Cosmic rays in galaxies: plasma instabilities, transport, and observations

Christoph Pfrommer[1]

in collaboration with

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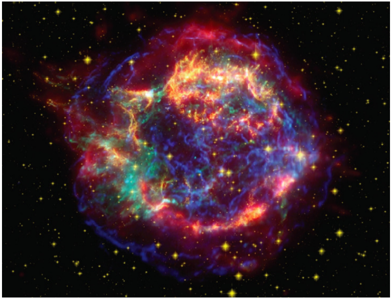
Understanding the Most Energetic Cosmic Accelerators: Advances in Theory and Simulation, Princeton, Oct 2020

Cosmic ray transport and feedback in galaxies



supernova Cassiopeia A

X-ray: NASA/CXC/SAO; Optical: NASA/STScI;
Infrared: NASA/JPL-Caltech/Steward/O.Krause et al.

- galactic supernova remnants drive shock waves, turbulence, accelerate electrons + protons, amplify magnetic fields

Cosmic ray transport and feedback in galaxies

super wind in M82

NASA/JPL-Caltech/STScI/CXC/UofA

- galactic supernova remnants drive shock waves, turbulence, accelerate electrons + protons, amplify magnetic fields
- supernovae, radiation and cosmic rays (CRs) drive gas out of galaxies via outflows

AIP

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NASA/JPL-Caltech/STScI/CXC/UofA

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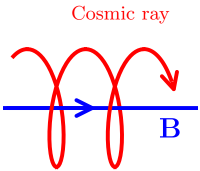
NASA/JPL-Caltech/STScI/CXC/UofA

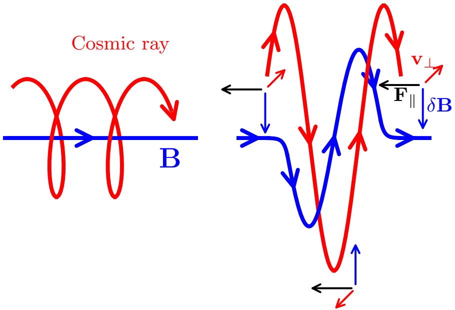
- galactic supernova remnants drive shock waves, turbulence, accelerate electrons + protons, amplify magnetic fields
- supernovae, radiation and cosmic rays (CRs) drive gas out of galaxies via outflows
- critical for explaining low star conversion efficiency in dwarfs → physics of galaxy formation
- need to study cosmic-ray driven plasma instabilities → CR acceleration, transport and feedback

Interactions of CRs and magnetic fields



sketch: Jacob

Interactions of CRs and magnetic fields



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- **gyro resonance:** $\omega - k_{||} v_{||} = n\Omega$

  Doppler-shifted MHD frequency is a multiple of the CR gyrofrequency

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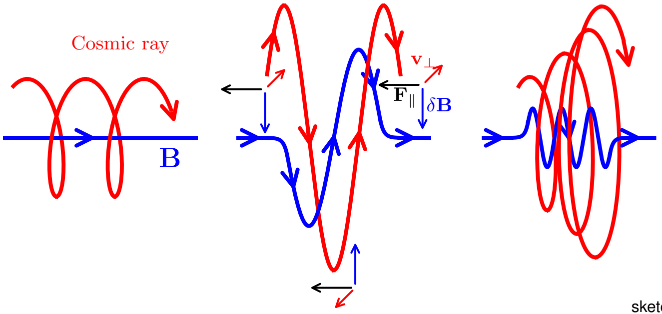

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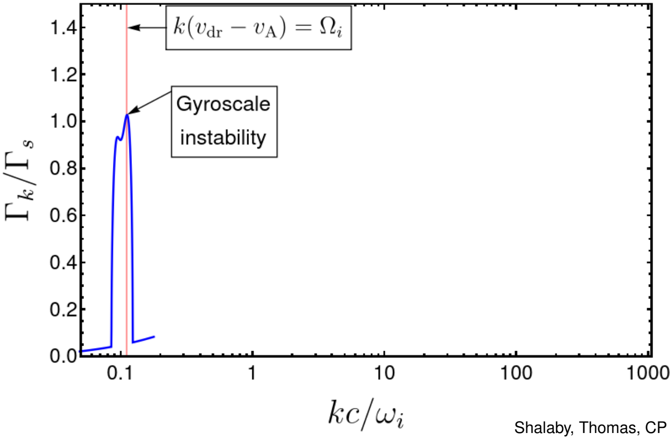
- **gyro resonance:** $\omega - k_{||} v_{||} = n\Omega$
  Doppler-shifted MHD frequency is a multiple of the CR gyrofrequency
- CRs scatter on magnetic fields $\rightarrow$ isotropization of CR momenta

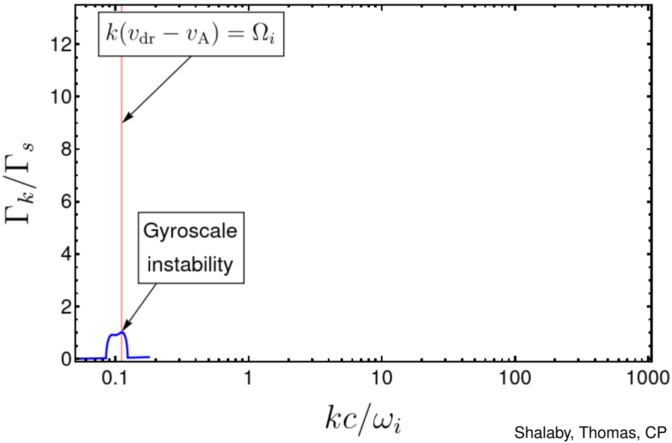
CR driven instabilities – growth rates



Shalaby, Thomas, CP (subm.)

- gyro-resonant instability of gyrotropic CR population

CR driven instabilities – growth rates



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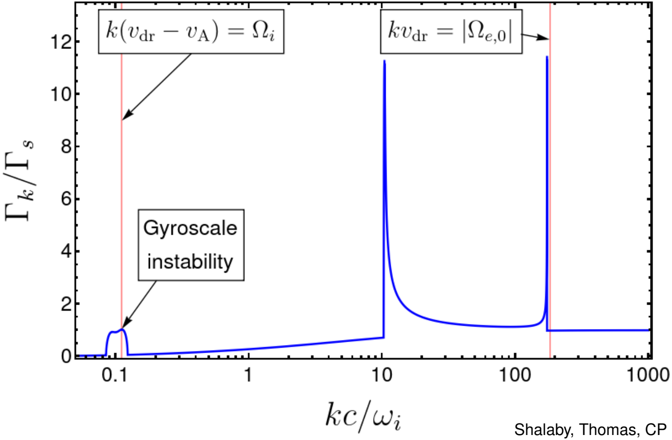

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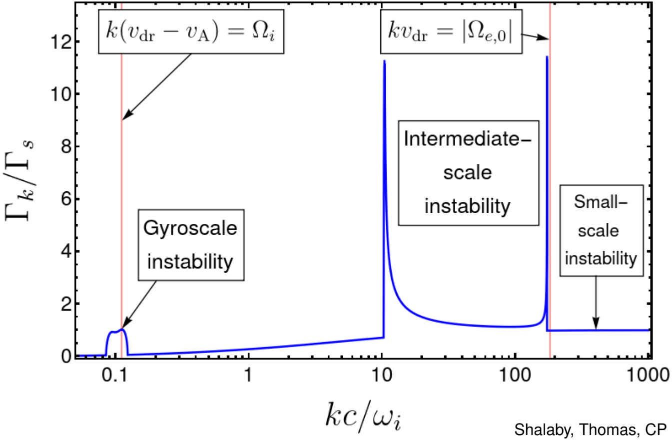
- **new intermediate-scale instability** of gyrotropic CR population

CR driven instabilities – growth rates



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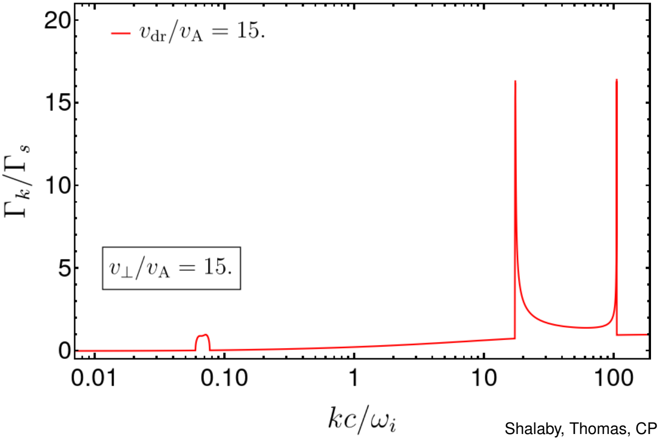
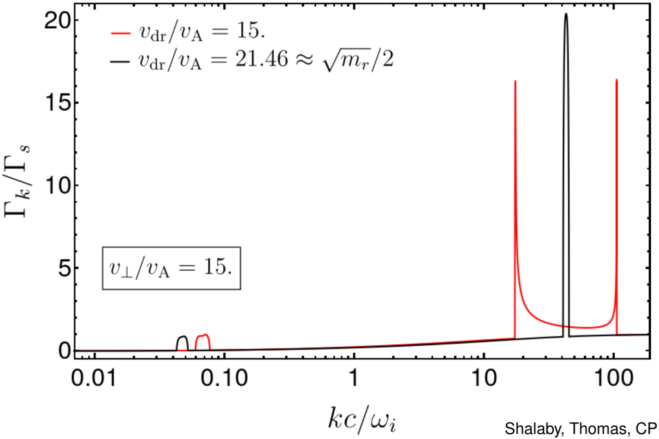
CR driven intermediate-scale instability



Shalaby, Thomas, CP (subm.)

- low CR drift speed: two instability peaks

CR driven intermediate-scale instability



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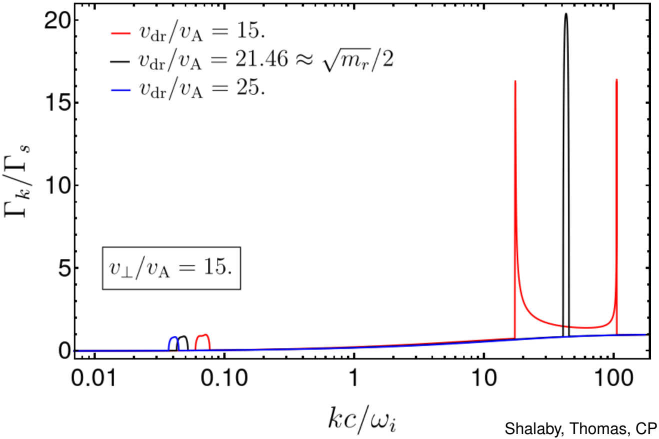
- for CR drift speed $v_{\rm dr} \approx \sqrt{\frac{m_{\rm i}}{m_{\rm e}}}\frac{v_{\rm A}}{2}$: two instability peaks merge

CR driven intermediate-scale instability



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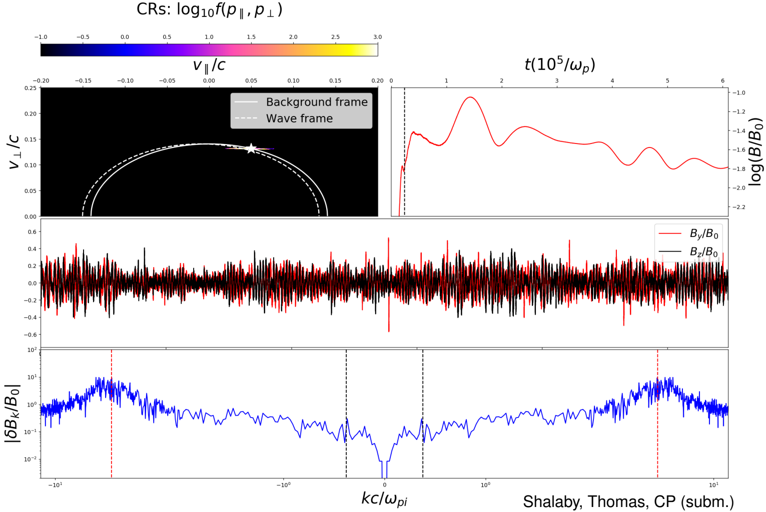
- for $v_{\rm dr} > \sqrt{\frac{m_{\rm i}}{m_{\rm e}}}\frac{v_{\rm A}}{2}$: intermediate-scale instability quenched

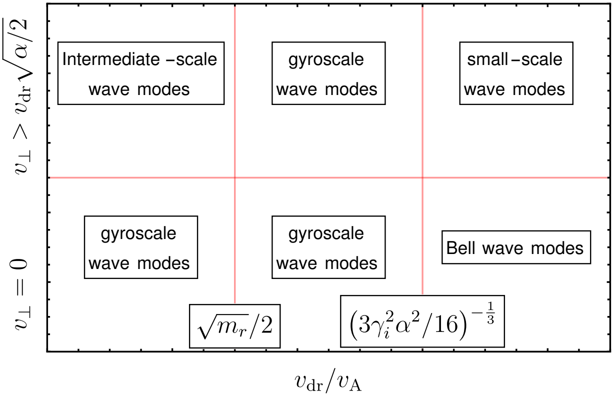
Cosmic ray driven instabilities

Growth of the intermediate-scale and the gyro-resonant instability



Shalaby, Thomas, CP (subm.)

Regimes of CR driven instabilities



Shalaby, Thomas, CP (subm.)

- where $\alpha = \frac{n_{cr}}{n_i}$ is the CR number fraction, $m_r = \frac{m_i}{m_e}$ is the mass ratio, and γ_i is the Lorentz factor of CR ions

The intermediate-scale instability

Properties of the intermediate-scale instability:

- growth rate $\Gamma_{\text{inter}} \gg \Gamma_{\text{gyro}}$ and excites broad spectral support
- unstable modes are background ion-cyclotron waves in the comoving CR frame
- condition for growth: $$\frac{v_{\text{dr}}}{v_{\text{A}}} < \frac{1}{2}\sqrt{\frac{m_{\text{i}}}{m_{\text{e}}}}$$

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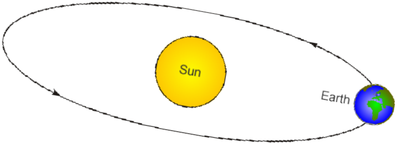
Implication of this new instability:

- couples CRs more tightly to background plasma and strengthens CR feedback in galaxies and galaxy clusters
- enables electron injection into diffusive shock acceleration
- decelerates CR escape from the sites of particle acceleration $\rightarrow$ brighter gamma-ray halos

Cosmic ray transport: an extreme multi-scale problem



Milky Way-like galaxy:

$$r_{\rm gal} \sim 10^4\ \mathrm{pc}$$

gyro-orbit of GeV cosmic ray:

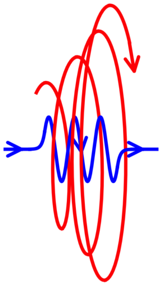
$$r_{\rm cr} = \frac{p_\perp}{e\,B_{\mu\mathrm{G}}} \sim 10^{-6}\ \mathrm{pc} \sim \frac{1}{4}\ \mathrm{AU}$$

$\Rightarrow$ need to develop a **fluid theory for a collisionless, non-Maxwellian component!**

Zweibel (2017), Jiang & Oh (2018), Thomas & CP (2019)

CR streaming and diffusion

- **CR streaming instability:** Kulsrud & Pearce 1969
  - if $v_{cr} > v_a$, CR flux excites and amplifies an Alfvén wave field in resonance with the gyroradii of CRs
  - scattering off of this wave field limits the (GeV) CRs' bulk speed $\sim v_a$
  - wave damping: transfer of CR energy and momentum to the thermal gas



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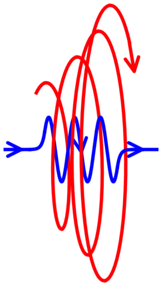

$\rightarrow$ *CRs exert pressure on thermal gas via scattering on Alfvén waves*

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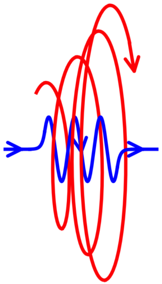
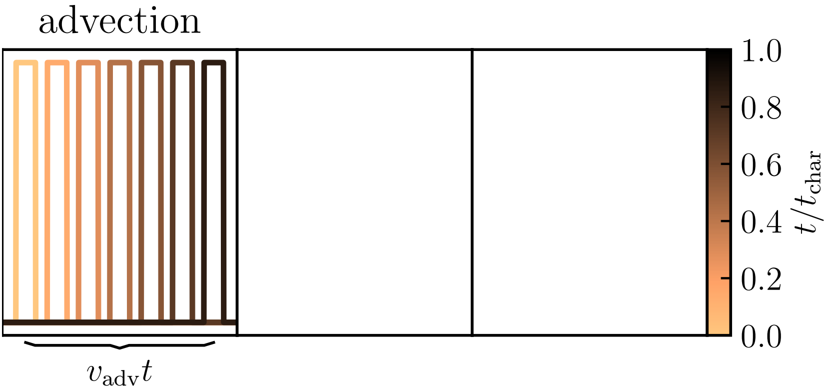


$\rightarrow$ *CRs exert pressure on thermal gas via scattering on Alfvén waves*

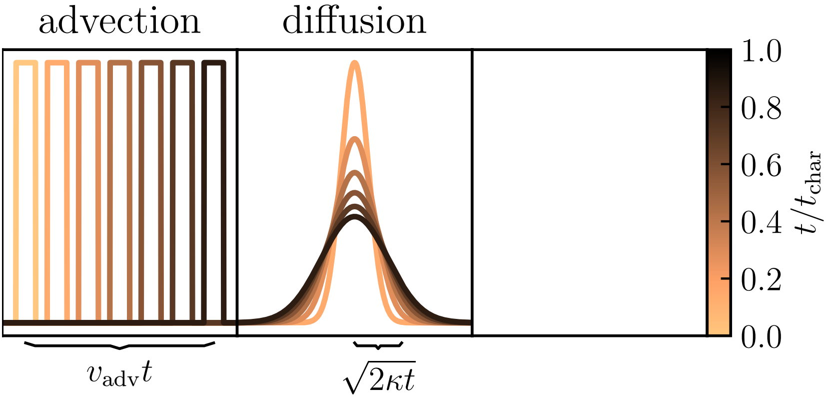
weak wave damping: strong coupling $\rightarrow$ CR stream with waves
strong wave damping: less waves to scatter $\rightarrow$ CR diffusion prevails

Modes of CR propagation



Thomas, CP, Enßlin (2020)

Modes of CR propagation



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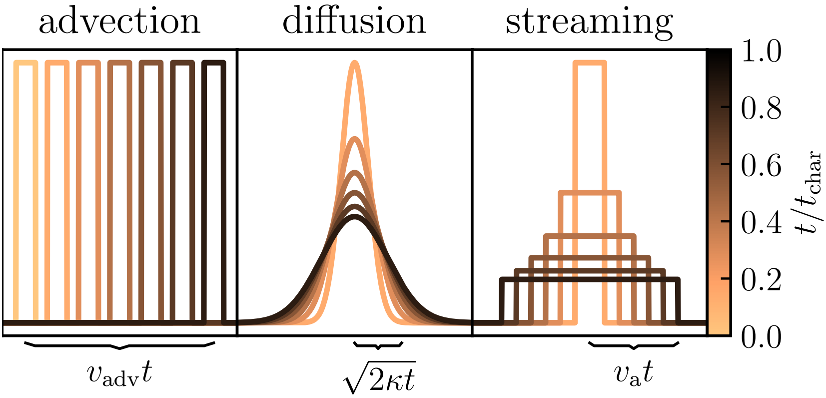
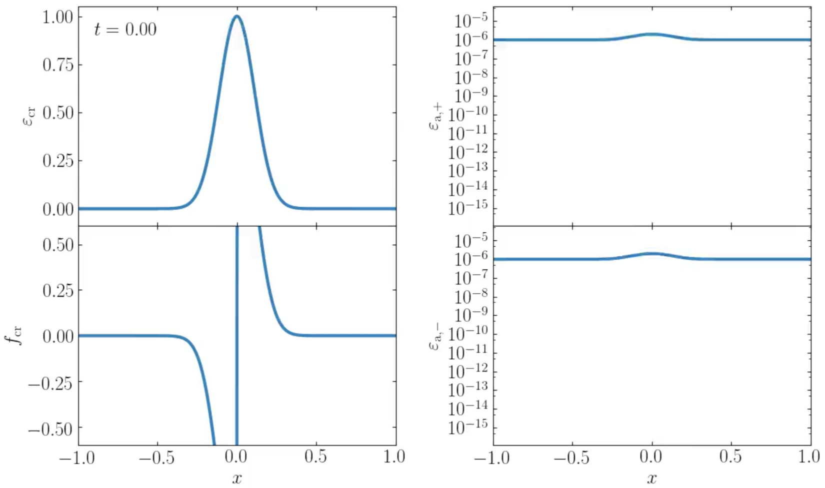


Thomas, CP, Enßlin (2020)

Non-equilibrium CR streaming and diffusion

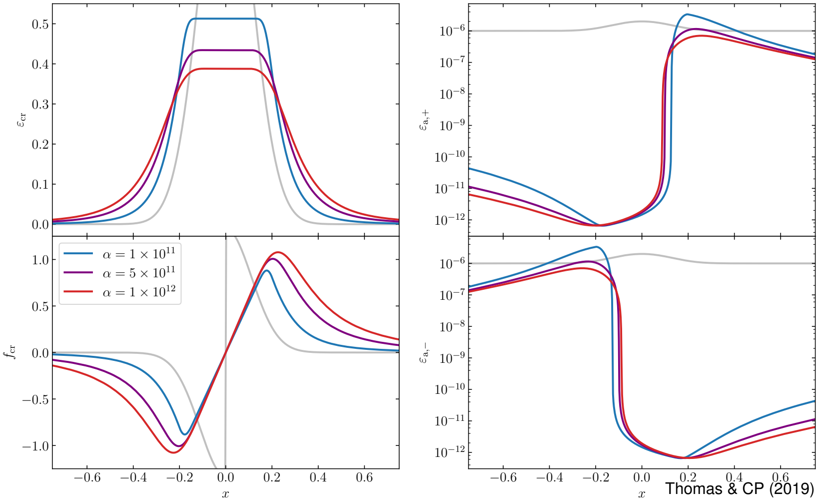
Coupling the evolution of CR and Alfvén wave energy densities



Thomas & CP (2019)

Non-equilibrium CR streaming and diffusion

Varying damping rate of Alfvén waves modulates the diffusivity of solution

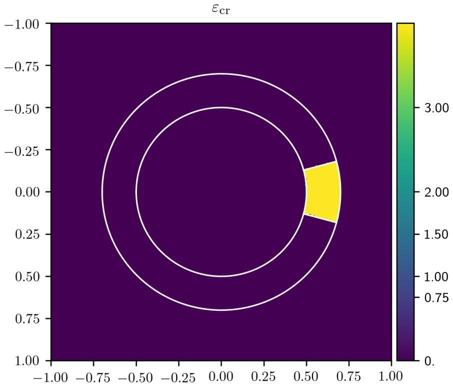


Thomas & CP (2019)

Anisotropic CR streaming and diffusion – AREPO

CR transport mediated by Alfvén waves and coupled to magneto-hydrodynamics

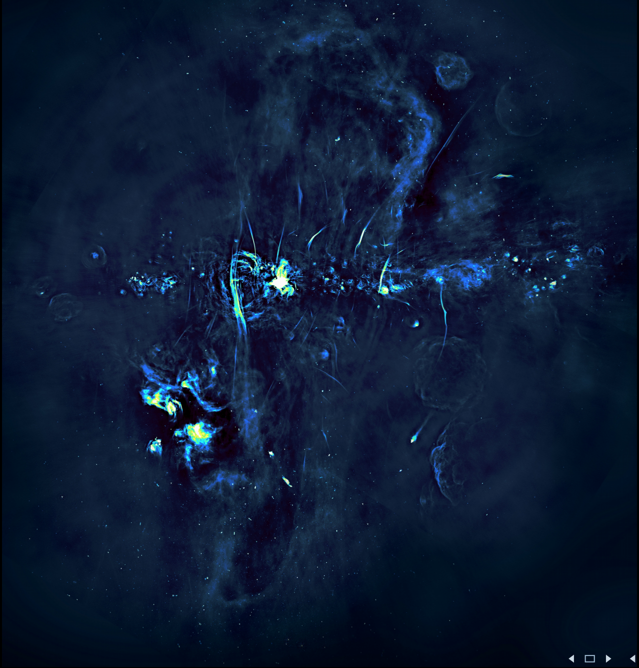
- CR streaming and diffusion along magnetic field lines in the self-confinement picture
- moment expansion similar to radiation hydrodynamics
- accounts for kinetic physics: non-linear Landau damping, gyro-resonant instability, . . .
- Galilean invariant and causal transport
- energy and momentum conserving



Thomas, CP, Pakmor (subm.)

MeerKAT image of the Galactic Center

Haywood+ (Nature, 2019)



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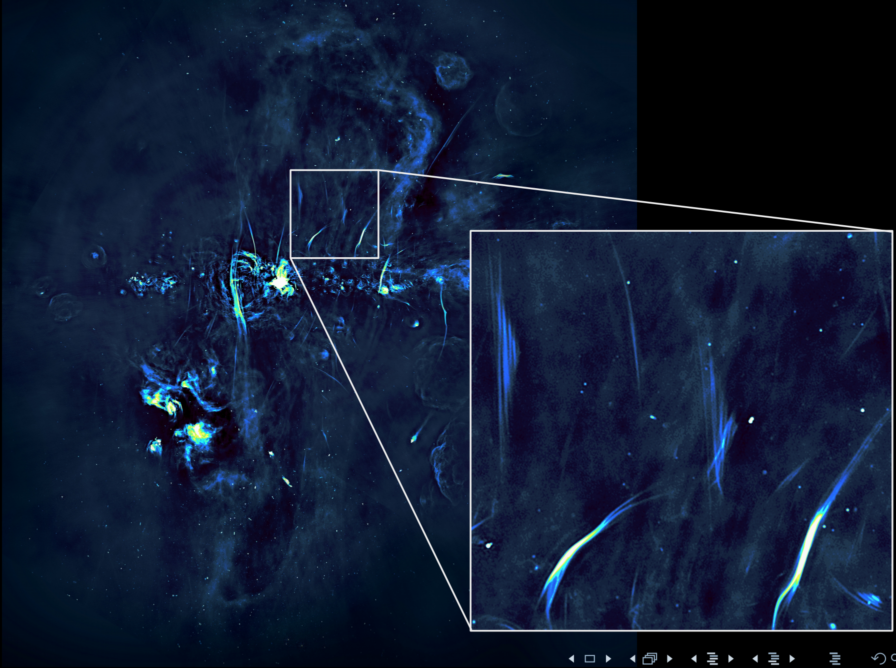
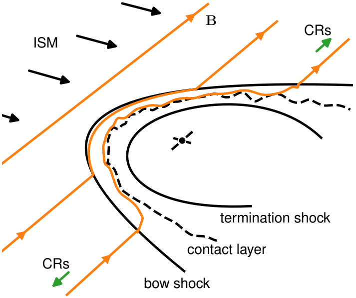


Radio synchrotron harps: the model

shock acceleration scenario



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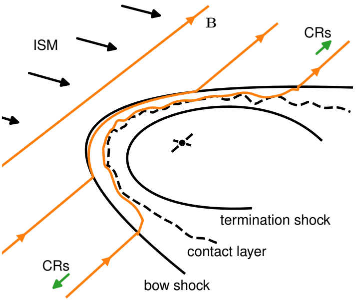
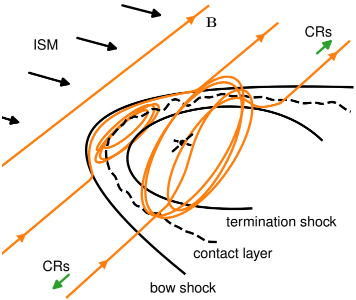


magnetic reconnection at pulsar wind



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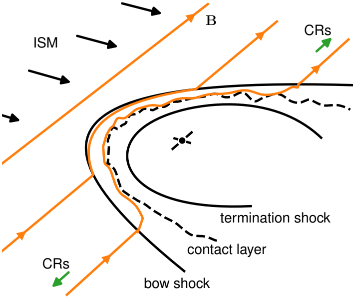
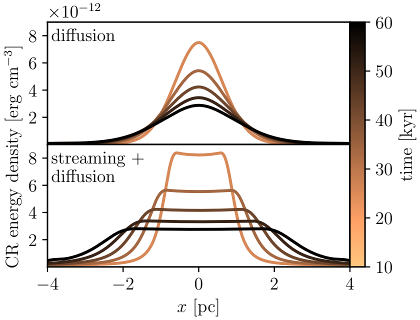
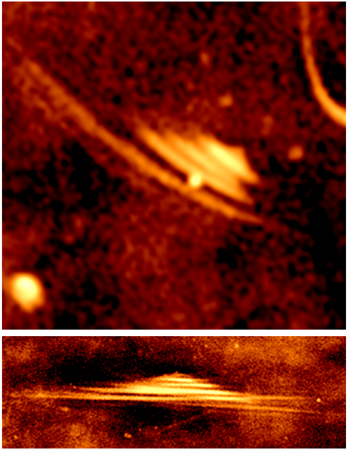


Thomas, CP, Enßlin (2020)

CR diffusion vs. streaming + diffusion



Radio synchrotron harps: testing CR propagation



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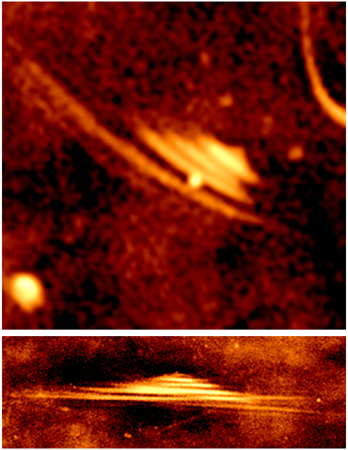
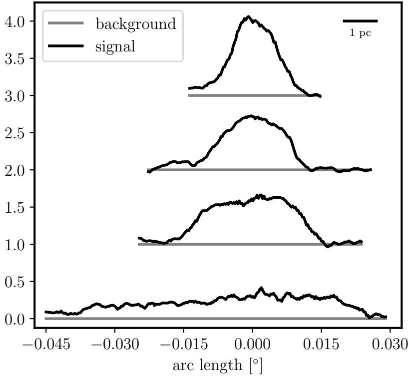


Haywood+ (Nature, 2019)

lateral radio profiles



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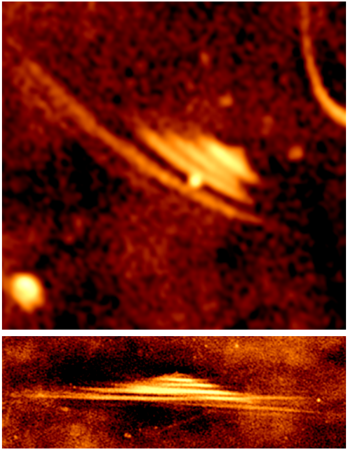

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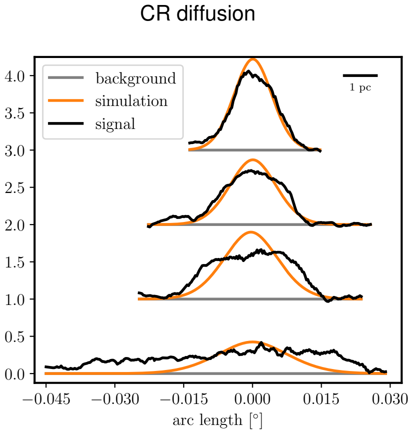

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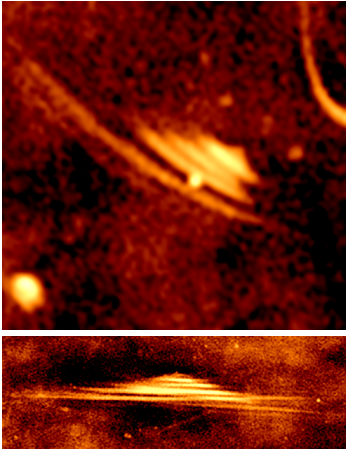

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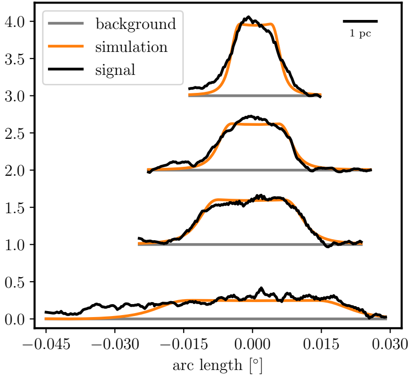
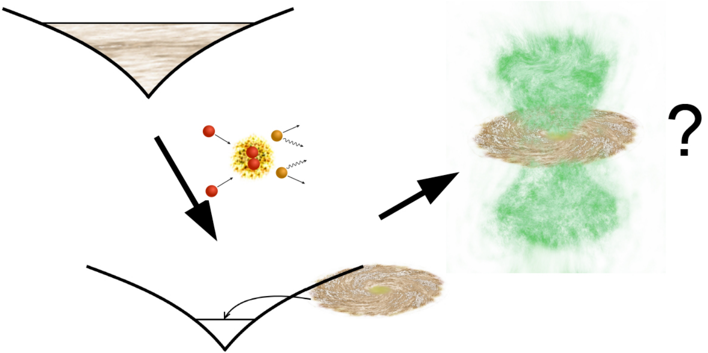
CR streaming and diffusion



Thomas, CP, Enßlin (2020)

Cosmic rays in galaxy formation



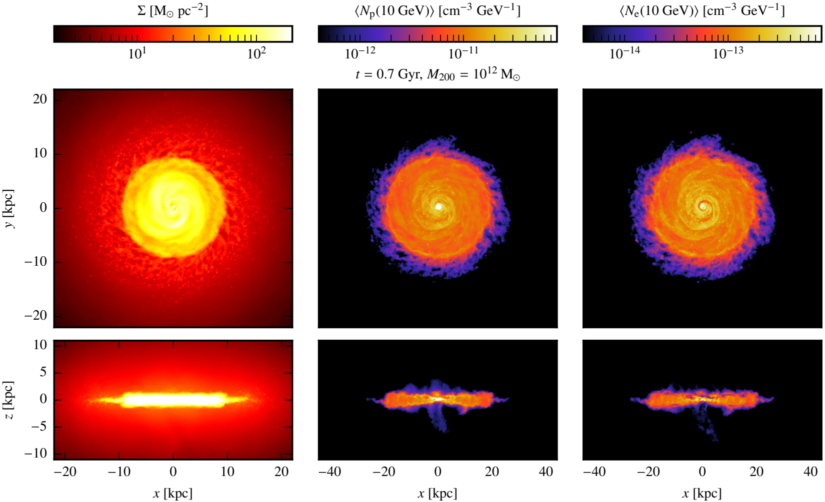
Werhahn, CP, Girichidis+ (in prep.)

Cosmic rays and non-thermal emission in simulated galaxies: I. & II.

MHD + CR advection + anisotropic diffusion: $\{10^{10}, 10^{11}, 10^{12}\}\ \mathrm{M}_\odot$

steady-state spectra of CR protons, primary & secondary electrons

From a starburst galaxy to a Milky Way analogy



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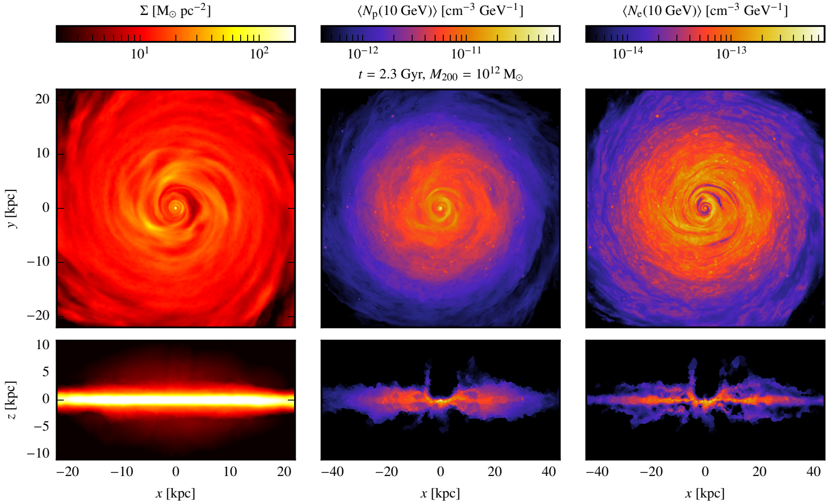
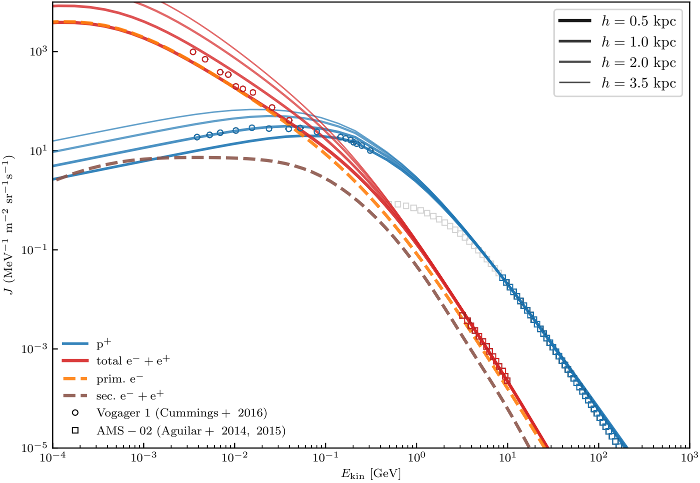


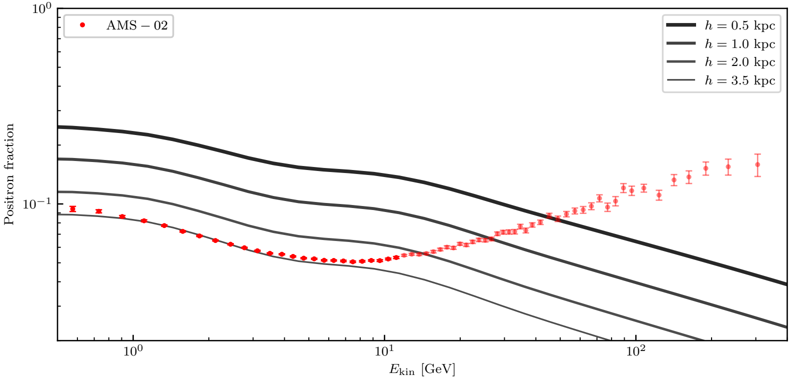
Werhahn, CP+ (in prep.)

Comparing CR spectra to Voyager and AMS-02 data



Werhahn, CP+ (in prep.)

Comparing the positron fraction to AMS-02 data



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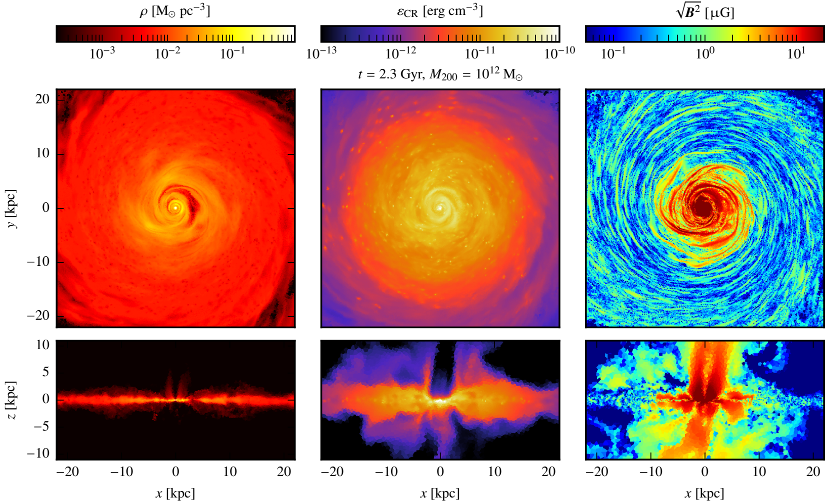
Simulation of a starburst galaxy



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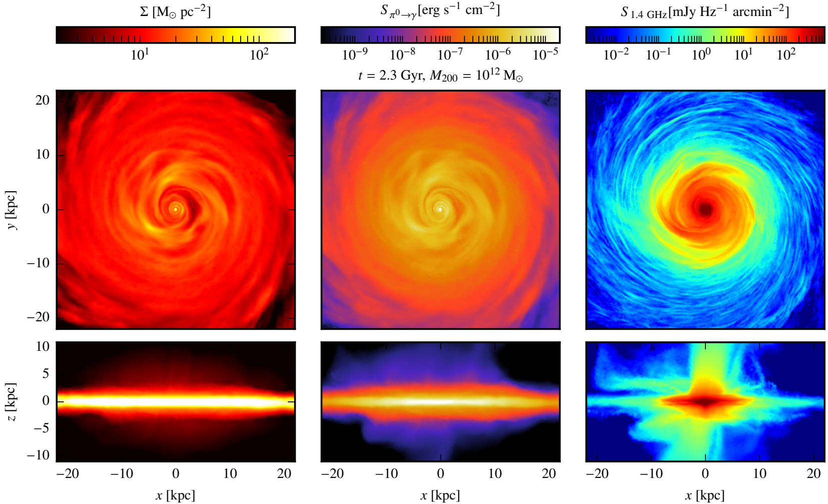
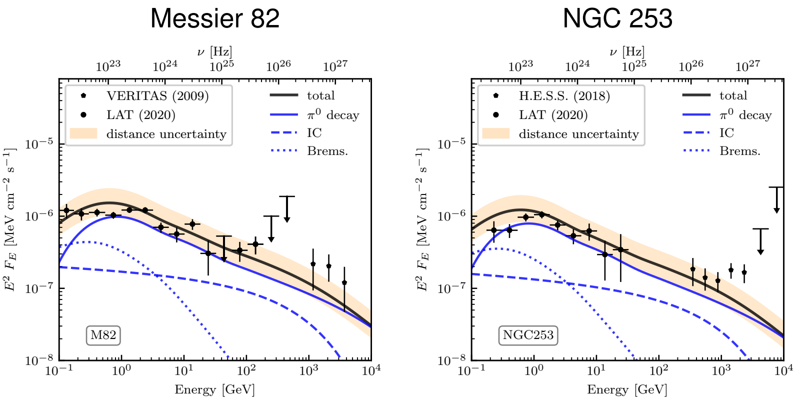


Werhahn, CP+ (in prep.)

Gamma-ray spectra of starburst galaxies

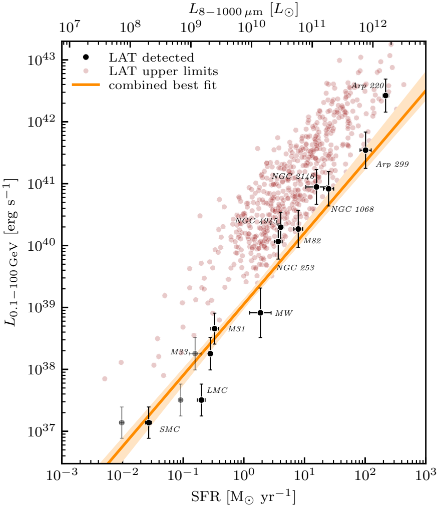


Werhahn, CP+ (in prep.)

- gamma-ray spectra in starbursts dominated by pion decay
- CR protons propagate in Kolmogorov turbulence: $\kappa \propto E^{0.3}$

Far infra-red – gamma-ray correlation

Universal conversion: star formation → cosmic rays → gamma rays



Ajello+ (2020)

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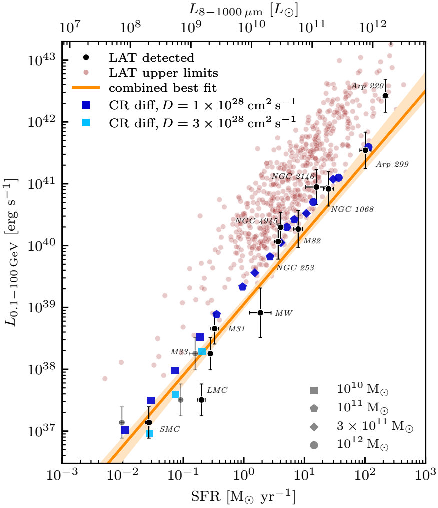

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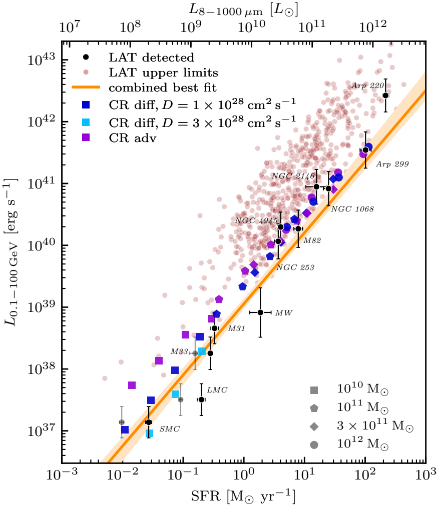

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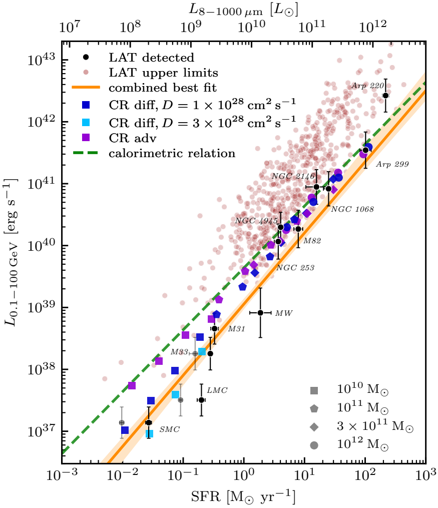

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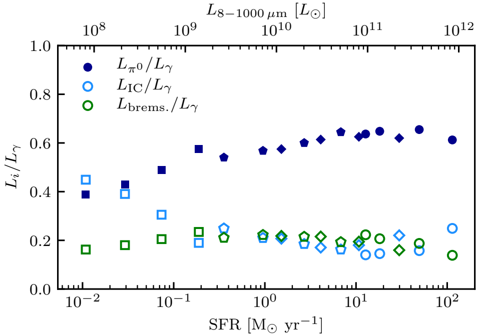
Far infra-red – gamma-ray correlation

Contributions of hadronic and leptonic emission to the gamma-ray luminosity



Werhahn, CP+ (in prep.)

- gamma-ray emission in starbursts dominated by pion decay
- leptonic component (primarily inverse Compton) dominates at low star formation rates

Conclusions

CR-driven plasma instabilities:

- discovery of new *intermediate-scale instability*, which grows faster than the gyro-resonant instability
- implications for CR transport and feedback in galaxies, electron injection into diffusive shock acceleration, and CR escape from acceleration sites

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- novel theory of CR transport mediated by Alfvén waves and coupled to magneto-hydrodynamics
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CR-induced signatures in galaxies

- Voyager's high electron-to-proton ratio at low energies explained by Coulomb losses of steady-state spectra
- leptonic gamma-ray contribution important at low star formation rates

CRAGSMAN: The Impact of Cosmic RAys on Galaxy and CluSter ForMAtioN

This project has received funding from the European Research Counsil (ERC) under the European Union’s Horizon 2020 research and innovation program (grant agreement No CRAGSMAN–646955).

Literature for the talk

Cosmic ray instabilities and transport:

- Shalaby, Thomas, Pfrommer, *A new cosmic ray-driven instability,* 2020, submitted, `arXiv:2010.11197`.
- Thomas & Pfrommer, *Cosmic-ray hydrodynamics: Alfvén-wave regulated transport of cosmic rays,* 2019, MNRAS, 485, 2977.
- Thomas, Pfrommer, Enßlin, *Probing cosmic ray transport with radio synchrotron harps in the Galactic center,* 2020, ApJL, 890, L18.

Cosmic rays in galaxies:

- Werhahn, Pfrommer, Girichidis, Puchwein, Pakmor, *Cosmic rays and non-thermal emission in simulated galaxies. I. Electron and proton spectra explain Voyager-1 data,* in prep.
- Werhahn, Pfrommer, Girichidis, Winner, *Cosmic rays and non-thermal emission in simulated galaxies. II. γ-ray maps, spectra and the far infrared-γ-ray relation,* in prep.

Additional slides

Analogies of CR and radiation hydrodynamics

CRs and radiation are relativistic fluids

regime	CR transport	radiation HD analogy
• tangled $\boldsymbol{B}$, strong scattering	CR diffusion	diffusive transport in clumsy medium
• resolved $\boldsymbol{B}$, strong scattering	CR streaming with $\boldsymbol{v}_\mathrm{a}$	Thomson scattering ($\tau \gg 1$) $\to$ advection with $\boldsymbol{v}$
• weak scattering	CR streaming and diffusion	flux-limited diffusion/ M1 closure ($\tau \gtrsim 1$)
• no scattering	CR propagation with c	vacuum propagation

Jiang & Oh (2018), Thomas & CP (2019)

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but: *CR hydrodynamics is charged RHD*

$\rightarrow$ ***take gyrotropic average and account for anisotropic transport***

CR vs. radiation hydrodynamics

- capitalize on analogies of CR and radiation hydrodynamics (Jiang & Oh 2018)
  derive two-moment equations from CR Vlasov equation (Thomas & CP 2019)

AIP

CR vs. radiation hydrodynamics

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- lab-frame equ's for CR energy and momentum density, $\varepsilon_{\rm cr}$ and $\boldsymbol{f}_{\rm cr}/c^2$

$$\frac{\partial \varepsilon_{\rm cr}}{\partial t} + \boldsymbol{\nabla}\cdot\boldsymbol{f}_{\rm cr} = -\boldsymbol{w}_\pm\cdot\frac{\boldsymbol{bb}}{3\kappa_\pm}\cdot\left[\boldsymbol{f}_{\rm cr} - \boldsymbol{w}_\pm(\varepsilon_{\rm cr}+P_{\rm cr})\right] - \boldsymbol{v}\cdot\boldsymbol{g}_{\rm Lorentz} + S_\varepsilon$$

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Alfvén wave velocity in lab frame: $\boldsymbol{w}_\pm = \boldsymbol{v} \pm \boldsymbol{v}_{\rm a}$,
CR scattering frequency $\bar{\nu}_\pm = c^2/(3\kappa_\pm)$

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- lab-frame equ's for radiation energy and momentum density, ε and $\boldsymbol{f}/c^2$ (Mihalas & Mihalas, 1984, Lowrie+ 1999):

$$\frac{\partial \varepsilon}{\partial t} + \boldsymbol{\nabla}\cdot\boldsymbol{f} = -\sigma_{\mathrm{s}}\boldsymbol{v}\cdot\left[\boldsymbol{f} - \boldsymbol{v}\cdot(\varepsilon\mathbf{1} + \mathbf{P})\right] + S_{\mathrm{a}}$$

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$$\frac{1}{c^2}\frac{\partial \boldsymbol{f}}{\partial t} + \boldsymbol{\nabla}\cdot\mathbf{P} = -\sigma_{\rm s}\left[\boldsymbol{f} - \boldsymbol{v}\cdot(\varepsilon\mathbf{1}+\mathbf{P})\right] + S_{\rm a}\boldsymbol{v}$$

- ***problem:*** *CR lab-frame equation requires resolving rapid gyrokinetics!*

CR vs. radiation hydrodynamics

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$$\frac{1}{c^2}\frac{\partial \boldsymbol{f}}{\partial t} + \boldsymbol{\nabla} \cdot \mathbf{P} = -\sigma_{\mathrm{s}} \left[\boldsymbol{f} - \boldsymbol{v} \cdot (\varepsilon \mathbf{1} + \mathbf{P})\right] + S_{\mathrm{a}} \boldsymbol{v}$$

- ***solution:*** *transform in comoving frame and project out gyrokinetics!*

Alfvén-wave regulated CR transport

- comoving equ's for CR energy and momentum density (along $\boldsymbol{B}$), $\varepsilon_{\rm cr}$ and $f_{\rm cr}/c^2$, and Alfvén-wave energy densities $\varepsilon_{\rm a,\pm}$ (Thomas & CP 2019)

$$\frac{\partial \varepsilon_{\rm cr}}{\partial t} + \boldsymbol{\nabla}\cdot[\boldsymbol{v}(\varepsilon_{\rm cr}+P_{\rm cr}) + \boldsymbol{b} f_{\rm cr}] = \boldsymbol{v}\cdot\boldsymbol{\nabla} P_{\rm cr} - \frac{v_{\rm a}}{3\kappa_+}[f_{\rm cr} - v_{\rm a}(\varepsilon_{\rm cr}+P_{\rm cr})] + \frac{v_{\rm a}}{3\kappa_-}[f_{\rm cr} + v_{\rm a}(\varepsilon_{\rm cr}+P_{\rm cr})],$$

$$\frac{\partial f_{\rm cr}/c^2}{\partial t} + \boldsymbol{\nabla}\cdot\left(\boldsymbol{v} f_{\rm cr}/c^2\right) + \boldsymbol{b}\cdot\boldsymbol{\nabla} P_{\rm cr} = -(\boldsymbol{b}\cdot\boldsymbol{\nabla}\boldsymbol{v})\cdot(\boldsymbol{b} f_{\rm cr}/c^2) - \frac{1}{3\kappa_+}[f_{\rm cr} - v_{\rm a}(\varepsilon_{\rm cr}+P_{\rm cr})] - \frac{1}{3\kappa_-}[f_{\rm cr} + v_{\rm a}(\varepsilon_{\rm cr}+P_{\rm cr})],$$

$$\frac{\partial \varepsilon_{\rm a,\pm}}{\partial t} + \boldsymbol{\nabla}\cdot[\boldsymbol{v}(\varepsilon_{\rm a,\pm}+P_{\rm a,\pm}) \pm v_{\rm a}\boldsymbol{b}\varepsilon_{\rm a,\pm}] = \boldsymbol{v}\cdot\boldsymbol{\nabla} P_{\rm a,\pm} \pm \frac{v_{\rm a}}{3\kappa_\pm}[f_{\rm cr} \mp v_{\rm a}(\varepsilon_{\rm cr}+P_{\rm cr})] - S_{\rm a,\pm}.$$