# Graduate mini couse:
# “High Energy Astrophysics – Selected Topics”

Assignment #4

Due 1pm, Thu Mar 27

This assignment is due before class on Mar 27; that is, at 1:00pm **sharp**. Assignments may be handed in in person at class; may be placed at our offices 1303 (Huirong) or 1304 (Christoph). The instructors will have office hours 2-3pm on Wed.

Show your work, and good luck!

## Question 1 - The Greisen-Zatsepin-Kuz’min (GZK) cutoff [15 pts]

Last year, the Pierre Auger and HiRes collaborations have confirmed the existence of the so-called GZK cutoff that forms the end of the cosmic ray spectrum. It is due to photo-pion production in the rest frame of the ultra-high energy cosmic ray (UHECR) particle which itself is decelerated by this reaction.

### 1 (a)

(9 pts) Compute the cutoff energy by assuming that a pion and protons form at rest in the center of momentum system (CMS) of the interacting photon and CR proton. For simplicity, assume that we have mono-energetic photons from the cosmic microwave background with an energy that is given by Wien’s displacement law. Calculate alongside the Lorentz factor that the CMB photons experience when they are boosted into the CMS.

### 1 (b)

(3 pts) The cutoff energy $E \simeq 6 \times 10^{19}$ eV that you calculated in the previous section corresponds to the well-known value; with the minor problem that the reaction that we calculated would not happen, at least not with a decent cross section. In reality, you excite a delta resonance (a peak in the total cross section of the proton-photon reaction with mass $1.23\,\mathrm{GeV}c^{-2}$) at a photon energy of $E_\gamma \simeq 0.3$ GeV in the center of momentum frame of the reaction (compare this to the photon energy from your calculation). What would this imply for the resulting cutoff energy and how can you still reconcile this with the value that you calculated?

### 1 (c)

(3 pts) Estimate the mean-free path of the UHECR at the GZK cutoff. Assume a cross-section of $\sigma_{\gamma\mathrm{p}} \simeq 0.2$ mbarn and assume that the UHECR loses a energy fraction of 0.1 in this reaction. You might want to calculate the number density of CMB photons, that is given by its general form,

$$n = \frac{g}{(2\pi\hbar)^3} \int_0^\infty \frac{4\pi p^2 \mathrm{d}p}{\exp[\varepsilon(p)/kT] \pm 1}, \tag{1}$$

where $g$ is the statistical weight, $\varepsilon(p)$ the dispersion relation of photons, and you have to chose the appropriate sign in the denominator.

## Question 2 - Relativistic decay kinematics [15 pts]

**2 (a)**

(8 pts) Consider the decay of a massive particle of mass $M$ into two daughter particles of masses $m_{\rm a}$ and $m_{\rm b}$ in the CMS. Derive the energy of each these particles in the lab system as a function of the involved particle masses and the energy of the initial particle, $E_M$:

$$E_{\rm a,b} = \frac{E_M}{M}\left[\frac{M^2 + m_{\rm a,b}^2 - m_{\rm b,a}^2}{2M} + \sqrt{1 - \frac{M^2c^4}{E_M^2}}\, p'(M, m_{\rm a}, m_{\rm b})c^{-1}\cos\theta'\right], \tag{2}$$

where CMS quantities are denoted with a prime and $p'(M, m_{\rm a}, m_{\rm b})$ is implicitly given by energy conservation, $(p'^2c^{-2} + m_{\rm a}^2)^{1/2} + (p'^2c^{-2} + m_{\rm b}^2)^{1/2} = M$.

*Hint:* Using conservation laws in the CMS, show that you can express the energy of one daughter particle solely by the masses of the involved particles:

$$E'_{\rm a,b} = \frac{M^2 + m_{\rm a,b}^2 - m_{\rm b,a}^2}{2M}\, c^2. \tag{3}$$

**2 (b)**

(4 pts) Assuming $M$ decays isotropically, i.e. the emission probability is equally distributed in the CMS frame, derive the energy distribution of the produced particle.

**2 (c)**

(3 pts) Discuss the individual cases of the energy distribution where (i) the decay products have equal mass, (ii) a ultra-relativistic particle is decaying ($M^2 \ll E_M^2$), and (iii) both decay particles have mass zero (which applies e.g. to the decay $\pi^0 \to 2\gamma$).