Self-interacting dark matter

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in collaboration with

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Searching for dark matter (DM)

correct relic density $\rightarrow$ DM annihilation in the Early Universe

DM production:
\[ \chi \chi \rightarrow q q \]

DM annihilation:
\[ \chi \chi \rightarrow q q \]

DM scattering:
\[ \chi \chi \rightarrow q q \]

DM production: particle colliders

DM annihilation: indirect detection

DM scattering: direct detection

(slide concept Feng)
Outline

1. Standard WIMPS
   - Chemical decoupling
   - Kinetic decoupling
   - Smallest protohalos

2. Self-interacting WIMPS
   - Sommerfeld effect
   - Small-scale problems
   - A solution to all $\Lambda$CDM problems

3. Cosmological simulations
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The WIMP “miracle”

“freeze-out”: annihilation rate drops below expansion rate $H$

\[ \frac{dn_\chi}{dt} + 3Hn_\chi = -\langle \sigma v \rangle (n^2_\chi - n^2_{\chi,\text{eq}}), \quad \langle \sigma v \rangle : \chi\chi \rightarrow \text{SM SM} \]
The WIMP “miracle”

- “freeze-out”: annihilation rate drops below expansion rate $H$ → number density of Weakly Interacting Massive Particles:

$$\frac{dn_\chi}{dt} + 3Hn_\chi = -\langle \sigma v \rangle \left(n_\chi^2 - n_{\chi, eq}^2\right), \quad \langle \sigma v \rangle : \chi\chi \rightarrow \text{SM SM}$$

- assuming a particle $\chi$, initially in thermal equilibrium, with a relic density

$$\Omega_\chi \sim \frac{1}{m_{\text{Pl}} T_0 \langle \sigma v \rangle} \sim \frac{m_\chi^2}{m_{\text{Pl}} T_0 g_\chi^4},$$

$$m_\chi \sim m_{\text{weak}} \sim 100 \text{ GeV} \quad g_\chi \sim g_{\text{weak}} \sim 0.6 \quad \{ \Omega_\chi \sim 0.1 \}$$
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$$m_\chi \sim m_{\text{weak}} \sim 100 \text{ GeV} \quad \{\Omega_\chi \sim 0.1\}$$

- remarkable coincidence: particle physics independently predicts particles with the right density to be dark matter
**Freeze-out ≠ decoupling!**

- **WIMP** interactions with *heat bath* of SM particles:

  ![Diagram showing WIMP interactions with SM particles](image)

  - Boltzmann suppression of $n_\chi$:
    - scattering process more frequent
    - continue even after *chemical decoupling* ("freeze-out") at $T_{cd} \sim m_\chi/25$
  
  - **Kinetic decoupling** much later: $\tau(T_{kd}) \equiv N_{coll}/\Gamma_{el} \sim H^{-1}(T_{kd})$
    - random walk in momentum space: $N_{coll} \sim m_\chi/T$  
      (Schmid+ 1999, Green+ 2005)

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Kinetic decoupling

- evolution of phase space density $f_\chi$ given by the full Boltzmann equation in FRW space time:

$$E \left( \partial_t - H \mathbf{p} \cdot \nabla \mathbf{p} \right) f_\chi = C \left[ f_\chi \right]$$

- 1\textsuperscript{st} moment ($\int d^3p$) recovers the familiar continuity equation:

$$\frac{d n_\chi}{dt} + 3 H n_\chi = - \langle \sigma v \rangle \left( n_\chi^2 - n_{\chi,eq}^2 \right)$$

- consider the 2\textsuperscript{nd} moment ($\int d^3p \mathbf{p}^2$) and introduce

$$T_\chi n_\chi \equiv \int \frac{d^3p}{(2\pi)^3} \frac{\mathbf{p}^2}{3m_\chi} f_\chi(\mathbf{p})$$

→ analytic treatment possible without assumptions about $f_\chi(\mathbf{p})$

Thermal history of WIMPs

- resulting ODE for $T_\chi$

$$\frac{dy}{dx} = 2 \frac{m_\chi c(T)}{H \tilde{g}^{-1/2}} \left(1 - \frac{T_\chi}{T}\right)$$

example:

$m_\chi = 100$ GeV

$|\mathcal{M}|^2 \sim g_Y^4 (E_\chi/m_\chi)^2$

- fast transition allows definition of $T_{kd}$:

$$T_\chi = \begin{cases} 
T & \text{for } T \gtrsim T_{kd}, \\
T_{kd} (a_{kd}/a)^2 & \text{for } T \lesssim T_{kd}
\end{cases}$$

Bringmann & Hofmann (2007), Bringmann (2009)
The smallest protohalos

- **free streaming** of WIMPS after \( t_{kd} \) at the thermal speed of decoupling erases small-scale fluctuations (Green+ 2005)

- initial coupling between WIMPS and the radiation field → **acoustic oscillations** in the power-spectrum at the horizon scale of kinematic decoupling (Loeb & Zaldarriaga 2005, Bertschinger 2006)
The smallest protohalos

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- **Cutoff** in the power spectrum corresponds to smallest gravitationally bound objects in the universe

- Strong dependence on particle physics properties, no “typical” value of $M_{\text{cut}} \sim 10^{-6} M_\odot$ (Profumo+ 2006)
Consequences

- **indirect detection experiments through WIMP annihilation:**

\[ \Phi_{\text{SM}} \propto \langle \rho^2 \rangle = (1 + \text{BF}) \langle \rho \chi \rangle^2, \]
\[ \text{BF} \propto \log \left( \frac{M_{\text{halo}}}{M_{\text{min}}} \right) \]
(Pinzke+ 2011, Gao+ 2012, Ludlow+ 2014)

- **flux depends on astrophysics, particle physics, detector properties:**

\[ N_\gamma = \left[ \int_{\text{LOS}} \rho^2 \, dI_\chi \right] \frac{\langle \sigma v \rangle}{2M_\chi^2} \left[ \int_{E_{\text{th}}}^{M_\chi} \left( \frac{dN_\gamma}{dE} \right) \right]_{\text{SUSY}} A_{\text{eff}}(E) \, dE \, \frac{\Delta \Omega}{4\pi} \, \tau_{\text{exp}} \]
Consequences

- indirect detection experiments through WIMP annihilation:

\[
\Phi_{\text{SM}} \propto \langle \rho^2 \rangle = (1 + BF) \langle \rho \chi \rangle^2,
\]

\[
BF \propto \log(M_{\text{halo}}/M_{\text{min}})
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(Pinzke+ 2011, Gao+ 2012, Ludlow+ 2014)

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\]

- fluctuations in the event rate of direct detection experiments

- gravitational lensing of substructures $\rightarrow$ flux anomalies

- Lyman-$\alpha$ forest . . .
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Self-interacting dark matter
WIMPS with long-range forces

- Annihilation
- Self-scattering
- Scattering

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Sommerfeld effect

- **kinematics:** non-relativistic DM particle $\chi$ interacts with light force carrier $\phi$ ($m_\phi \ll m_\chi$)
- **repeated exchange of $\phi$:** each “rung” of ladder contributes at $O(\alpha/\nu)$
  $\rightarrow$ resummation necessary

- **long range interaction:**
  potential distorts wave function
  
  $$\left(-\frac{\nabla^2}{m_\chi} + V\right) \psi(r) = m_\chi \nu^2 \psi(r)$$

  $\Rightarrow \sigma = S(\nu) \sigma_{\chi\chi \rightarrow \phi\phi}$, with $S(\nu) = |\psi(0)|^2$

- **short-range interaction:**
  standard QFT result

Arkani-Hamed+ (2009)

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Enhancement factor

- Coulomb potential: analytic solution
  \[ S(v) = \frac{\pi \alpha / v}{1 - \exp(-\pi \alpha / v)} \quad \text{as} \quad v \to 0 \quad \frac{\pi \alpha}{v} \]

- Yukawa potential: numerical solution
  \[ S(v) \propto v^{-1} \quad \text{on resonance} \]
  \[ S(v) \propto v^{-2} \quad \text{saturation for small} \quad v \quad \text{for} \quad m_{\phi} \lesssim 100 \text{ MeV}, \phi \quad \text{can only decay into leptons (e, } \mu) \]
  \[ \rightarrow \text{appearance of resonances near bound states} \]
Enhancement factor

- **Coulomb potential:** analytic solution
  \[ S(\nu) = \frac{\pi \alpha / \nu}{1 - \exp(-\pi \alpha / \nu)} \]
  \[ \nu \to 0 \quad \frac{\pi \alpha}{\nu} \]

- **Yukawa potential:** numerical solution
  \[ S \propto \nu^{-1} \]
  \[ S \propto \nu^{-2} \]
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  → appearance of resonances near bound states

  - off resonance: \( S \propto \nu^{-1} \)
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Enhancement factor

- **Coulomb potential**: analytic solution

\[ S(\nu) = \frac{\pi \alpha/\nu}{1 - \exp(-\pi \alpha/\nu)} \quad \nu \rightarrow 0 \rightarrow \frac{\pi \alpha}{\nu} \]

- **Yukawa potential**: numerical solution
  - appearance of resonances near bound states
  - off resonance: \( S \propto \nu^{-1} \)
  - on resonance: \( S \propto \nu^{-2} \)
  - saturation for small \( \nu \)

- for \( m_{\phi} \lesssim 100 \text{ MeV} \), \( \phi \) can only decay into leptons (e, \( \mu \))

\[ \rightarrow \text{leptophilic DM} \]

Lattanzi, Silk (2009)
\( \Lambda \)CDM cosmology

**a great success**

story on **large scales**

- **Springel**+ (2006)
- **Kuhlen**+ (2012)

Supernova Cosmology Project

No Big Bang

- Union2.1 SN Ia Compilation
- BAO
- CMB

Cosmic
Cluster
Galactic

- non-linear (simulation)
- linear (analytic)

Baryon Acoustic Oscillations

- CDM
- ADM
- WDM (8 keV)

\( \Lambda \)CDM cosmology

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Self-interacting dark matter
1. Missing satellites?

Moore+ (1999)

→ many more satellites in simulations of MW-sized galaxies than observed
ΛCDM small-scale problems

1. Missing satellites?
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2. Cusps or cores?
   - Blok+ (2001)
   - Cuspy inner density profiles predicted by simulations not found in observations

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Standard WIMPs
Self-interacting WIMPS
Cosmological simulations

Sommerfeld effect
Small-scale problems
A solution to all $\Lambda$CDM problems

$\Lambda$CDM small-scale problems

1. Missing satellites?
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     → many more satellites in simulations of MW-sized galaxies than observed

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3. Too big to fail?
   - Boylan-Kolchin+ (2011)
     → most massive sub-halos in simulations too dense to host observed brightest dwarf satellites

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Inner DM profile in galaxy groups and clusters

Star formation efficiency $\frac{M_*}{M_{200}}/(\Omega_b/\Omega_m)$

[Graph showing star formation efficiency vs. $\theta_{Ein}$ (arcsecond)]

- Galaxies
- Groups
- N13 clusters

Salp.+NFW
Salp.+Contract
Chab.+NFW
Chab.+Contract
Salp.+Core

Sonnentfeld+ 2012 "Jackpot"
Sonnentfeld+ 2015
Oguri+ 2014
Grillo+ 2012
Treu & Koopmans 2004

Newman+ (2015)
Solutions?

many possibilities, no consensus reached yet:

- **astrophysical solutions:**
  increased gas entropy, suppress cooling efficiency, SN feedback, large velocity anisotropy, other baryonic feedback, increased stochasticity of galaxy formation, small MW mass, . . .

- **dark matter solutions:**
  warm DM, interacting DM, DM from late decays, large annihilation rates, condensates, . . .

- **all have shortcomings** and/or solve at most 2 problems at the time!
Solutions?

*velocity-dependent self-interacting dark matter:*

- scattering cross-section for **Yukawa potential** Khrapak+ (2003)
  \[ \sigma_{\chi \chi} = \text{const.} \] unnatural from particle physics viewpoint!

- elastic DM self-scattering is completely analogous to screened Coulomb scattering in a plasma
velocity-dependent self-interacting dark matter:

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- cored profiles possible without violating astrophysical constraints
  Feng+ (2010), Loeb & Weiner (2011)

- **N-body simulations**: “too big to fail” problem avoided
  Vogelsberger+ (2012)

- what about missing satellites?

Loeb & Weiner (2011)
Our model

van den Aarssen, Bringmann, C.P. (2012)

- assume **light vector mediator** coupling to dark matter and neutrinos:

\[ \mathcal{L}_{\text{int}} \supset -g_\chi \bar{\chi} V \chi - g_\nu \bar{\nu} \nu \]

- **annihilation**
  - → relic density
  - → indirect $4\nu$ detection signal from galactic center(?)

- **self-scattering**
  - → changes inner density and velocity profiles of dwarf galaxies

- **scattering**
  - → large $M_{\text{min}}$
"Cusp vs. core" and "too big to fail" problems

- demand correct relic density
  → unique relation between \((v_{\text{max}}, \sigma_{\text{max}})\) and \((m_\chi, m_\nu)\)

\[
\begin{align*}
\sigma_{\text{max}} / m_\chi \ [\text{cm}^2 \text{ g}^{-1}] \\
v_{\text{max}} \ [\text{km s}^{-1}]
\end{align*}
\]

ruled out by astrophysics

not enough flattening of cuspy profiles

van den Aarssen, Bringmann & Pfrommer (2012)
DM scattering off standard model particles

- free-streaming of WIMPs after kinetic decoupling creates cutoff in power spectrum
- acoustic oscillations leads to similar cutoff
- cutoff scale is set by size of horizon at KD: late KD $\rightarrow$ high $M_{\text{min}}$
- $M_{\text{min}} = \max(M_{fs}, M_{ao})$: only objects with $M \geq M_{\text{min}}$ form
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Scalar mediator:
- Scatters off $\phi, \mu^\pm, e^\pm$
- Saturation at $M_{\text{min}} \sim 10^3 \, M_\odot$
- $\nu$'s negligible: $|M_{\phi l \rightarrow \phi l}|^2 \propto m_l^2$

Vector mediator:
- $\nu$'s contribute: $|M_{\nu l \rightarrow \nu l}|^2 \propto E_{\nu}^2$
- $M_{\text{min}}$ increases to $\mathcal{O}(10^{11} \, M_\odot)$

van den Aarssen+ (2012)
“Missing satellites” problem

- now compute $M_{\text{min}}$ from kinetic decoupling temperature . . .

In this simple phenomenological model, it is possible to simultaneously solve all small-scale problems of $\Lambda$CDM!
Cored central density profiles of clusters

- velocity-dependent DM self-scattering cores out central density slopes in clusters with rate
  \[ \Gamma \sim \frac{\rho}{m_\chi} \langle \sigma v \rangle \sim H \]
  
- ellipticals/clusters, \( f_s = 10 - 100 \):
  \[ \Gamma \sim \frac{f_s \rho}{m_\chi} \frac{\langle \sigma v \rangle}{f_s \max} \]

Loeb & Weiner (2011)
Cored central density profiles of clusters

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- ellipticals/clusters, \( f_s = 10 - 100 \):

\[ \Gamma \sim \frac{f_s \rho}{m_\chi} \frac{\langle \sigma v \rangle}{f_s} \]

- using \( \rho \sim 1/r \) for \( r \ll r_s \):

\[ \frac{r_{\text{core}}}{r_{200}} \bigg|_{\text{cluster}} \sim \frac{1}{f_s} \frac{r_{\text{core}}}{r_{200}} \bigg|_{\text{dwarf}} \sim \frac{1}{f_s 10} \Rightarrow r_{\text{core}}(10^{15} M_\odot) \sim O(1-10 \text{ kpc}) \]

- need simulations to understand interplay of hierarchical evolution and determination of cluster-\( r_{\text{core}} \): merging history → scatter
small-scale problems of $\Lambda$CDM can be solved by a DM model with:

- velocity-dependent self-interactions mediated by (sub-)MeV vector:
  - transforms cusps to cores and solves “too big to fail” problem

- much later kinetic decoupling than in standard case follows naturally for vector mediator coupling to neutrinos:
  - potentially solves “missing satellites” problem

- predicts cores in clusters on scales $\mathcal{O}(1 – 10 \text{ kpc})$

→ need further model building and simulations to confirm
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SIDM simulations: models

Vogelsberger, Zavala, Cyr-Racine, Pfrommer, Bringmann, Sigurdson, in prep.
SIDM simulations: large-scale structure

Vogelsberger, Zavala, Cyr-Racine, Pfrommer, Bringmann, Sigurdson, in prep.
SIDM simulations: Milky Way-sized halos

Vogelsberger, Zavala, Cyr-Racine, Pfrommer, Bringmann, Sigurdson, in prep.
SIDM simulations: power spectrum and mass function

Vogelsberger, Zavala, Cyr-Racine, Pfrommer, Bringmann, Sigurdson, in prep.
SIDM simulations: density profile of MW-sized halo

Vogelsberger, Zavala, Cyr-Racine, Pfrommer, Bringmann, Sigurdson, in prep.
SIDM simulations: subhalo abundances

Vogelsberger, Zavala, Cyr-Racine, Pfrommer, Bringmann, Sigurdson, in prep.
SIDM simulations: internal subhalo structure

Vogelsberger, Zavala, Cyr-Racine, Pfrommer, Bringmann, Sigurdson, in prep.
If DM searches (production, indirect, and direct experiments) continue to deliver null results, we need to search for alternative windows:

- **small-scale features of ΛCDM cosmology**: abundances, density profiles, ... in the most DM-dominated objects (dwarfs, clusters)
- **particle physics model building** that addresses anomalies (beam dump experiments, ...)
- **develop effective theory for structure formation** that connects particle physics properties to effective parameters of structure formation