = 5 kpc, $v_A = 0$ (dotted), 15 (dashed), 20 (thin solid), 30 km s^{-1} (thick solid). In each case the interstellar ratio and the ratio modulated to 500 MV is shown. Data: from Webber et al. (1996).

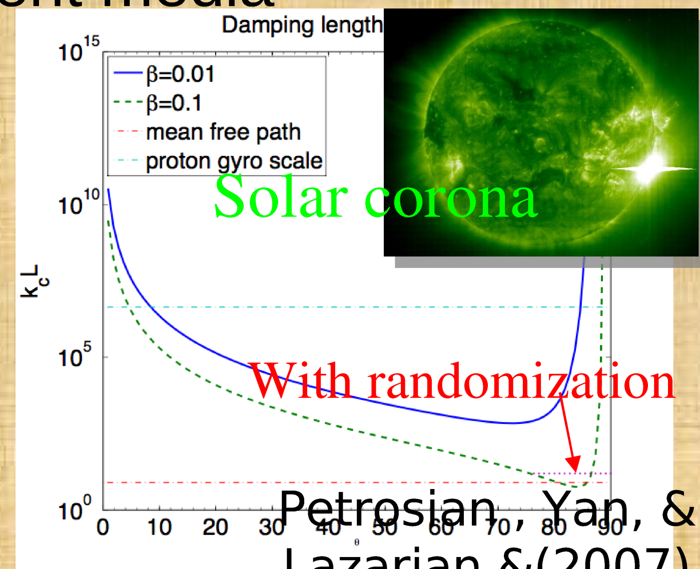
Examples of anisotropic Damping of fast modes

Cutoff scale in different media



Yan & Lazarian (07)

Pitch angle



Petrosian, Yan, & Lazarian &(2007)

Pitch angle

- Damping depends on medium.
- Anisotropic damping results in quasi-slab geometry.
- Field line wandering should be accounted for.

We need nonlinear theory

- Long standing problem: 90 degree scattering $K_{res} = \Omega/v_{||} \rightarrow \infty$, the scale is below the dissipation scale of turbulence → No scattering at 90^o? → $\lambda_{||} \rightarrow \infty$?!
- Perpendicular diffusion .

A key assumption in Quasilinear theory:

guiding center is unperturbed $Z_0 = v\mu t$;

→

Nonlinear theory:

In reality, the guiding center is perturbed, especially on large scales,

$z = (v\mu \pm \Delta v_{||})t$.