Uncovering the cloak of invincibility: the draping of cluster magnetic fields over bullets

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Draping of solar wind field around Earth

- Happens very quickly (figure to right -- 600 s)
- Can induce magnetic field in even a neutral atmosphere
- Earth Magnetic Field reversals may not be catastrophic to life

Draping of Saturn’s field over Titan

- Observable with Cassini
- Emission from draped field

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Comets in solar wind

- Draping occurs and can distort velocity, magnetic fields in wind over significant distances

Wegmann (2002)
Applications to galaxy clusters

- Radio Bubbles, seen as voids in X-rays, are observed out to large distances and have very sharp interfaces.
- Hydrodynamic instabilities should disrupt them, conduction should dissipate the interfaces in $\sim 10^8$ yrs.
- Could bubble motions sweep up enough field to suppress instabilities and conduction?

NASA/IoA/A.Fabian et al.
Magnetic draping at work

- Rising radio bubbles in a hot atmosphere
- Shown is the log of the density for the non-draping versus draping case
- Hydrodynamical instabilities are suppressed

Ruszkowski et al. 2007
Mergers of galaxy clusters

- Minor mergers involve smaller cluster falling into more massive ones
- Stripping of small-mass ICM
- When/where does this occur?
- Consequences for enrichment, cold fronts, ...
Pulsar wind nebulae?
Previous work: Lyutikov 2004

- Analytics
- Particularly along stagnation line

\[ \frac{B}{\rho} = \frac{1}{\sqrt{1 - \frac{R_0^3}{L^3}}} \left( \frac{B}{\rho} \right)_0; \quad l \approx \frac{1}{\mathcal{M}^2 A R_0} \]
Asai et al (2004,5,6..)

• Numerics
• 2d, 3d
• ‘Kitchen sink’ - turbulent magnetic field, conduction,...
• Can draping effect conduction? Yes
Linear theory analysis

- Can such a thin layer have interesting dynamic effects?
- Three layers; velocity +/- U, magnetized layer of some thickness/strength

Dursi 2007
If $V_A$ is a few times relevant velocity, can stabilize against wavelengths an order of magnitude longer than thickness of layer.
$V_A = 0.2 \, U$

$V_A = 1.25 \, U$
Excellent agreement between theory and simulation!
Our Contributions

- 3D, AMR numerical experiments of draping of uniform magnetic fields in context relevant to galaxy clusters
- More careful analytic calculation in potential flow approximation to understand dynamics
- Analytic understanding of field strength in the draping layer, opening angle, deceleration due to magnetic tension, vorticity generation, instabilities in the perpendicular plane
3D simulations using FLASH

- AMR very useful for focusing resolution in near draped layer
- Large dynamic range between size of traversed region and thickness of layer
- Magnetic dynamics relatively straightforward
Sometimes, 2D just isn’t enough...
Magnetic energy density in 2D

Not only slingshots back the bullet, but squishes it, too...
Foreshadowed earlier

- Asai et al 2004, 2005 saw strong growth of magnetic field in 2d, but only commented on it
- Simulation was not run long enough to see that there is no steady state

Asai et al (2005)
\[ v = e_r \left( \frac{R^3}{r^3} - 1 \right) u \cos \theta + e_\theta \left( \frac{R^3}{2r^3} + 1 \right) u \sin \theta \]

Potential flow around solid sphere

3d AMR results
Exact MHD solution: kinetic approximation

\[ \text{curl}(v \times B) = 0 \quad \text{div} B = 0 \]

• given our potential flow solution it, looks straightforward to solve the usual Maxwell’s equations for the B-field...

• but sometimes things only look simple
Exact MHD solution: kinetic approximation

\[ \text{curl}(\mathbf{v} \times \mathbf{B}) = 0 \quad \text{div} \mathbf{B} = 0 \]

\[ B_\phi = \frac{B_0 \cos \phi}{\sqrt{1 - \frac{R^3}{r^3}}} \]

\[ B_r = \frac{r^3 - R^3}{r^3} \cos \theta \left[ C_1 \mp B_0 \sin \phi \int_\xi^r \frac{p(r, \theta) r'^4 \, dr'}{(r'^3 - R^3 - p(r, \theta)^2 r')^{3/2} \sqrt{r'^3 - R^3}} \right] \]

\[ B_\theta = \frac{2r^3 + R^3}{r^{5/2} \sqrt{r^3 - R^3}} \left[ C_2 \pm 2B_0 \sin \phi \int_\xi^r \frac{r'^3 \left(r'^3 + 2R^3\right) \sqrt{r'^3 - R^3} \, dr'}{(2r'^3 + R^3)^2 \sqrt{r'^3 - R^3 - p(r, \theta)^2 r'}} \right] \]
Approximate MHD solution near the sphere

\[ \text{curl} (v \times B) = 0 \quad \text{div } B = 0 \]

\[ B_r = \frac{2}{3} B_0 \sin \phi \frac{\sin \theta}{1 + \cos \theta} \sqrt{\frac{3s}{R}} \]

\[ B_\theta = B_0 \sin \phi \sqrt{\frac{R}{3s}} \]

\[ B_\phi = B_0 \cos \phi \sqrt{\frac{R}{3s}} \quad \text{and } s = r - R \]
Potential flow around solid sphere

$B_x$, $B_y$, $B_z$ in the plane of the initial B-field

3d AMR results
$B_x$, $B_y$, $B_z$ in the plane transverse to the initial B-field

Potential flow around solid sphere

3d AMR results
Agreement with potential flow calculations

\[ \frac{B}{\rho} = \frac{1}{\sqrt{1 - \frac{R_0^3}{r^3}}} \left( \frac{B}{\rho} \right)_0; \quad l \approx \frac{1}{M_A^2} R_0 \]
Magnetic field strength in draped layer

- To first order, depends only on ram pressure
- Maximum magnetic field strength $\sim 2 \times$ ram pressure
Deceleration due to mag. tension

- Magnetic layer is strong enough (and curved enough) that it dominates deceleration
- Even in 3D case!
- ~ 4x stronger than viscous/turbulent drag

\[
\dot{\mathbf{u}}_T = -\frac{3}{8} \frac{\rho u^2}{\langle \rho_c \rangle R} C_G
\]
Opening angle of drape

- Comparison with 9 3D simulation
- Correlation a little rattier than other quantities --
- Largest scales in simulations, some effects of boundary conditions

\[ \tan \theta = \frac{v_A}{u} \]
Opening angle \( \sim v_A/U \)
Opening angle $\sim v_A/U$
Opening angle $\sim v_A/U$
Vorticity generation

\[ \rho \frac{dv}{dt} = -\nabla P + j \times B = -\nabla \left( P + \frac{B^2}{8\pi} \right) + \frac{1}{4\pi} (B \cdot \nabla) B \]

equilibrium: balance between magnetic tension and magnetic + thermal pressure

\[ \frac{d}{dt} \left( \frac{\omega}{\rho} \right) = \left( \frac{\omega}{\rho} \cdot \nabla \right) \mathbf{v} + \frac{1}{4\pi \rho^2} \nabla \times (B \cdot \nabla) B + \frac{1}{\rho^3} \nabla \rho \times \left[ \nabla \left( P + \frac{B^2}{8\pi} \right) - \frac{1}{4\pi} (B \cdot \nabla) B \right] \]

vorticity is frozen into the flow if source terms negligible
Generation of Vorticity

• Magnetic contact layer induces vorticity in fluid elements which cross it
• Operates primarily in plane along field lines
• Much less vorticity generation in other plane
Instabilities in transverse plane

- Material is stripped off the object,
- Flow accelerated there due to Bernoulli effect
Instabilities in transverse plane

\[ \omega_{RT}^2 = \frac{\langle \rho_c \rangle - \rho_0}{\langle \rho_c \rangle + \rho_0} \dot{u}_T k \]

\[ \omega_{KH} = \frac{\sqrt{\langle \rho_c \rangle \rho_0}}{\langle \rho_c \rangle + \rho_0} \Delta u k \]

\[ \frac{\omega_{KH}^2}{\omega_{RT}^2} \approx \frac{12\pi}{C_G} \frac{k}{k_0} \geq \frac{12\pi}{C_G} \approx 20 \]

\[ L_{KH} = \frac{2\pi u}{\omega_{KH}} \approx \frac{2R}{3} \sqrt{\frac{\langle \rho_c \rangle}{\rho_0} \frac{k_0}{k}} \leq 15 \]

Kelvin-Helmholtz instability is responsible for disintegration of the core
Long-term behavior

- Evolution of core after it has swept past roughly its own mass
- Mixed material ‘fills up’ drape
- Highly constrained in other plane!
Conclusions

• Very quickly drape strong magnetized layer

• Even thin layer can have interesting effects protecting object (bubble or bullet) against indignities of shearing into environment

• Pushing forward analytics, guided by simple numerical experiments

• Rich astrophysical applications